Flexible Dispute Derivations with Forward and Backward Arguments for Assumption-Based Argumentation

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Abstract. Assumption-based argumentation (ABA) is one of the main general frameworks for structured argumentation. Dispute derivations for ABA allow for evaluating claims in a dialectical manner: i.e. on the basis of an exchange of arguments and counter-arguments for a claim between a proponent and an opponent of the claim. Current versions of dispute derivations are geared towards determining (credulous) acceptance of claims w.r.t. the admissibility-based semantics that ABA inherits from abstract argumentation. Relatedly, they make use of backwards or top down reasoning for constructing arguments. In this work we define flexible dispute derivations with forward as well as backward reasoning allowing us, in particular, to also have dispute derivations for finding admissible, complete, and stable assumption sets rather than only determine acceptability of claims. We give an argumentation-based definition of such dispute derivations and a more implementation friendly alternative representation in which disputes involve exchange of claims and rules rather than arguments. These can be seen as elaborations on, in particular, existing graph-based dispute derivations on two fronts: first, in also allowing for forward reasoning; second, in that all arguments put forward in the dispute are represented by a graph and not only the proponents.

Keywords: Argumentation · Assumption-based argumentation · Dispute derivations

1 Introduction

Assumption-based argumentation [3,4,12,16,34] (ABA) is one of the main formalisms for structured argumentation [2], also very much related to ASPIC+ [18,27,28]. ABA frameworks are built from a deductive system consisting of a language and set of rules. ABA arguments are then proofs in such a deductive system. Certain elements of the language are singled out as assumptions and a total mapping is provided associating each assumption to its so called contrary. Assumptions, and thus arguments using such

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assumptions, can be attacked by arguments for their contraries. For flat ABA, which we will be focusing on this work (and has, as far as we are aware, also been the focus of all other work on reasoning methods for ABA), semantics can be equivalently defined at the level of assumption sets as well as arguments. In either case one ultimately obtains sets of assumptions which can be deemed reasonable to the same degree that the arguments that can be built from them are reasonable according to the classical admissibility-based semantics of abstract argumentation [14].

One of the main reasoning methods which has been devised for (flat) ABA is that of dispute derivations [10, 11, 15, 17, 20, 33]. These build on one of if not *the* main native (vs reduction-based [9]) method for reasoning in abstract argumentation; namely, argumentation games (see e.g. [5, 7, 8, 13, 21–23, 26, 29, 31, 35]). Dispute derivations are conceived of as a game between a proponent and opponent, where starting from some goal claim the proponent searches for an argument proving the goal. This search reveals assumptions on which the proof depends, which can be attacked by the opponent by arguments for their contraries. Such arguments from the opponent can in turn be attacked by the proponent by searching for further arguments and so on. Dispute derivations can be seen as hybrid syntactic-semantic methods for searching for only those arguments needed to answer a query and are thus related to the issue of selecting such relevant arguments in structured argumentation more general [1, 6, 19, 30, 32, 36].

Although reduction-based methods also for reasoning in ABA (as for abstract argumentation) have to date proved to be much more efficient than dispute derivations [24, 25], dispute derivations remain interesting for a number of reasons. The main of these is that reducing argumentation to other formalisms often undermines the purpose of using argumentation in the first place; which is presumably to allow for a dialectic evaluation of claims in terms of arguments and counter-arguments. Dispute derivations deliver such "dialectic explications". This makes them especially suitable when information is limited and unreliable; also, for approximate, dynamic, and interactive reasoning.

As detailed, there have been several versions of dispute derivations to date. But all have in common that they are conceived primarily as decision procedures for determining credulous acceptance of a claim w.r.t. the admissible-based semantics (in the first versions of dispute derivations focus was on grounded, admissible, and ideal semantics; in later [10, 11] versions the ideal semantics is dropped); i.e. whether there is an admissible (and hence complete and preferred) assumption set from which the claim can be proven. Related to this, they make use of backwards reasoning: both the proponent and opponent make use of top down or backwards reasoning to search for their arguments.

Top down reasoning is often enough. In particular, for the focused task of determining credulous acceptance of claims; yet, another fundamental paradigm in reasoning is forward or bottom up from established claims to further claims. In the context of dispute derivations such reasoning becomes relevant for more global tasks as e.g. determining acceptance of several claims or, relatedly, determining complete assumption sets rather than only credulous acceptance. Moreover, while for determining credulous acceptance computing e.g. complete assumption sets is not necessary, more revealing explications can often be obtained. In particular, computing complete assumption sets allows, as the name of the semantics suggests, a more "complete" picture of sets of assumptions which are congruous with a claim of interest. Furthermore, forward reasoning allows a straightforward generalisation of dispute derivations also for the stable semantics, this semantics not having been considered in previous work on disputes for ABA.

So in this work we add forward reasoning to dispute derivations. This allows us, in particular, to define dispute derivations for finding admissible, complete, and stable assumption sets as well as for determining acceptance of claims w.r.t. these semantics. We do so in several steps. We start in Sect. 3 by considering dispute derivations from an implementation independent and purely argumentation-based perspective: i.e. in terms of the arguments that are exchanged by the proponent and opponent. In particular, especially for forward reasoning how much of a dispute is "remembered" and made use of in further dispute steps is crucial. We provide a definition of flexible dispute derivations (with forward and backward arguments) based on structured dispute derivations from [33] and then graph-based dispute derivations from [10] in Sect. 3.2. The only thing that distinguishes these variants of dispute derivations is precisely how much of previous dispute steps is made use of in further steps. We note that there is room for improvement in this regard, particularly for the purpose of forward reasoning, and thus propose a novel variant of flexible dispute derivations in Sect. 3.3.

We then in Sect. 4 change gear to a more implementation focused perspective more in line with existing work on ABA disputes and give an alternative representation of our novel variant of dispute derivations from Sect. 3.3. In this version disputes involve the exchange of claims and rules rather than arguments; in particular, the opponents and proponents arguments are represented in a shared graph consisting in the dependency relations between rules and statements put forward during the dispute. Thus we further generalise [10] in which only the proponents arguments are represented as a graph, while the opponents are not. In Sect. 5 we then provide details on an interactive interface we implemented for our dispute derivations that is freely available. Section 2 contains the background needed for our work and Sect. 6 the conclusions.

2 Formal Background

Definition 1. *An ABA framework is a tuple* $\mathscr{F} = (\mathscr{L}, \mathscr{R}, \mathscr{A}, \overline{})$ *where*

- $(\mathscr{L}, \mathscr{R})$ *is a deductive system, with a language* $\mathscr{L}$ *and a set of inference rules* $\mathscr{R}$,
- $\mathscr{A} \subseteq \mathscr{L}$ *is a (non-empty) set, whose elements are referred to as* assumptions,
- $\overline{}$ *is a total mapping from* $\mathscr{A}$ *into* $\mathscr{L}$, *where* $\overline{a}$ *is the* contrary *of* a.

We also define for a set of statements $S \subseteq \mathscr{L}$, $\overline{S} = \{\overline{u} \in \mathscr{L} \mid u \in (S \cap \mathscr{A})\}$. As in previous work on dispute derivations, here we also restrict our attention to flat ABA: i.e. frameworks where there is no rule $h \leftarrow B \in \mathscr{R}$ s.t. $h \in \mathscr{A}$. In all of this work we will consider the ABA framework to be fixed and thus not define notions relative to an ABA framework. Elements of $\mathscr{L}$ we will refer to as statements, sometimes as claims.

Arguments have been defined in several different ways for ABA. For a comprehensive definition we define arguments in ASPIC+ [27] style:

Definition 2. *For an ABA* $\mathscr{F} = (\mathscr{L}, \mathscr{R}, \mathscr{A}, \overline{})$, *an* argument *is defined as follows.*

(i) *$a = s$ is an argument if $s \in \mathscr{L}$. Then $Prem(a) = \{s\}$, $Asm(a) = \{s\} \cap \mathscr{A}$, $Conc(a) = s$, $TopSub(a) = \{s\}$, $Sub(a) = \{s\}$.*

(ii) *$a = s \leftarrow a_1, \ldots, a_n$ is an argument if $a_1, \ldots, a_n$ are arguments such that there exists $s \leftarrow Conc(a_1), \ldots, Conc(a_n) \in \mathscr{R}$. Then $Prem(a) = Prem(a_1) \cup \ldots \cup Prem(a_n)$, $Asm(a) = Asm(a_1) \cup \ldots \cup Asm(a_n) \cup (\{s\} \cap \mathscr{A})$, $Conc(a) = s$, $TopSub(a) = \{s\} \cup \{s \leftarrow a'_1, \ldots, a'_n \mid a'_1 \in TopSub(a_1), \ldots, a'_n \in TopSub(a_n)\}$, $Sub(a) = Sub(a_1) \cup \ldots \cup Sub(a_n) \cup TopSub(a)$.*

For instance let $a = p \leftarrow b, [q \leftarrow r, s]$ be an argument built from rules $p \leftarrow b, q$ and $q \leftarrow r, s$ with only $b \in \mathscr{A}$. Then we have that the premisses of the argument are $Prem(a) = \{b, r, s\}$, the assumptions $Asm(a) = \{b\}$, the conclusion $Conc(a) = p$, the top-sub-arguments $TopSub(a) = \{p;\ p \leftarrow b, q;\ p \leftarrow b, [q \leftarrow r, s]\}$, and the sub-arguments $Sub(a) = \{b;\ r;\ s;\ q \leftarrow r, s\} \cup TopSub(a)$. We extend the above notions to sets of arguments in the obvious manner; e.g. for a set of arguments A, $Prem(A) = \bigcup_{a \in A} Prem(a)$.

