TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic ∴ Group

Dr. Hannes Strass

M.A. Jonas Karge

Knowledge Representation and Reasoning

Winter 2024 Term

Exercises 10

27–31/01/2025

Problem 1.
From the probability axioms, we can deduce several further rules for computing probabilities. Consider the following two:

R_1 $P(\neg A) = 1 - P(A)$.

R_2 $P(A) = 0$, if A is a contradiction.

Derive these from the Kolmogorov axioms.

Problem 2.
Suppose that $W = \{w_1, w_2, w_3\}$. Define m as follows:

$$\begin{aligned} m(w_1) &= 1/4 \\ m(w_1, w_2) &= 1/4 \\ m(w_2, w_3) &= 1/2 \\ m(U) &= 0 \textit{ if } U \textit{ is not one of } \{w_1\}, \{w_1, w_2\}, \textit{or } \{w_2, w_3\}. \end{aligned}$$

Provide all values for:

$Bel_m(w_1)$, $Bel_m(w_2)$, $Bel_m(w_3)$, $Bel_m(w_1, w_2)$,
$Bel_m(w_1, w_3)$, $Bel_m(w_2, w_3)$, $Bel_m(w_1, w_2, w_3)$

as well as

$Plaus_m(w_1)$,$Plaus_m(w_2)$,$Plaus_m(w_3)$,$Plaus_m(w_1, w_2)$,
$Plaus_m(w_1, w_3)$, $Plaus_m(w_2, w_3)Plaus_m(w_1, w_2, w_3)$.

Problem 3.
From the lecture, recall the following result for combining the masses of two tests:

$$\begin{aligned} (m_1 \oplus m_2)(\{hep\}) &= 0.8 \\ (m_1 \oplus m_2)(\{hep, cirr\}) &= 0.12 \\ (m_1 \oplus m_2)(W) &= 0.08 \end{aligned}$$

Illustrate each step of the computation to derive the above result.

Problem 4.

In a multi-sensor system, we need to combine evidences from multiple sensors. Assume that we have two sensors and three observable events with $W = A, B, C$. From the sensors, we get the following pieces of information that indicate the likelihood of observing each event:

$$\begin{aligned} Sensor_1 &: m_1(A) = 0.95, \quad m_1(B) = 0.05, \quad m_1(C) = 0 \\ Sensor_2 &: m_2(A) = 0, \quad m_2(B) = 0.1, \quad m_2(C) = 0.9 \end{aligned} \tag{1}$$

Do the following:
(i) Compute $m_1 \oplus m_2(A)$, $m_1 \oplus m_2(B)$, $m_1 \oplus m_2(C)$,
(ii) Argue whether or not your result poses a problem for the combination rule.