Don't Repeat Yourself: Termination of the Skolem Chase on Disjunctive Existential Rules

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Motivation

- Reasoning over *Knowledge Bases*

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- Expressivity of *Disjunctive Existential Rules*

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- Reasoning over *Knowledge Bases*
- Expressivity of *Disjunctive Existential Rules*
- Efficiency of the *Skolem Chase* (ASP Solvers)

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A *(disjunctive existental) rule* ρ is an expression of the form

$$\forall\vec{x}\forall\vec{y}.[B_\rho(\vec{x},\vec{y}) \rightarrow \bigvee_{i=1}^{n} \exists\vec{z}_i.H^i_\rho(\vec{x}_i,\vec{z}_i)]$$

where B_ρ and H^i_ρ are conjunctions of atoms without function symbols or constants; $\vec{x}, \vec{y}$, and $\vec{z}_i$ are pairwise disjoint lists of variables; and $\bigcup_{i=1}^{n} \vec{x}_i = \vec{x}$.

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Is σ entailed by $\langle R, I \rangle$?

How to solve this in general?

Definition

A BCQ $\sigma := \exists \vec{z}.\varphi(\vec{z})$ is *entailed* by a knowledge base $\mathcal{K}$ if, for each first order model M of $\mathcal{K}$, there exists a substitution θ such that $\varphi\theta \subseteq M$.

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Proposition

BCQ entailment is undecidable [Beeri and Vardi, 1981].

Do we really need every model?

Definition

A *universal model set* [Bourhis et al., 2016] of a knowledge base $\mathcal{K}$ is a set of models $\mathcal{U}$, such that for each model M for $\mathcal{K}$ there exists a homomorphism from some model in $\mathcal{U}$ to M.

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Proof.
"$\Rightarrow$":
If σ is entailed by $\mathcal{K}$, then it is entailed for every model of $\mathcal{K}$.

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3. By (1) and (2): $\tau \circ \theta$ is a substitution with $\varphi(\tau \circ \theta) \subseteq M$.

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4. By (3): σ is entailed by $\mathcal{K}$.

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A BCQ $\sigma := \exists \vec{z}.\varphi(\vec{z})$ is entailed by a knowledge base $\mathcal{K}$ iff it is entailed by each model in some universal model set of $\mathcal{K}$.

We study an algorithm that should compute a finite universal model set containing only finite models.

Chasing a Universal Model Set

General Chase Procedure

- Input: Knowledge Base $\mathcal{K}$
- Procedure: Apply rules in $\mathcal{K}$ exhaustively, until no new facts are obtained. Consider the head disjuncts individually by branching out on them.
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Those applications can be implemented using ASP-solvers.

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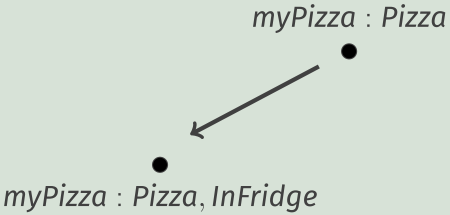

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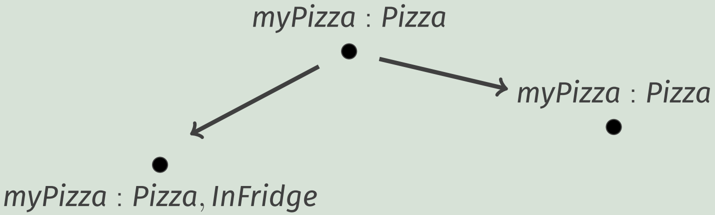

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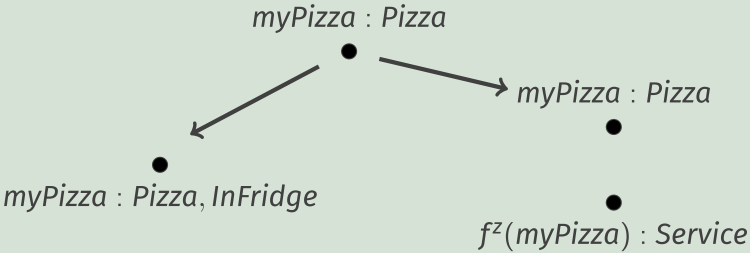

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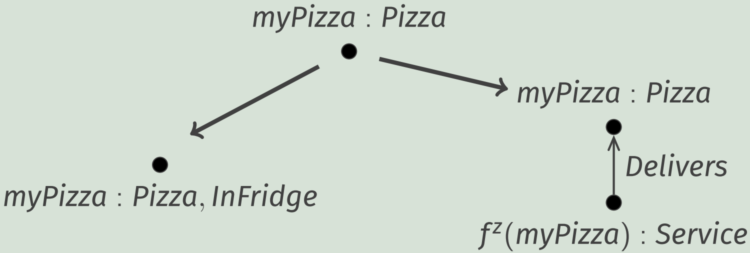