We denote all arguments in $\mathscr{F}$ as $Args$. An argument a is complete if $Prem(a) \subseteq \mathscr{A}$. This is what is usually called an argument for ABA; what we have defined are "potential arguments". The reason for the latter being that these are what dispute derivations work on. Related to this, note that our notion of sub-arguments, differently to what is the case in ASPIC+, includes all sub-arguments; not only those with the same premisses as the main argument. Given that statements and rules can be thought of as (potential) simple arguments we notationally and otherwise will usually not distinguish between such simple arguments and the statements and rules underlying them.

Attacks in (flat) ABA can be defined between assumption sets, between arguments, as well as in the form of hybrid attacks between assumptions and arguments. This leads to equivalent assumption, argument, and hybrid views respectively of the semantics. Dispute derivations are based on a hybrid view and so we here review this perspective.

Definition 3. *The notions of attack we need are:*

- *An argument a attacks a set of assumptions U' if $Conc(a) = \overline{u'}$ for a $u' \in U'$.*
- *A set of assumptions U attacks a set of assumptions U' if there is a (complete) argument a with $Prem(a) \subseteq U$ that attacks U'. In particular, if $U' = \{u'\}$ (i.e. U' is a singleton set with only the assumption u') we say simply that U attacks u'.*
- *A set of assumptions U attacks an argument a' if there is a (complete) argument a with $Prem(a) \subseteq U$ that attacks $Asm(a')$.*

Definition 4. *The definitions of the semantics we mainly consider in this work are:*

- *A set of assumptions is* admissible *if it does not attack itself and it attacks all complete arguments that attack it.*
- *A set of assumptions is* complete *if it is admissible and contains all assumptions it defends, where $U \subseteq \mathscr{A}$ defends $u \in \mathscr{A}$ if U attacks all complete arguments that attack u.*
- *A set of assumptions is* stable *if it does not attack itself and attacks all assumptions it does not contain.*

A set of statements S is (credulously) acceptable w.r.t. a semantics σ if there is a σ assumption set U w.r.t. which $S \subseteq Conc(A)$ for $A \subseteq Args$ with $Asm(A) \subseteq U$.

3 Argument-Based Flexible Dispute Derivations

In this section we develop rather abstract (not implementation focused) definitions of flexible dispute derivations, first of all, following structured dispute derivations [33] and then graph-based dispute derivations [10] (Sect. 3.2). We call these StFlexDDs and GrFlexDDs for short. We focus on the common aspects of these, at first sight, rather different looking versions of dispute derivations by considering how the disputes evolve in terms of the arguments put forward by the proponent and opponent.

We identify certain shortcomings (inherited from their non-flexible counterparts) in the manner in which StFlexDDs and GrFlexDDs make use of the arguments constructed in previous steps in the disputes. These shortcomings are particularly relevant for incorporating forward reasoning into dispute derivations, since forward reasoning builds on established claims. We thus then propose a different form of dispute derivations which we call simply flexible dispute derivations or FlexDDs for short (Sect. 3.3). Although from an argument-based perspective FlexDDs seem quite complex, we will see in Sect. 4 that in fact they lead to an equally natural yet implementation friendly alternative representation where claims and rules are put forward rather than arguments.

3.1 Argument and Dispute State Expansions

Basic moves both from the proponent and opponent in flexible dispute derivations involve expansions of arguments which we define as follows:

Definition 5. *An* expansion *of* $A = \{a_1, \ldots, a_n\} \subseteq Args$ *w.r.t. an argument* $a' \in Args$ *with* $Conc(a_1) \cup \ldots \cup Conc(a_n) \subseteq Prem(a')$ *is obtained from* a' *by replacing at least one* $s_i \in Prem(a')$ *for which* $s_i = Conc(a_i)$ *with* a_i *for each* $1 \leq i \leq n$*. We denote it* $a' \lessdot A$*. When* $n = 1$*, we will often denote the expansion as* $a' \lessdot a_1$*.*

Thus a *forward expansion* of a set of arguments A w.r.t. $\mathscr{R}$ (now taken as a set of 1-step arguments) is of the form $r \lessdot A$ with $r \in \mathscr{R}$. A *backward expansion* of an argument a w.r.t. $\mathscr{R}$ amounts to an expansion of the form $a \lessdot r$ with $r \in \mathscr{R}$.

Disputes consist of sequences of dispute states which we define simply as tuples $(\mathscr{B}, \mathscr{P})$ where $\mathscr{B} \subseteq Args$ are the arguments considered by the opponent and $\mathscr{P} \subseteq Args$ those considered by the proponent. The different types of moves which the proponent and opponent can make in a dispute amount to "expanding" either $\mathscr{B}$ or $\mathscr{P}$. The expansion is by an argument a with i) $a = u \in \mathscr{A}$, ii) $a = h \leftarrow B$, iii) $a = h \leftarrow B \lessdot A'$ or iv) $a = a' \lessdot h \leftarrow B$ for $A' \subseteq \mathscr{B}$ and $a' \in \mathscr{B}$, or $A' \subseteq \mathscr{P}$, $a' \in \mathscr{P}$ respectively, $h \leftarrow B \in \mathscr{R}$.

There are several viable options for defining such expansions. These correspond to differences in how much of the arguments put forward during a dispute is "remembered" and considered in future expansions by the proponent and opponent. The different variants of flexible dispute derivations we consider in this work, i.e. StFlexDDs, GrFlexDDs, and FlexDDs will differ precisely on the underlying notion of expansion.

Table 1. Auxiliary notation for argument-based flexible dispute derivations. All defined w.r.t. a dispute state $(\mathscr{B}, \mathscr{P})$.

Notation	Description
$\mathscr{D} = Asm(\mathscr{P})$	Defenses
$\mathscr{C} = \{u \in \mathscr{A} \mid \overline{u} \in Conc(\mathscr{P})\}$	Culprits
$\mathscr{R}^- = \{h \leftarrow B \in \mathscr{R} \mid B \cap \mathscr{C} \neq \emptyset\}$	Blocked rules (culprits in bodies)
$\mathscr{R}^\sim = \{h \leftarrow B \in \mathscr{R} \mid (\{h\} \cup B) \cap (\overline{B} \cup \mathscr{C} \cup \overline{\mathscr{D}}) \neq \emptyset\}$	Rules blocked for the proponent (either inconsistent; otherwise culprits or contraries of defenses in head or body)
$\mathscr{P}^* = \{a \in \mathscr{P} \mid Prem(a) \subseteq \mathscr{A}\}$	Proponents complete arguments
$\mathscr{B}^{*/-} = \{a \in \mathscr{B} \mid Prem(a) \subseteq (\mathscr{A} \setminus \mathscr{C})\}$	Opponents complete unblocked arguments
$\mathscr{P}^+ = \{a \in \mathscr{P} \setminus \mathscr{P}^* \mid \neg\exists a' \neq a \in \mathscr{P} \text{ s.t. } Conc(a') = Conc(a) \text{ and } a' \in \mathscr{P}^* \text{ or } a \in Sub(a')\}$	Maximal incomplete proponent arguments
$\mathscr{P}^\#_{\gamma \cup \overline{\mathscr{C}}} = \{a \in \mathscr{P}^+ \mid Conc(a) \in \gamma \cup \overline{\mathscr{C}}\}$	Maximal incomplete proponent arguments for goals and contraries of culprits
$\mathscr{B}^{!/-}_S = \{a \in \mathscr{B} \mid Asm(a) \cap \mathscr{C} = \emptyset, Conc(a) \in S\}$	Unblocked arguments with conclusions in $S \subseteq \mathscr{L}$
$\mathscr{A}^! = \{u \in \mathscr{A} \mid u \in Asm(\mathscr{B}^{!/-}_{\overline{\mathscr{D}}})\}$	Candidates for culprits
$\mathscr{I} = \{u \in \mathscr{A} \setminus \mathscr{C} \mid \overline{u} \notin Conc(\mathscr{B}^{*/-})\}$	Assumptions defended at the dispute state

3.2 Argument-Based Flexible Dispute Derivations Following Structured and Graph-Based Dispute Derivations

Flexible Structured Dispute Derivations. Dispute derivations consist of a sequence of dispute states which are tuples of the form $(\mathscr{B}, \mathscr{P})$ where $\mathscr{B} \subseteq Args$ are the opponents and $\mathscr{P} \subseteq Args$ the proponents arguments. Dispute derivations are also defined for a set of goals $\gamma \subseteq \mathscr{L}$ which we assume to be consistent; i.e. $\gamma \cap \overline{\gamma} = \emptyset$. Note that we consider a set of goals here rather than a single goal as in previous versions of dispute derivations.

In Table 1 we give definitions of several auxiliary notions needed to define the possible moves in dispute derivations. These are all defined w.r.t. a dispute state $(\mathscr{B}, \mathscr{P})$.

Dispute derivations consist of a sequence of dispute advancements either by the proponent or the opponent and a termination condition indicating when the dispute has concluded. Each of the advancements consist of a move by the proponent or opponent, there being several conceivable "backward" and "forward" moves that accord with ABA semantics. We give thus a very general definition of dispute advancements including all such conceivable moves in what follows. The moves can be restricted in several ways to obtain, together with tailored termination conditions, restricted dispute variants which, for instance, are sound w.r.t. the admissible, complete, or stable semantics.

