

Complexity Theory
Exercise 5: Space Complexity
30th November 2022

Exercise 5.1. Let A_{LBA} be the word problem of deterministic linear bounded automata. Show that A_{LBA} is PSPACE-complete.

$$A_{LBA} = \{ \langle \mathcal{M}, w \rangle \mid \mathcal{M} \text{ is a (deterministic) LBA and } w \in L(\mathcal{M}) \}$$

Exercise 5.2. Consider the Japanese game *go-moku* that is played by two players X and O on a 19x19 board. Players alternately place markers on the board, and the first one to have five of its markers in a row, column, or diagonal wins.

Consider the generalised version of go-moku on an $n \times n$ board. Say that a *position* of go-moku is a placement of markers on such a board as it could occur during the game, together with a marker which player moves next. We define

$$GM = \{ \langle B \rangle \mid B \text{ is a position of go-moku where X has a winning strategy} \}.$$

Show that GM is in PSPACE.

Exercise 5.3. Show that the universality problem of nondeterministic finite automata

$$ALL_{NFA} = \{ \langle \mathcal{A} \rangle \mid \mathcal{A} \text{ an NFA accepting every valid input} \}$$

is in PSPACE.

Hint:

δογλινομιασπλ ρομπδεσ: εινασπλ: αββπλ δαυιτφ,δ πρεοιειμ.

πιατ π ε Γ(Λ). Ππει, ησε ππς ιατ το ειλε α non-deterministic algorithm whose space consumption is πολε πιατ π Γ(Λ) ε Σ* απδ Λ ηαδ η στατεσ, ππει ππειε εχιστε α word π ε Σ* οτ ιεππειρ ατ ποστ δπ, απδ

Exercise 5.4. Show that the composition of logspace reductions again yields a logspace reduction.

Exercise 5.5. Show that the word problem A_{NFA} of non-deterministic finite automata is NL-complete.

Exercise 5.6. Show that

$$BIPARTITE = \{ \langle G \rangle \mid G \text{ a finite bipartite graph} \}$$

is in NL. For this show that $\overline{BIPARTITE} \in NL$ and use $NL = \overline{NL}$. **Hint:**

δπωμ πιατ α ειαβπ Γ ε βιβαριτιε π απδ ομπλ π π ποεε ποτ κομπαρ α ελεγε οτ οδδ ιεππειρ.