Exercise 10.1. Show that \textbf{MAJSAT} is in \textit{PP}.

\textbf{MAJSAT} = \{ \phi \mid \phi \text{ is some propositional logic formula that is satisfied by more than half of its assignments} \}

Exercise 10.2. Show \textit{BPP} = \textit{coBPP}.

* Exercise 10.3. Show \textit{BPP}^{\textit{BPP}} = \textit{BPP}.

Exercise 10.4. Find the error in the following proof that shows \textit{PP} = \textit{BPP}: Let \textit{L} \in \textit{PP}.

Then there exists a poly-time bounded PTM accepting \textit{L} with error probability smaller than \( \frac{1}{2} \). Using error amplification, we can make this error arbitrarily small, and in particular smaller than \( \frac{1}{3} \). Hence, \textit{L} \in \textit{BPP}.

Exercise 10.5. Let \( \mathcal{M} \) be a polynomial-time probabilistic Turing machine. We say that \( \mathcal{M} \) has error probability smaller than \( \frac{1}{3} \) if and only if

\[ \Pr[\mathcal{M} \text{ accepts } w] < \frac{1}{3} \quad \text{or} \quad \Pr[\mathcal{M} \text{ accepts } w] \geq \frac{2}{3} \]

for all inputs \( w \). Show that deciding whether a polynomial-time probabilistic TM has error probability smaller than \( \frac{1}{3} \) is undecidable.