For StFlexDDs[1] a proponent dispute state advancement from a dispute state $(\mathscr{B}, \mathscr{P})$ is a dispute state $(\mathscr{B}, \mathscr{P}')$ with $\mathscr{P}' = \mathscr{P} \cup \{a\} \neq \mathscr{P}$, $X_1 \subseteq \overline{\mathscr{A}}$, $X_2 \subseteq \mathscr{A}$ where either

[1] Note that in [10,33] the rules blocked for the proponent and opponent are identical (i.e. $\mathscr{R}^-$), while we use the stronger notion of blocked rules for the proponent $\mathscr{R}^\sim$.

P-B-$\langle\overline{\mathscr{A}^!}\cup X_1\rangle$: i) $a = a' \lessdot h \leftarrow B$ for $h \leftarrow B \in \mathscr{R}\setminus\mathscr{R}^{\sim}$, $a' \in \mathscr{P}^{\#}_{\gamma\cup\overline{\mathscr{C}}}$; or
ii) $a = h \leftarrow B$ for $h \leftarrow B \in \mathscr{R}\setminus\mathscr{R}^{\sim}$ with $h \in (\overline{\mathscr{A}^!}\cup X_1)\setminus\overline{\mathscr{D}}$;
P-F–$\langle(\overline{\mathscr{A}^!}\cap\mathscr{A})\cup X_2\rangle$: i) $a = h \leftarrow B \lessdot A$ for $A \subseteq \mathscr{P}^*$, $h \leftarrow B \in \mathscr{R}\setminus\mathscr{R}^{\sim}$; or
ii) $a = u$ for $u \in ((\overline{\mathscr{A}^!}\cap\mathscr{A})\cup X_2)\setminus(\{\overline{u}\}\cup\mathscr{C}\cup\overline{\mathscr{D}})$.

An opponent dispute state advancement from a dispute state $(\mathscr{B},\mathscr{P})$ is a dispute state $(\mathscr{B}',\mathscr{P})$ with $\mathscr{B}' = \mathscr{B}\cup\{a\} \neq \mathscr{B}$, $Y_1 \subseteq \overline{\mathscr{A}}$, and $Y_2 \subseteq \mathscr{A}$ where either

O-B-$\langle\overline{\mathscr{D}}\cup Y_1\rangle$: i) $a = a' \lessdot h \leftarrow B$ for $a' \in \mathscr{B}^{!/-}_{\overline{\mathscr{D}}\cup Y_1}$, $h \leftarrow B \in \mathscr{R}\setminus\mathscr{R}^{-}$; or
ii) $a = h \leftarrow B$ for a $h \leftarrow B \in \mathscr{R}\setminus\mathscr{R}^{-}$ with $h \in \overline{\mathscr{D}}\cup Y_1$;
O-F-$\langle(\overline{\mathscr{D}}\cap\mathscr{A})\cup Y_2\rangle$: i) $a = h \leftarrow B \lessdot A$ for $A \subseteq \mathscr{B}^{*/-}$, $h \leftarrow B \in \mathscr{R}\setminus\mathscr{R}^{-}$; or
ii) $a = u$ for $u \in (\overline{\mathscr{D}}\cap\mathscr{A})\cup Y_2\setminus\mathscr{C}$.

Each of the types of moves in disputes, e.g. P-B-$\langle\overline{\mathscr{A}^!}\cup X_1\rangle$ which represents a backward move from the proponent, depend on a parameter, here $X_1 \subseteq \overline{\mathscr{A}}$. When $X_1 = \overline{\mathscr{A}}$, the move P-B is "least constrained". P-B is "most constrained" when $X_1 = \{\}$. The latter we denote as P-B-$\langle\overline{\mathscr{A}^!}\rangle$. The least constrained moves give us the most general possible dispute advancements, which we denote "free style" (DF) dispute advancements. The most constrained moves gives us dispute advancements which are sound and complete (when $\mathscr{L}$ is finite and $\mathscr{R}$ is acyclic) for credulous acceptance w.r.t. the admissible semantics. These, which we denote DAB, follow previous versions of dispute derivations as in [33] and [10]. The dispute advancements we consider in this work, including also for complete and stable semantics, are summarised in Table 2. Here e.g. for dispute advancements of type DAB, the proponent can move in P-B-$\langle\overline{\mathscr{A}^!}\rangle$ manner: both making P-B-$\langle\overline{\mathscr{A}^!}\rangle$-i or P-B-$\langle\overline{\mathscr{A}^!}\rangle$-ii moves. On the other hand, the proponent can move in P-F-$\langle\overline{\mathscr{A}^!}\cap\mathscr{A}\rangle$-ii but not in P-F-$\langle\overline{\mathscr{A}^!}\cap\mathscr{A}\rangle$-i manner. The dispute advancement types listed in Table 2 are just a few of the most obvious of several possible combinations. Note that DAB $\subseteq$ DABF (i.e. DAB moves are DABF moves), DABF $\subseteq$ DC, DABF $\subseteq$ DS, DC $\subseteq$ DF, and DS $\subseteq$ DF (also, usually $\subsetneq$).

Table 2. Dispute advancements with DAB for credulous acceptance w.r.t. the admissible semantics, DABF for credulous acceptance w.r.t. the admissible semantics but including "conservative" forward moves of the proponent, DC for the complete semantics, DS for the stable semantics, and DF for "free style". Columns "Proponent" and "Opponent" represent allowed moves by the proponent and opponent respectively.

Advancement	Proponent	Opponent
DAB	P-B-$\langle\overline{\mathscr{A}^!}\rangle$, P-F-$\langle\overline{\mathscr{A}^!}\cap\mathscr{A}\rangle$-ii	O-B-$\langle\overline{\mathscr{D}}\rangle$, O-F-$\langle\overline{\mathscr{D}}\cap\mathscr{A}\rangle$-ii
DABF	P-B-$\langle\overline{\mathscr{A}^!}\rangle$, P-F-$\langle\overline{\mathscr{A}^!}\cap\mathscr{A}\rangle$	O-B-$\langle\overline{\mathscr{D}}\rangle$, O-F-$\langle\overline{\mathscr{D}}\cap\mathscr{A}\rangle$-ii
DC	P-B-$\langle\overline{\mathscr{A}^!}\rangle$, P-F-$\langle(\overline{\mathscr{A}^!}\cap\mathscr{A})\cup\mathscr{I}\rangle$	O-B-$\langle\overline{\mathscr{D}}\cup\overline{\mathscr{I}}\rangle$, O-F-$\langle(\overline{\mathscr{D}}\cup\overline{\mathscr{I}})\cap\mathscr{A}\rangle$-ii
DS	P-B-$\langle\overline{\mathscr{A}^!}\rangle$, P-F-$\langle\mathscr{A}\rangle$	O-B-$\langle\overline{\mathscr{D}}\rangle$, O-F-$\langle\overline{\mathscr{D}}\cap\mathscr{A}\rangle$-ii
DF	P-B-$\langle\overline{\mathscr{A}}\rangle$, P-F-$\langle\mathscr{A}\rangle$	O-B-$\langle\overline{\mathscr{A}}\rangle$, O-F-$\langle\mathscr{A}\rangle$

Table 3. Termination conditions. TA for admissible, TC for complete, and TS for stable.

Cond.	Proponent winning	Opponent cannot move	Proponent cannot move
TA	$\gamma \cup \overline{\mathscr{C}} \subseteq Conc(\mathscr{P}^*)$, $\mathscr{B}^{!/-}_{\mathscr{D}} \cap \mathscr{B}^{*/-} = \emptyset$	O-B-$\langle\overline{\mathscr{D}}\rangle$ + O-F-$\langle\overline{\mathscr{D}} \cap \mathscr{A}\rangle$-ii or O-F-$\langle\mathscr{A}\rangle$	P-B-$\langle\overline{\mathscr{A}^!}\rangle$ + P-F-$\langle\overline{\mathscr{A}^!} \cap \mathscr{A}\rangle$-ii or P-F-$\langle\mathscr{A}\rangle$
TC	$\gamma \cup \overline{\mathscr{C}} \subseteq Conc(\mathscr{P}^*)$, $\mathscr{B}^{!/-}_{\mathscr{D}} \cap \mathscr{B}^{*/-} = \emptyset$, $\mathscr{I} \setminus \mathscr{D} = \emptyset$	O-B-$\langle\overline{\mathscr{D}}\rangle$ + O-F-$\langle\overline{\mathscr{D}} \cap \mathscr{A}\rangle$-ii or O-F-$\langle\mathscr{A}\rangle$	P-B-$\langle\overline{\mathscr{A}^!}\rangle$ + P-F-$\langle(\overline{\mathscr{A}^!} \cap \mathscr{A}) \cup \mathscr{I}\rangle$ or P-F-$\langle\mathscr{A}\rangle$
TS	$\gamma \cup \overline{\mathscr{C}} \subseteq Conc(\mathscr{P}^*)$, $\mathscr{B}^{!/-}_{\mathscr{D}} \cap \mathscr{B}^{*/-} = \emptyset$, $\mathscr{D} \cup \mathscr{C} = \mathscr{A}$	O-B-$\langle\overline{\mathscr{D}}\rangle$ + O-F-$\langle\overline{\mathscr{D}} \cap \mathscr{A}\rangle$-ii or O-F-$\langle\mathscr{A}\rangle$	P-B-$\langle\overline{\mathscr{A}^!}\rangle$ + P-F-$\langle\mathscr{A}\rangle$

The termination conditions we consider in this work are summarised in Table 3. There is, first of all, a condition that has to be satisfied at a dispute state $(\mathscr{B}, \mathscr{P})$ for the proponent to be winning. This is in the column "Proponent winning". Then the proponent wins if this condition is satisfied and the opponent cannot move in either of the two possible combinations of moves in the column "Opponent cannot move". The opponent wins if the "Proponent winning" condition is not satisfied and the proponent cannot move in either of the two possible combinations of moves in the column "Proponent moves". So, for the termination condition for the admissible semantics TA, we have that the proponent wins if $\gamma \cup \overline{\mathscr{C}} \subseteq Conc(\mathscr{P}^*)$, $\mathscr{B}^{!/-}_{\mathscr{D}} \cap \mathscr{B}^{*/-} = \emptyset$ and the opponent cannot advance further either in DAB manner: O-B-$\langle\overline{\mathscr{D}}\rangle$+O-F-$\langle\overline{\mathscr{D}} \cap \mathscr{A}\rangle$-ii; or in forwards DF manner: O-F-$\langle\mathscr{A}\rangle$. The opponent wins if $\gamma \cup \overline{\mathscr{C}} \setminus Conc(\mathscr{P}^*) \neq \emptyset$ or $\mathscr{B}^{!/-}_{\mathscr{D}} \cap \mathscr{B}^{*/-} \neq \emptyset$ and the proponent cannot advance further either in DAB manner: P-B-$\langle\overline{\mathscr{A}^!}\rangle$+P-F-$\langle\overline{\mathscr{A}^!} \cap \mathscr{A}\rangle$-ii or in forwards DF manner: P-F-$\langle\mathscr{A}\rangle$.

