Dresden University of Technology

Computational Logic ∴ Group

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Algorithmic Game Theory

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Exercises 3

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Problem 1.
Consider a $2 \times n$ zero-sum game where the best response to every pure strategy is unique. Suppose that the top row and leftmost column define a pure-strategy equilibrium of the game.

1. Show that this equilibrium can be found by iterated elimination of strictly dominated strategies.

   Hint: Consider first the case $n = 2$.

2. Does the claim in (a) still hold if some pure strategy has more than one best response?

Problem 2.
Recall the functional problem End-of-the-Line from the lecture:

We are given a directed graph $G = (V, E)$ over $V = \{0, 1\}^n$, where every node of the graph has in-degree and out-degree at most one. The edge relation E is represented by two Boolean circuits P (for predecessor) and S (for successor) such that

$$(u, v) \in E \text{ iff } u = P(v) \text{ and } S(u) = v.$$

The designated node $0^n \in V$ has in-degree zero and out-degree one (is a *source*); by convention, $P(0^n) = 0^n$ and $S(0^n) \neq 0^n$.

We are asked to find another unbalanced node (which is guaranteed to exist), more precisely a *sink* (a node with in-degree one and out-degree zero) or another source.

1. Prove the following statements:

   (a) $v \in V$ has out-degree zero iff $P(S(v)) \neq v$.

   (b) $v \in V$ has in-degree zero iff $S(P(v)) \neq v$.

   (c) $v \in V$ with $v \neq 0^n$ is a solution iff either $P(S(v)) \neq v$ or $S(P(v)) \neq v$.

2. Use the statement of (1c) to solve the following instance of END-OF-THE-LINE for $n = 3$:

$$
\begin{aligned}
P: \quad & P_1 \equiv y_1 \wedge y_2 \\
& P_2 \equiv y_1 \wedge \neg y_2 \\
& P_3 \equiv (y_1 \vee y_2) \wedge \neg y_3 \\
S: \quad & S_1 \equiv x_1 \vee x_2 \\
& S_2 \equiv x_1 \vee \neg x_2 \\
& S_3 \equiv \neg x_3
\end{aligned}
$$

Problem 3.
Consider the following game:

(Player1, Player2)	C	K
C	(6, 6)	(2, 7)
K	(7, 2)	(0, 0)

- Find all mixed Nash equilibria and the corresponding payoffs.
- Argue without computations why the following correlated strategy is a correlated equilibrium:

(Player1, Player2)	C	K
C	0	$\frac{1}{2}$
K	$\frac{1}{2}$	0

- Show formally that the following correlated strategy is a correlated equilibrium:

(Player1, Player2)	C	K
C	$\frac{1}{3}$	$\frac{1}{3}$
K	$\frac{1}{3}$	0