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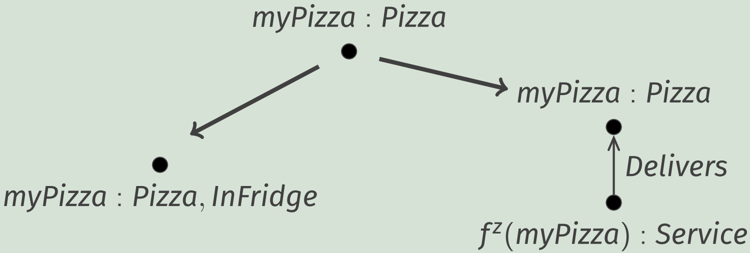

The BCQ $\exists z.(Service(z) \wedge Delivers(z, myPizza))$ is not entailed.

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Example

The following rule set is not terminating.

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Why not use RMFA?
RMFA is not sound for the disjunctive skolem chase. (We can still use some ideas.)

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General idea of MFA: Compute chase on critical instance $I_R^\star$ and check for cyclic terms.

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Theorem

A rule set R without disjunctions is terminating if and only if $\langle R, I_R^\star \rangle$ is terminating [Marnette, 2009].

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Consider the rule set R and its critical instance $I_R^\star$:

$$R = \{\, Pizza(x) \to Last(x) \vee (NextOrder(x, f^z(x)) \wedge Pizza(f^z(x)))\,\}$$
$$I_R^\star = \{\, Pizza(\star), Last(\star), NextOrder(\star, \star)\,\}$$

The knowledge base $\langle R, I_R^\star\rangle$ is terminating but R is not.

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Example

The following rule set is terminating but it does not terminate if we replace the disjunction by a conjunction:

$$Pizza(x) \rightarrow Last(x) \vee (NextOrder(x, f^z(x)) \wedge Pizza(f^z(x)))$$
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Theorem

In the context of a rule set: If a trigger λ is blocked, then λ is not applicable to any fact set occurring in the chase on any instance.

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For a rule set R, we define DMFA(R) to be the smallest fact set such that $I^{\star}_{R} \subseteq$ DMFA(R) and, for every trigger $\langle \rho, \theta \rangle$ with $\rho \in R$ that is active w.r.t. DMFA(R) and not blocked, we have $sk(H^{i}_{\rho})\theta \subseteq$ DMFA(R) for all $1 \leq i \leq branch(\rho)$.

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Proof.
We show that R is not DMFA if R is not terminating.

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Compute a generalized Chase result

Example

R:

$$Pizza(x) \rightarrow Last(x) \vee (NextOrder(x, f^z(x)) \wedge Pizza(f^z(x)))$$
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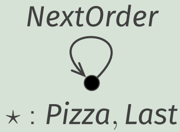
DMFA(R):



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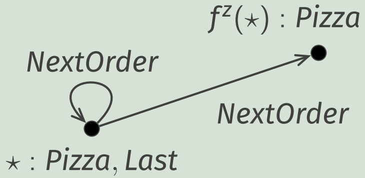

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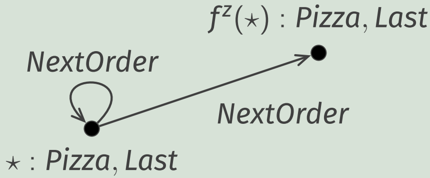

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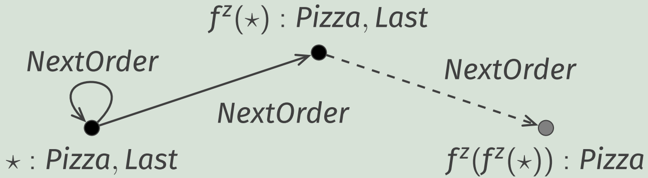

Hierarchy of Acyclicity Notions

Disjunctive Existential Rule Sets

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Terminating w.r.t. restricted chase

Hierarchy of Acyclicity Notions

Disjunctive Existential Rule Sets

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Terminating w.r.t.
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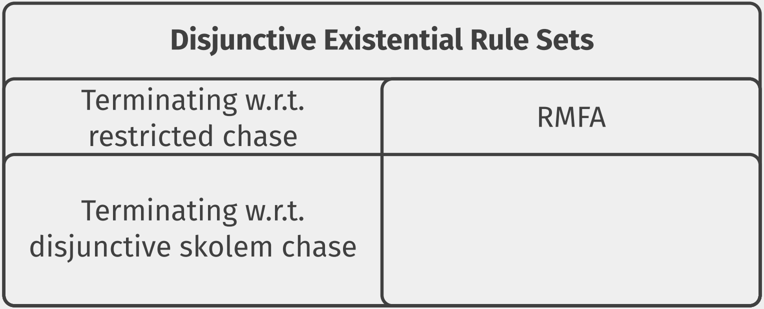