A dispute derivation variant then depends on allowed moves M and termination criteria C. For simplicity we allow that termination criteria make reference to moves which may not be allowed at a specific dispute variant; i.e. although moves are restricted these are all conceived as subsets of dispute variants where advancements are as in DF and hence checking for DF moves (and any other subset) is possible. As already indicated, dispute variants are defined for a set of goals $\gamma \subseteq \mathscr{L}$ (s.t. $\gamma \cap \overline{\gamma} = \emptyset$). They consist of a sequence of dispute states starting at $(\{\}, \gamma)$. At each step the last dispute state is selected and advanced either according to the proponent or opponent and the allowed moves M. The dispute derivation ends at a dispute state satisfying the termination criteria C.

Example 1. Consider the ABA framework from Example 6.2 in [33] with $\mathscr{A} = \{a, b, c, d, e, f\}$, where $\overline{a} = q$, $\overline{b} = f$, $\overline{c} = u$, $\overline{d} = v$, $\overline{e} = v$, $\overline{f} = v$. Also:

$$\mathscr{R} = \{p \leftarrow a, u;\ \ q \leftarrow b, r;\ \ q \leftarrow c, s;\ \ q \leftarrow c, t;\ \ u \leftarrow a;\ \ s \leftarrow;\ \ t \leftarrow d;\ \ t \leftarrow e\}.$$

A DAB + TA StFlexDD following the structured dispute derivation of Fig. 7 in [33] is shown in Table 4. Note first of all, that in order to follow structured dispute derivations as in [33] the opponent must, for every statement that it (backward-) expands

Table 4. A DAB + TA StFlexDD for Example 1. Labels $-$, $*$, #, ! are used to distinguish blocked, complete, maximal incomplete, and opposing arguments respectively. Only complete and maximal incomplete arguments for goals and contraries of culprits of the proponent are shown. The dispute derivation ends with the opponent not being able to advance further in O-B-$\langle\overline{\mathscr{D}}\rangle$+O-F-$\langle\overline{\mathscr{D}}\cap\mathscr{A}\rangle$-ii manner.

Step and move type	$\mathscr{P}$	$\mathscr{B}$	$(\gamma\cup\overline{\mathscr{C}})\setminus Conc(\mathscr{P}^*)$	$\mathscr{D}$	$\mathscr{C}$
0	$\{^{\#}p\}$	$\{\}$	$\{p\}$	$\{\}$	$\{\}$
1 (P-B-i, $p\leftarrow a,u$)	$\{^{\#}p\leftarrow a,u\}$	$\{\}$	$\{p\}$	$\{a\}$	$\{\}$
2 (O-B-ii, $q\leftarrow b,r$)	$\{^{\#}p\leftarrow a,u\}$	$\{^{!}q\leftarrow b,r\}$	$\{p\}$	$\{a\}$	$\{\}$
3 (O-B-ii, $q\leftarrow c,s$)	$\{^{\#}p\leftarrow a,u\}$	$\{^{!}q\leftarrow b,r;\ ^{!}q\leftarrow c,s\}$	$\{p\}$	$\{a\}$	$\{\}$
4 (O-B-ii, $q\leftarrow c,t$)	$\{^{\#}p\leftarrow a,u\}$	$\{^{!}q\leftarrow b,r;\ ^{!}q\leftarrow c,s;\ ^{!}q\leftarrow c,t\}$	$\{p\}$	$\{a\}$	$\{\}$
5 (P-B-ii, $u\leftarrow a$)	$\{^{\#}p\leftarrow a,u;\ ^{*}u\leftarrow a\}$	$\{^{!}q\leftarrow b,r;\ ^{-!}q\leftarrow c,s;$ $^{-!}q\leftarrow c,t\}$	$\{p\}$	$\{a\}$	$\{c\}$
6 (P-B-i, $u\leftarrow a$)	$\{^{*}u\leftarrow a;\ ^{*}p\leftarrow a,[u\leftarrow a]\}$	$\{^{!}q\leftarrow b,r;\ ^{-!}q\leftarrow c,s;$ $^{-!}q\leftarrow c,t\}$	$\{\}$	$\{a\}$	$\{c\}$
7 (P-F-ii, f)	$\{^{*}u\leftarrow a;\ ^{*}p\leftarrow a,[u\leftarrow a];\ ^{*}f\}$	$\{^{-!}q\leftarrow b,r;\ ^{-!}q\leftarrow c,s;$ $^{-!}q\leftarrow c,t\}$	$\{\}$	$\{a,f\}$	$\{b,c\}$

Table 5. A DF + TA StFlexDD for Example 2. Only complete and maximal incomplete arguments of the proponent are shown. The dispute derivation ends with the opponent not being able to advance further in O-F-$\langle\mathscr{A}\rangle$ manner.

Step and move type	$\mathscr{P}$	$\mathscr{B}$	$(\gamma\cup\overline{\mathscr{C}})\setminus Conc(\mathscr{P}^*)$	$\mathscr{D}$	$\mathscr{C}$
0	$\{\ ^{\#}p\ \}$	$\{\}$	$\{p\}$	$\{\}$	$\{b,c\}$
1 (P-B-i, $p\leftarrow a$)	$\{\ ^{*}p\leftarrow a\ \}$	$\{\}$	$\{\}$	$\{a\}$	$\{b,c\}$
2 (O-F-ii, a)	$\{\ ^{*}p\leftarrow a\ \}$	$\{^{*}a\}$	$\{\}$	$\{a\}$	$\{b,c\}$
3 (O-F-i, $p\leftarrow a$)	$\{\ ^{*}p\leftarrow a\ \}$	$\{\ ^{*}a;\ \ ^{*}p\leftarrow a\ \}$	$\{\}$	$\{a\}$	$\{b,c\}$

on (e.g. q in the example in steps 2–4), expand the statement with every non-blocked rule. This is not necessary in StFlexDDs. Secondly, note that structured dispute derivations from [33] include a tracking mechanism whereby arguments that are not necessary for further evolution of the dispute derivation are discarded. As we strive for a general definition which allows us to consider several manners of expanding the opponents and proponents argument we do not do this here. For a direct implementation of StFlexDDs one could e.g. only store complete and maximal incomplete arguments of the proponent; also, one could remove arguments from the opponent which have been fully backward expanded. In fact, to simplify the example, we do not show all the proponents arguments in Table 4.

Nevertheless, we note in the dispute derivation from Table 4 redundancy in the moves. In particular, $u\leftarrow a$ is used twice by the proponent and it is only the second use that makes c a culprit. The reason is that following [33] the proponent is only "aware" of its arguments, but not of their internal structure.

Example 2. Consider the ABA framework with $\mathscr{A}=\{a,b,c\}$, where $\overline{a}=t, \overline{b}=p, \overline{c}=p$; and

$$\mathscr{R}=\{p\leftarrow a;\ \ t\leftarrow b;\ \ t\leftarrow c;\ \ t\leftarrow u;\ \ u\leftarrow v;\ \ v\leftarrow u\}.$$

A DF + TA StFlexDD (which also satisfies TC and TS) is shown in Table 5. Note that any DAB + TA dispute derivation for the same example will not terminate because of the circularity in the rules $u \leftarrow v$ and $v \leftarrow u$. Even when replacing these circular rules with a very long chain of rules starting at $t \leftarrow u$ and ending e.g. with a rule with one of b or c in the body, one gets a much shorter dispute derivation using forward moves. Note nevertheless again here the redundancy in particular in the need for the opponent to essentially repeat the moves by the proponent.

For flexible dispute derivations following structured dispute derivations and the variants we consider in this section we have the following results generalising the results for structured dispute derivations (for credulous reasoning w.r.t. the admissible semantics; i.e. DAB+TA in our context) from [33] in our more flexible setting:

Theorem 1. *$DF + \{TA, TC, TS\}$ StFlexDDs are sound for the admissible, complete, and stable semantics respectively. This means e.g. for $DF + TA$ that if there is a $DF + TA$ StFlexDD ending with a dispute state $(\mathscr{B}, \mathscr{P})$ and the proponent as winner, then $\mathscr{D}$ is an admissible assumption set w.r.t. which γ is acceptable.*

Corollary 1. *$\{DAB, DABF, DC, DS\} + \{TA, TC, TS\}$ StFlexDDs are sound for the admissible, complete, and stable semantics respectively.*

Theorem 2. *If $\mathscr{L}$ is finite and $\mathscr{R}$ is acyclic, $DAB + TA$ StFlexDDs are complete for credulous acceptance w.r.t. the admissible semantics. I.e. if γ is acceptable for some admissible assumption set, then there is a $DAB + TA$ StFlexDD ending with a dispute state $(\mathscr{B}, \mathscr{P})$ and the proponent as winner, s.t. $\mathscr{D}$ is an admissible assumption set w.r.t. which γ is acceptable. Moreover, $DC + TC$ StFlexDDs are complete for the complete semantics and $DS + TS$ StFlexDDs are complete for the stable semantics. E.g. for the complete semantics: if γ is acceptable for some complete assumption set U, then there is a $DC + TC$ StFlexDD ending with a dispute state $(\mathscr{B}, \mathscr{P})$ and the proponent as winner, s.t. $\mathscr{D} = U$. Finally, $\{DC, DS, DF\} + TA$ StFlexDDs are complete for the admissible semantics.*

Corollary 2. *If $\mathscr{L}$ is finite and $\mathscr{R}$ is acyclic, $\{DABF, DC, DS, DF\} + TA$ StFlexDDs are complete for credulous acceptance w.r.t. the admissible semantics. Also, $DF + TC$ StFlexDDs are complete for the complete semantics and $DF + TS$ StFlexDDs are complete for the stable semantics.*

Flexible Graph-Based Dispute Derivations. We only need to change the notion of expansion of the opponents, respectively proponents arguments in the definition of dispute advancements to get GrFlexDDs. Specifically, we need the following notions:

Definition 6. *Let $A \subseteq Args$ and $a \in Args$. Then $A \rtimes \{a\}$ is the rule minimal (also called non bloated in [10]) closure of $A \cup \{a\}$ under sub-arguments and argument expansions. Here, first of all, $A' \subseteq Args$ is closed under sub-arguments if $A' = Sub(A')$. Moreover, A' is closed under expansions if $a' = a'' \lessdot A''$ for some $a'' \in A'$, $A'' \subseteq A'$, then also $a' \in A'$. Also, A' is rule minimal if there are no $h \leftarrow B, h' \leftarrow B' \in Sub(A')$ s.t. $h = h'$ but $B \neq B'$. Then, assuming A is closed under sub-arguments, closed under argument expansions,*

Table 6. A DAB + TA GrFlexDD for Example 3 (ABA framework from Example 1). Only maximal arguments of the proponent (for goals and contraries of culprits) and the opponent (for contraries of defenses) are shown. The dispute derivation ends with the opponent not being able to advance further in O-B-$\langle\overline{\mathscr{D}}\rangle$+O-F-$\langle\overline{\mathscr{D}}\cap\mathscr{A}\rangle$-ii manner.

Step and move type	$\mathscr{P}$	$\mathscr{B}$	$\gamma\cup\overline{\mathscr{C}}\setminus Conc(\mathscr{P}^*)$	$\mathscr{D}$	$\mathscr{C}$
0	$\{\,^{\#}p\,\}$	$\{\}$	$\{p\}$	$\{\}$	$\{\}$
1 (P-B-i, $p\leftarrow a,u$)	$\{^{\#}p\leftarrow a,u\,\}$	$\{\}$	$\{p,u\}$	$\{a\}$	$\{c\}$
2 (O-B-ii, $q\leftarrow b,r$)	$\{\,^{\#}p\leftarrow a,u\,\}$	$\{\,^{!}q\leftarrow b,r\,\}$	$\{p,u\}$	$\{a\}$	$\{c\}$
3 (P-B-i, $u\leftarrow a$)	$\{^{*}u\leftarrow a;\ \ ^{*}p\leftarrow a,[u\leftarrow a]$ $\}$	$\{\,^{!}q\leftarrow b,r\,\}$	$\{\}$	$\{a\}$	$\{c\}$
4 (P-F-ii, f)	$\{^{*}u\leftarrow a;\ \ ^{*}p\leftarrow a,[u\leftarrow a];$ $^{*}f\,\}$	$\{^{-!}q\leftarrow b,r\,\}$	$\{\}$	$\{a,f\}$	$\{b,c\}$

and rule minimal, $A\rtimes\{a\}$ is the closure under sub-arguments and argument expansions of $A\cup\{a\}$ if this closure is also rule minimal, while otherwise $A\rtimes\{a\}=A$ (i.e. expansions which bloat the argument set are disallowed).

On the other hand, $A:\{a\}$ is the argument rule minimal union of A and a. Here $A'\subseteq Args$ is argument rule minimal if for each $a'\in A'$, $\{a'\}$ is rule minimal (such an a' is also called non-flabby in [10]*). Then, assuming A is argument rule minimal, $A:\{a\}=A\cup\{a\}$ if $A\cup\{a\}$ is argument rule minimal, while otherwise $A:\{a\}=A$ (i.e. a must be rule minimal, aka non-flabby).*

In GrFlexDDs a proponent dispute state advancement from a dispute state $(\mathscr{B},\mathscr{P})$ is a dispute state $(\mathscr{B},\mathscr{P}')$ with $\mathscr{P}'=\mathscr{P}\rtimes\{a\}\neq\mathscr{P}$, $X_1\subseteq\overline{\mathscr{A}}$, $X_2\subseteq\mathscr{A}$ with P-B-$\langle\overline{\mathscr{A}^{!}}\cup X_1\rangle$ and P-F–$\langle(\overline{\mathscr{A}^{!}}\cap\mathscr{A})\cup X_2\rangle$ moves defined as before. An opponent dispute state advancement from a dispute state $(\mathscr{B},\mathscr{P})$ is a dispute state $(\mathscr{B}',\mathscr{P})$ with $\mathscr{B}'=\mathscr{B}:\{a\}\neq\mathscr{B}$, $Y_1\subseteq\overline{\mathscr{A}}$, and $Y_2\subseteq\mathscr{A}$ with O-B-$\langle\overline{\mathscr{D}}\cup Y_1\rangle$ and O-F-$\langle(\overline{\mathscr{D}}\cap\mathscr{A})\cup Y_2\rangle$ moves defined as previously.

Example 3. Consider again the ABA framework from Example 1. To compare, a DAB + TA GrFlexDD following more or less that in Table 4 is shown in Table 6. Note that here c becomes a culprit already at step 1, while in the DAB + TA StFlexDD of Table 4 this happens at step 5 (since only then is there an argument in $\mathscr{P}$ with conclusion $\overline{c}=u$). Also, $u\leftarrow a$ only needs to be used once by the proponent, while in the dispute derivation of Table 4 this occurs twice. In the end the dispute becomes shorter by 3 steps.

Example 4. Consider a slightly more complex version of the ABA framework from Example 16 in [10] with $\mathscr{A}=\{a,b,c,d\}$, where $\overline{a}=t$, $\overline{b}=r$, $\overline{c}=t$, $\overline{d}=c$. Also:

$$\mathscr{R}=\{p\leftarrow q;\ \ q\leftarrow a;\ \ r\leftarrow p;\ \ t\leftarrow b;\ \ t\leftarrow p,s;\ \ t\leftarrow q,u,d\}.$$

A DAB + TA GrFlexDD based on the graph-based dispute derivation of Table 8 in [10] (the first 4 steps correspond to the whole dispute derivation in [10], except that here $t\leftarrow b$ is invoked by the opponent rather than simply b) is shown in Table 7. Note the redundancy in steps 6–7 of the opponent where the argument $p\leftarrow[q\leftarrow a]$ is constructed again. Also $q\leftarrow a$ is used in step 7 and then again in step 9.

Table 7. A DAB + TA GrFlexDD for Example 4. Only maximal arguments of the proponent (for goals and contraries of culprits) and the opponent (for contraries of defenses) are shown. The dispute derivation ends with the opponent not being able to advance further in O-B-$\langle\overline{\mathscr{D}}\rangle$+O-F-$\langle\overline{\mathscr{D}}\cap\mathscr{A}\rangle$-ii manner.