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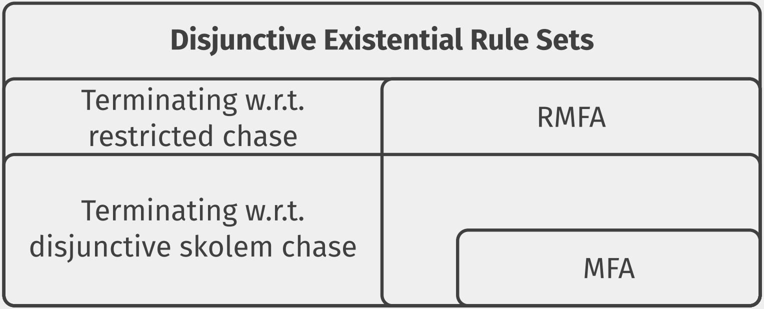

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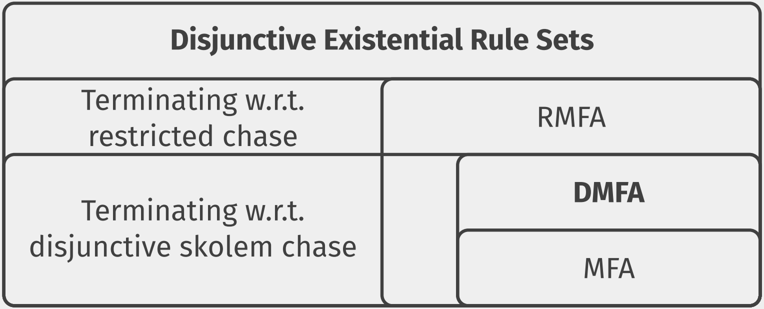

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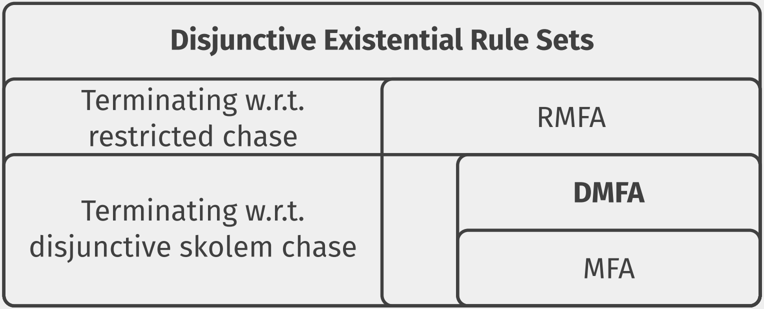

Example

The following singleton rule set is contained in the blank space:

$$P(y, x) \wedge Q(y) \rightarrow \exists z.(P(x, z) \wedge P(z, x))$$

Complexity of the DMFA Check

Theorem

Checking if a rule set R is DMFA is 2ExpTime-complete.

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(Hardness): Reduction from MFA (MFA and DMFA coincide for rule sets without disjunctions)
[Cuenca Grau et al., 2013, Calì et al., 2010].

Complexity of BCQ entailment for DMFA Ruleset

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Let R be a rule set that is DMFA and let $\mathcal{K}$ be a knowledge base featuring R. BCQ entailment for $\mathcal{K}$ is coN2ExpTime*-complete.*

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(Hardness): Reduction from the word problem of N2ExpTime-bounded turing machines similar to the proof for RMFA [Carral et al., 2017, Calì et al., 2010].

Results

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RESULTS

- DMFA is a novel acyclicity notions tailored towards the disjunctive skolem chase.
- DMFA is in between MFA and RMFA (as expected).
- Checking DMFA is 2EXPTIME-complete and checking BCQ entailment for a rule set that is DMFA is coN2EXPTIME-complete.

Future Work

In theory:

1. Develop a cyclicity notion for the disjunctive skolem chase.

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In practice:

1. Evaluate DMFA on real world knowledge bases.

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In theory:

1. Develop a cyclicity notion for the disjunctive skolem chase.
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In practice:

1. Evaluate DMFA on real world knowledge bases.
2. Use an ASP based implementation of the disjunctive skolem chase for reasoning with description logics.

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Technical References

The sources of the presentation can be found on Gitlab:
https://gitlab.com/m0nstR/defence-grosser-beleg

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https://github.com/elauksap/focus-beamertheme
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