Step and move type	$\mathscr{P}$	$\mathscr{B}$	$\gamma\cup\overline{\mathscr{C}}\setminus Conc(\mathscr{P}^*)$	$\mathscr{D}$	$\mathscr{C}$
0	$\{{}^{\#}p\}$	$\{\}$	$\{p\}$	$\{\}$	$\{\}$
1 (P-B-i, $p\leftarrow q$)	$\{{}^{\#}p\leftarrow q\}$	$\{\}$	$\{p\}$	$\{\}$	$\{\}$
2 (P-B-i, $q\leftarrow a$)	$\{{}^{*}p\leftarrow[q\leftarrow a]\}$	$\{\}$	$\{\}$	$\{a\}$	$\{\}$
3 (O-B-ii, $t\leftarrow b$)	$\{{}^{*}p\leftarrow[q\leftarrow a]\}$	$\{{}^{*!}t\leftarrow b\}$	$\{\}$	$\{a\}$	$\{\}$
4 (P-B-ii, $r\leftarrow p$)	$\{{}^{*}p\leftarrow[q\leftarrow a];$ ${}^{*}r\leftarrow[p\leftarrow[q\leftarrow a]]\}$	$\{{}^{-*!}t\leftarrow b\}$	$\{\}$	$\{a\}$	$\{b\}$
5 (O-B-ii, $t\leftarrow p,s$)	$\{{}^{*}p\leftarrow[q\leftarrow a];$ ${}^{*}r\leftarrow[p\leftarrow[q\leftarrow a]]\}$	$\{{}^{-*!}t\leftarrow b;\ {}^{!}t\leftarrow p,s\}$	$\{\}$	$\{a\}$	$\{b\}$
6 (O-B-i, $p\leftarrow q$)	$\{{}^{*}p\leftarrow[q\leftarrow a];$ ${}^{*}r\leftarrow[p\leftarrow[q\leftarrow a]]\}$	$\{{}^{-*!}t\leftarrow b;\ {}^{!}t\leftarrow[p\leftarrow q],s$ $\}$	$\{\}$	$\{a\}$	$\{b\}$
7 (O-B-i, $q\leftarrow a$)	$\{{}^{*}p\leftarrow[q\leftarrow a];$ ${}^{*}r\leftarrow[p\leftarrow[q\leftarrow a]]\}$	$\{{}^{-*!}t\leftarrow b;$ ${}^{!}t\leftarrow[p\leftarrow[q\leftarrow a]],s\}$	$\{\}$	$\{a\}$	$\{b\}$
8 (O-B-ii, $t\leftarrow q,u,d$)	$\{{}^{*}p\leftarrow[q\leftarrow a];$ ${}^{*}r\leftarrow[p\leftarrow[q\leftarrow a]]\}$	$\{{}^{-*!}t\leftarrow b;$ ${}^{!}t\leftarrow[p\leftarrow[q\leftarrow a]],s;$ ${}^{!}t\leftarrow q,u,d\}$	$\{\}$	$\{a\}$	$\{b\}$
9 (O-B-ii, $q\leftarrow a$)	$\{{}^{*}p\leftarrow[q\leftarrow a];$ ${}^{*}r\leftarrow[p\leftarrow[q\leftarrow a]]\}$	$\{{}^{-*!}t\leftarrow b;$ ${}^{!}t\leftarrow[p\leftarrow[q\leftarrow a]],s;$ ${}^{!}t\leftarrow[q\leftarrow a],u,d\}$	$\{\}$	$\{a\}$	$\{b\}$

We again obtain soundness and completeness results generalising the results for graph-based dispute derivations (for credulous reasoning) from [10]:

Theorem 3. *DF* + $\{TA, TC, TS\}$ *GrFlexDDs are sound for the admissible, complete, and stable semantics respectively.*

Corollary 3. $\{DAB, DABF, DC, DS\}$+$\{TA, TC, TS\}$ *GrFlexDDs are sound for the admissible, complete, and stable semantics respectively.*

Theorem 4. *If* $\mathscr{L}$ *is finite DAB + TA GrFlexDDs are complete for credulous acceptance w.r.t. the admissible semantics. Moreover, DC + TC GrFlexDDs are complete for the complete semantics and DS + TS GrFlexDDs are complete for the stable semantics. Finally,* $\{DC, DS, DF\}$ *+ TA GrFlexDDs are complete for the admissible semantics.*

Corollary 4. *If* $\mathscr{L}$ *is finite,* $\{DABF, DC, DS, DF\}$ *+ TA GrFlexDDs are complete for credulous acceptance w.r.t. the admissible semantics. Also, DF + TC GrFlexDDs are complete for the complete semantics and DF + TS GrFlexDDs are complete for the stable semantics.*

3.3 Flexible Dispute Derivations

In the previous section we presented definitions of argument-based flexible variants of structured and graph-based dispute derivations. The objective was, first of all, to give a general definition showing the common aspects between the, at the first sight, different looking forms of dispute derivations while also incorporating flexibility in the order and types of moves allowed. At the same time, our definition allows to make clear the differences between structured and graph-based dispute derivations (and their flexible variants) in terms of how much of the arguments put forward during a dispute is stored and made use of in later steps of the dispute. We have seen that in this regard GrFlexDDs, while improving on StFlexDDs, still have some redundancy in that, firstly, the opponent does not make use of the proponents arguments when putting forward its own arguments. Also, there is redundancy in the moves of the opponent w.r.t. previous moves of itself (see in particular Example 4). These issues become especially pressing in the context of dispute derivations with forward moves as forward reasoning, more than backward reasoning, relies on previous moves.

We now propose FlexDDs to remedy the above mentioned issues. Again, we only need to change the definition of expansions of the opponents and proponents arguments in dispute advancements. Once more, we first need a definition:

Definition 7. *Let $A \subseteq Args$, $a \in Args$. Then $A \bowtie \{a\}$ is the closure of $A \cup \{a\}$ under sub-arguments and argument expansions.*

Thus $A \bowtie \{a\}$ is a more relaxed version of $A \rtimes \{a\}$ used in the definition of GrFlexDDs for the proponents dispute advancements.

In FlexDDs a proponent dispute state advancement from a dispute state $(\mathscr{B}, \mathscr{P})$ is a dispute state $(\mathscr{B}', \mathscr{P}')$ with $\mathscr{P}' = \mathscr{P} \rtimes \{a\} \neq \mathscr{P}$, $\mathscr{B}' = \mathscr{B} \bowtie \{a\}$, $X_1 \subseteq \overline{\mathscr{A}}$, $X_2 \subseteq \mathscr{A}$ with P-B-$\langle\overline{\mathscr{A}^!} \cup X_1\rangle$ and P-F–$\langle(\overline{\mathscr{A}^!} \cap \mathscr{A}) \cup X_2\rangle$ moves defined as before. An opponent dispute state advancement from a dispute state $(\mathscr{B}, \mathscr{P})$ is a dispute state $(\mathscr{B}', \mathscr{P})$ with $\mathscr{B}' = \mathscr{B} \bowtie \{a\} \neq \mathscr{B}$, $Y_1 \subseteq \overline{\mathscr{A}}$, $Y_2 \subseteq \mathscr{A}$ with O-B-$\langle\overline{\mathscr{D}} \cup Y_1\rangle$ and O-F-$\langle(\overline{\mathscr{D}} \cap \mathscr{A}) \cup Y_2\rangle$ moves defined as previously. So, main changes w.r.t. GrFlexDDs are that the proponents moves also have an effect on the opponents arguments. Also, $\mathscr{B}' = \mathscr{B} \bowtie \{a\}$ rather than $\mathscr{B}' = \mathscr{B} : \{a\}$ is used for updating the opponents arguments. We thus, first of all, follow [10] in restricting the set of arguments of the proponent to be rule minimal. This has been argued for convincingly in [10] for both conceptual reasons (why have more than one justification line for a claim?) as well as computational reasons (guarantees completeness of disputes when $\mathscr{L}$ is finite even if $\mathscr{R}$ contains cycles).

In [10] then the authors have also argued for the opponents arguments to be rule minimal partly again for conceptual reasons but even more so for computational reasons. Regarding the conceptual arguments of the authors, we note that, in any case, all possible rule minimal arguments attacking the defenses of the proponent need to be considered in dispute derivations. Thus the opponents arguments are not globally rule minimal (as the proponents are). Regarding the computational reasons, while it is true that restricting attention to the arguments of the opponent that are rule minimal guarantees completeness also if $\mathscr{R}$ contains cycles (assuming $\mathscr{L}$ is finite), we will show that this is not necessary. In fact, treatment of the proponents and opponents expansions in an (almost) symmetric way leads to a definition of dispute derivations which avoids some of the remaining redundancy in moves of GrFlexDDs while staying complete when $\mathscr{L}$ is finite and $\mathscr{R}$ contains cycles. Moreover, as we will show in Sect. 4, our definition of FlexDDs leads naturally to an implementation where all arguments in dispute derivations are represented as a graph rather than only the proponents as in the implementation of [10].

Table 8. A DAB + TA FlexDD for Example 5 (ABA framework from Example 4). Here \$ labels arguments which are held by the proponent as well as the opponent. Only complete and maximal arguments for goals and contraries of culprits as well as maximal arguments for contraries of defenses are shown. The dispute derivation ends with the opponent not being able to advance further in O-B-$\langle\overline{\mathscr{D}}\rangle$+O-F-$\langle\overline{\mathscr{D}}\cap\mathscr{A}\rangle$-ii manner.

Step and move type	$\mathscr{B}$	$\gamma \cup \overline{\mathscr{C}} \setminus Conc(\mathscr{P}^*)$	$\mathscr{D}$	$\mathscr{C}$
0	$\{ {}^{\#\$}p \}$	$\{p\}$	$\{\}$	$\{\}$
1 (P-B-i, $p \leftarrow q$)	$\{ {}^{\#\$}p \leftarrow q \}$	$\{p\}$	$\{\}$	$\{\}$
2 (P-B-i, $q \leftarrow a$)	$\{ {}^{*\$}p \leftarrow [q \leftarrow a] \}$	$\{\}$	$\{a\}$	$\{\}$
3 (O-B-ii, $t \leftarrow b$)	$\{ {}^{*\$}p \leftarrow [q \leftarrow a];\ {}^{*!}t \leftarrow b \}$	$\{\}$	$\{a\}$	$\{\}$
4 (P-B-ii, $r \leftarrow p$)	$\{ {}^{*\$}p \leftarrow [q \leftarrow a];\ {}^{*\$}r \leftarrow [p \leftarrow [q \leftarrow a]];$ ${}^{-*!}t \leftarrow b \}$	$\{\}$	$\{a\}$	$\{b\}$
5 (O-B-ii, $t \leftarrow p, s$)	$\{ {}^{*\$}p \leftarrow [q \leftarrow a];\ {}^{*\$}r \leftarrow [p \leftarrow [q \leftarrow a]];$ ${}^{-*!}t \leftarrow b;\ {}^{!}t \leftarrow [p \leftarrow [q \leftarrow a]], s \}$	$\{\}$	$\{a\}$	$\{b\}$
6 (O-B-ii, $t \leftarrow q, u, d$)	$\{ {}^{*\$}p \leftarrow [q \leftarrow a];\ {}^{*\$}r \leftarrow [p \leftarrow [q \leftarrow a]];$ ${}^{-*!}t \leftarrow b;\ {}^{!}t \leftarrow [p \leftarrow [q \leftarrow a]], s;$ ${}^{!}t \leftarrow [q \leftarrow a], u, d \}$	$\{\}$	$\{a\}$	$\{b\}$

Example 5. Consider again the ABA framework from Example 4. A DAB + TA FlexDD following more or less the DAB + TA GrFlexDD from Table 7 is shown in Table 8. Note that here the steps 5–7 from Table 7 are performed in one step: step 5. Also, steps 8–9 from Table 7 are completed in step 6. A DC + TC (and DS + TS) FlexDD for the same example is shown in Table 9.

For FlexDDs we have the following results:

Theorem 5. *$DF + \{TA, TC, TS\}$ FlexDDs are sound for the admissible, complete, and stable semantics respectively.*

Corollary 5. *$\{DAB, DABF, DC, DS\} + \{TA, TC, TS\}$ FlexDDs are sound for the admissible, complete, and stable semantics respectively.*

Lemma 1. *If $\mathscr{L}$ is finite, the number of possible DF and hence also $\{DAB, DABF, DC, DS\}$ moves of the proponent and opponent in FlexDDs is also finite.*

Proof. We give the proof for the opponent. For the proponent it is analogous. Note first that the opponents moves involve adding an assumption (O-F-$\langle\mathscr{A}\rangle$-ii) or a rule to $\mathscr{B}$ (O-B-$\langle\overline{\mathscr{A}}\rangle$-ii), or expanding arguments backwards or forwards (O-B-$\langle\overline{\mathscr{A}}\rangle$-i or O-F-$\langle\mathscr{A}\rangle$-i) w.r.t. some rule. Now, once an assumption is put in $\mathscr{B}$ it cannot be added again by the requirement $\mathscr{B}' = \mathscr{B} \bowtie \{a\} \neq \mathscr{B}$. Also, if some rule r is used in one step (either by adding it to $\mathscr{B}$ or expanding some argument w.r.t. it, which means by closure under sub-arguments that then also r is in $\mathscr{B}'$), then r cannot be used in any other step. For O-B-$\langle\overline{\mathscr{A}}\rangle$-ii this is clear by the requirement $\mathscr{B}' = \mathscr{B} \bowtie \{a\} \neq \mathscr{B}$. For O-B-$\langle\overline{\mathscr{A}}\rangle$-i note that if $a' \in \mathscr{B}$ and $h \leftarrow B \in \mathscr{B}$ then $a' \lessdot h \leftarrow B$ is also already in $\mathscr{B}$ because $\mathscr{B}$ is required to be closed by argument expansions. Analogously for O-F-$\langle\mathscr{A}\rangle$-i moves.

Table 9. A DC + TC (and DS + TS) FlexDD for Example 5 (ABA framework from Example 4). Only complete and maximal arguments for goals and contraries of culprits as well as maximal arguments for contraries of defenses are shown. The dispute derivation ends with the opponent not being able to advance further in O-F-$\langle\mathscr{A}\rangle$ manner.

Step and move type	$\mathscr{B}$	$\gamma \cup \overline{\mathscr{C}} \setminus Conc(\mathscr{P}^*)$	$\mathscr{D}$	$\mathscr{C}$	$\mathscr{I} \setminus \mathscr{D}$
0	$\{\ ^{\#\$}p\ \}$	$\{p\}$	$\{\}$	$\{\}$	$\{a,b,c,d\}$
1 (P-B-i, $p \leftarrow q$)	$\{\ ^{\#\$}p \leftarrow q\ \}$	$\{p\}$	$\{\}$	$\{\}$	$\{a,b,c,d\}$
2 (P-B-i, $q \leftarrow a$)	$\{\ ^{*\$}p \leftarrow [q \leftarrow a]\ \}$	$\{\}$	$\{a\}$	$\{\}$	$\{b,c,d\}$
3 (O-B-ii, $t \leftarrow b$)	$\{\ ^{*\$}p \leftarrow [q \leftarrow a];\ \ ^{*!}t \leftarrow b\ \}$	$\{\}$	$\{a\}$	$\{\}$	$\{b,d\}$
4 (P-B-ii \ P-F-i, $r \leftarrow p$)	$\{^{*\$}p \leftarrow [q \leftarrow a];$ $^{*\$}r \leftarrow [p \leftarrow [q \leftarrow a]];$ $^{-*!}t \leftarrow b\}$	$\{\}$	$\{a\}$	$\{b\}$	$\{c,d\}$
5 (P-F-ii, c)	$\{^{*\$}p \leftarrow [q \leftarrow a];$ $^{*\$}r \leftarrow [p \leftarrow [q \leftarrow a]];$ $^{-*!}t \leftarrow b;\ \ ^{*\$}c\}$	$\{\}$	$\{a,c\}$	$\{b,d\}$	$\{\}$

Theorem 6. *If $\mathscr{L}$ is finite DAB + TA FlexDDs are complete for credulous acceptance w.r.t. the admissible semantics. Moreover, DC + TC FlexDDs are complete for the complete semantics and DS + TS FlexDDs are complete for the stable semantics. Finally, $\{DC, DS, DF\}$ + TA FlexDDs are complete for the admissible semantics.*

Corollary 6. *If $\mathscr{L}$ is finite, $\{DABF, DC, DS, DF\}$ + TA FlexDDs are complete for credulous acceptance w.r.t. the admissible semantics. Also, DF + TC FlexDDs are complete for the complete semantics and DF + TS FlexDDs are complete for the stable semantics.*

4 Rule-Based Flexible Dispute Derivations

Rule-based flexible dispute derivations, or RlFlexDDs for short, provide an alternative representation and implementation of FlexDDs. Relying on the observation contained in the proof of Lemma 1 (on which Theorem 6 depends), in RlFlexDDs the proponent and opponent put forward claims and rules rather than arguments. Moreover, they make use of the underlying (labelled) graph of the dependencies between statements and rules put forward by the proponent and opponent during a dispute. RlFlexDDs thus generalise the work of [10] which implements DAB+TA GrFlexDD disputes. As we have already indicated, in the dispute derivations of [10] the proponents arguments are represented as graph, while the opponents are not. Also, the opponent does not make use of the proponents arguments.

So, in RlFlexDDs a dispute state for a set of goals $\gamma \subseteq \mathscr{L}$ (s.t. $\gamma \cap \overline{\gamma} = \emptyset$) is a tuple $(\mathbb{B}, \mathbb{P})$ where $\mathbb{B} \subseteq (\mathscr{L} \cup \mathscr{R})$, and $\mathbb{P} \subseteq \mathbb{B}$. To define rule-based dispute advancements we define the auxiliary notation in Table 10; in large part encoding the analogous notions from Sect. 3.3 in the rule setting. Concretely, we have that e.g. $s \in \mathbb{P}^* \cap \mathscr{L}$ iff there is a complete argument for s using rules in $\mathbb{P}$. Also, $s \in \mathbb{B}^{*/-} \cap \mathscr{L}$ iff there is a complete argument for s using rules in $\mathbb{B}$ that does not use any culprit. On the other hand, $s \in \mathbb{B}^- \cap \mathscr{L}$ implies first of all that all arguments for s using non-blocked (i.e. without culprits in bodies) rules use rules only in $\mathbb{B}$. Also, that all such arguments (using rules only in $\mathbb{B}$) which are complete are blocked (i.e. make use of some culprit). As a consequence then, $s \in \mathbb{B}_S^{!/-} \cap \mathscr{L}$ if s is used in an argument for some $s' \in S$ (with $S \subseteq \mathscr{L}$) and the latter two conditions (for $s \in \mathbb{B}^-$) do not hold.

Table 10. Auxiliary notation for rule-based flexible dispute derivations. All notions w.r.t. a dispute state $(\mathbb{B}, \mathbb{P})$.

Notation	Description
$\mathscr{D} = \mathbb{P} \cap \mathscr{A}$	Defenses
$\mathscr{C} = \{u \in \mathscr{A} \mid \overline{u} \in \mathbb{P}\}$	Culprits
$\mathscr{J}_{\mathbb{B}} = \mathscr{R} \setminus \mathbb{B}$	Remaining rules for the opponent
$\mathscr{J}_{\mathbb{P}} = \mathscr{R} \setminus \mathbb{P}$	Remaining rules for the proponent
$\mathscr{J}_{\mathbb{B}}^- = \{h \leftarrow B \in \mathscr{J}_{\mathbb{B}} \mid B \cap \mathscr{C} \neq \emptyset\}$	Blocked remaining rules
$\mathscr{J}_{\mathbb{P}}^{\sim} = \{h \leftarrow B \in \mathscr{J}_{\mathbb{P}} \mid (\{h\} \cup B) \cap (\overline{B} \cup \mathscr{C} \cup \overline{\mathscr{D}}) \neq \emptyset\}$	Remaining rules blocked for the proponent
$(\mathbb{P} \cap \mathscr{L})^{\downarrow} = \{s \in \mathbb{P} \cap \mathscr{L} \mid \neg \exists h \leftarrow B \in \mathbb{P} \textit{ with } h = s\}$	Played unexpanded statements of the proponent
$(\mathbb{B} \cap \mathscr{L})^{\uparrow\uparrow} = \{s \in (\mathbb{B} \cap \mathscr{L}) \mid \neg \exists h \leftarrow B \in (\mathscr{J}_{\mathbb{B}} \setminus \mathscr{J}_{\mathbb{B}}^-) \textit{ with } h = s\}$	Played fully expanded statements
$\mathbb{B}^- = (\mathbb{B} \cap \mathscr{C}) \cup \{s \in (\mathbb{B} \cap \mathscr{L})^{\uparrow\uparrow} \setminus \mathscr{A} \mid \neg \exists h \leftarrow B \in (\mathbb{B} \cap \mathscr{R}) \setminus \mathbb{B}^- \textit{ with } h = s\} \cup \{h \leftarrow B \in \mathbb{B} \cap \mathscr{R} \mid B \cap \mathbb{B}^- \neq \emptyset\}$	Played blocked pieces
$\mathbb{P}^* = (\mathbb{P} \cap \mathscr{A}) \cup \{h \leftarrow B \in (\mathbb{P} \cap \mathscr{R}) \mid B \subseteq \mathbb{P}^*\} \cup \{s \in (\mathbb{P} \cap (\mathscr{L} \setminus \mathscr{A})) \mid \exists h \leftarrow B \in \mathbb{P}^* \text{ with } h = s\}$	Complete played pieces of the proponent
$\mathbb{B}^{*/-} = (\mathbb{B} \cap (\mathscr{A} \setminus \mathscr{C})) \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^-) \cap \mathscr{R} \mid B \subseteq \mathbb{B}^{*/-}\} \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^-) \cap (\mathscr{L} \setminus \mathscr{A}) \mid \exists h \leftarrow B \in \mathbb{B}^{*/-} \textit{ with } h = s\}$	Unblocked complete played pieces of the opponent
$\mathbb{B}_S^{!/-} = ((\mathbb{B} \setminus \mathbb{B}^-) \cap S) \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^-) \cap \mathscr{L} \mid \exists h \leftarrow B \in \mathbb{B}_S^{!/-} \cap \mathscr{R} \textit{ with } s \in B\} \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^-) \cap \mathscr{R} \mid h \in \mathbb{B}_S^{!/-}\}$	Unblocked pieces supporting statements in $S \subseteq \mathscr{L}$
$\mathscr{A}^! = \mathscr{A} \cap \mathbb{B}_{\overline{\mathscr{D}}}^{!/-}$	Candidates for culprits
$\mathscr{I} = \{u \in \mathscr{A} \setminus \mathscr{C} \mid \overline{u} \notin \mathbb{B}^{*/-}\}$	Currently defended assumptions

Note that $\mathbb{B}^-$ and $\mathbb{P}^*$ are monotonic; i.e. once some element is in the set they remain in the set. This means these sets can be computed incrementally as the dispute evolves. On the other hand, $\mathbb{B}^{*/-}$, and $\mathbb{B}_S^{!/-}$ are not monotonic; but once some element becomes blocked (is in $\mathbb{B}^-$) it cannot be in either $\mathbb{B}^{*/-}$ or $\mathbb{B}_S^{!/-}$ anymore. This means as elements become blocked, they do not need to be considered for computation of $\mathbb{B}^{*/-}$ and $\mathbb{B}_S^{!/-}$.

Now to the definition of dispute advancements for RlFlexDDs. First of all, a proponent dispute state advancement from a dispute state $(\mathbb{B},\mathbb{P})$ is a dispute state $(\mathbb{B}',\mathbb{P}')$ with $\mathbb{P}' = \mathbb{P} \cup T$, $\mathbb{B}' = \mathbb{B} \cup T$, $X_1 \subseteq \overline{\mathscr{A}}$, and $X_2 \subseteq \mathscr{A}$ where either

P-B-$\langle \overline{\mathscr{A}^!} \cup X_1 \rangle$: i) $T = \{h \leftarrow B\} \cup B$ for $h \leftarrow B \in \mathscr{J}_{\mathbb{P}} \setminus \mathscr{J}^{\sim}_{\mathbb{P}}$ with $h \in (\mathbb{P} \cap \mathscr{L})^{\downarrow}$; or
ii) $T = \{h\} \cup \{h \leftarrow B\} \cup B$ for $h \leftarrow B \in \mathscr{J}_{\mathbb{P}} \setminus \mathscr{J}^{\sim}_{\mathbb{P}}$ with $h \in (\overline{\mathscr{A}^!} \cup X_1) \setminus (\mathbb{P} \cup \overline{\mathscr{D}})$;

P-F–$\langle (\overline{\mathscr{A}^!} \cap \mathscr{A}) \cup X_2 \rangle$: i) $T = \{h\} \cup \{h \leftarrow B\}$ with $h \leftarrow B \in \mathscr{J}_{\mathbb{P}} \setminus \mathscr{J}^{\sim}_{\mathbb{P}}$ with $h \notin \mathbb{P}$ or $h \in (\mathbb{P} \cap \mathscr{L})^{\downarrow}$, $b \in \mathbb{P}^*$ for each $b \in B$; or
ii) $T = \{u\}$ for $u \in ((\overline{\mathscr{A}^!} \cap \mathscr{A}) \cup X_2) \setminus (\mathbb{P} \cup \{\overline{u}\} \cup \mathscr{C} \cup \overline{\mathscr{D}}\})$.

An opponent dispute state advancement from a dispute state $(\mathbb{B},\mathbb{P})$ is a dispute state $(\mathbb{B}',\mathbb{P})$ with $\mathbb{B}' = \mathbb{B} \cup T$, $Y_1 \subseteq \overline{\mathscr{A}}$, and $Y_2 \subseteq \mathscr{A}$ where either

O-B-$\langle \overline{\mathscr{D}} \cup Y_1 \rangle$: i) $T = \{h \leftarrow B\} \cup B$ for $h \leftarrow B \in \mathscr{J}_{\mathbb{B}} \setminus \mathscr{J}^{-}_{\mathbb{B}}$ with $h \in \mathbb{B}^{!/-}_{\overline{\mathscr{D}} \cup Y_1} \cap \mathscr{L}$; or
ii) $T = (\{h\} \cup \{h \leftarrow B\} \cup B)$ for a $h \leftarrow B \in \mathscr{J}_{\mathbb{B}} \setminus \mathscr{J}^{-}_{\mathbb{B}}$ with $h \in \overline{\mathscr{D}} \cup Y_1$;

O-F-$\langle (\overline{\mathscr{D}} \cap \mathscr{A}) \cup Y_2 \rangle$: i) $T = \{h\} \cup \{h \leftarrow B\}$ for $h \leftarrow B \in \mathscr{J}_{\mathbb{B}} \setminus \mathscr{J}^{-}_{\mathbb{B}}$ with $b \in \mathbb{B}^{*/-}$ for each $b \in B$; or
ii) $T = \{u\}$ for $u \in ((\overline{\mathscr{D}} \cap \mathscr{A}) \cup Y_2) \setminus (\mathscr{A} \cap \mathbb{B})$.

Different types of dispute advancements for RlFlexDDs are defined as for StFlexDDs, GrFlexDDs, and FlexDDs (i.e., with some abuse of notation, as in Table 2). Only for the termination conditions the definition needs to change slightly (concretely, the notion of "proponent winning") to reflect the change in notation. See Table 11.

Table 11. Termination conditions for RlFlexDDs. TA for admissible, TC for complete, and TS for stable.

Cond.	Proponent winning	Opponent cannot move	Proponent cannot move
TA	$\gamma \cup \overline{\mathscr{C}} \subseteq \mathscr{P}^*$, $(\overline{\mathscr{D}} \cap \mathbb{B}^{*/-}) = \emptyset$	O-B-$\langle \overline{\mathscr{D}} \rangle$ + O-F-$\langle \overline{\mathscr{D}} \cap \mathscr{A} \rangle$-ii or O-F-$\langle \mathscr{A} \rangle$	P-B-$\langle \overline{\mathscr{A}^!} \rangle$ + P-F-$\langle \overline{\mathscr{A}^!} \cap \mathscr{A} \rangle$-ii or P-F-$\langle \mathscr{A} \rangle$
TC	$\gamma \cup \overline{\mathscr{C}} \subseteq \mathscr{P}^*$, $(\overline{\mathscr{D}} \cap \mathbb{B}^{*/-}) = \emptyset$, $\mathscr{I} \setminus \mathscr{D} = \emptyset$	O-B-$\langle \overline{\mathscr{D}} \rangle$ + O-F-$\langle \overline{\mathscr{D}} \cap \mathscr{A} \rangle$-ii or O-F-$\langle \mathscr{A} \rangle$	P-B-$\langle \overline{\mathscr{A}^!} \rangle$ + P-F-$\langle (\overline{\mathscr{A}^!} \cap \mathscr{A}) \cup \mathscr{I} \rangle$ or P-F-$\langle \mathscr{A} \rangle$
TS	$\gamma \cup \overline{\mathscr{C}} \subseteq \mathscr{P}^*$, $(\overline{\mathscr{D}} \cap \mathbb{B}^{*/-}) = \emptyset$, $\mathscr{D} \cup \mathscr{C} = \mathscr{A}$	O-B-$\langle \overline{\mathscr{D}} \rangle$ + O-F-$\langle \overline{\mathscr{D}} \cap \mathscr{A} \rangle$-ii or O-F-$\langle \mathscr{A} \rangle$	P-B-$\langle \overline{\mathscr{A}^!} \rangle$ + P-F-$\langle \mathscr{A} \rangle$

Table 12. A DC + TC (and DS + TS) RlFlexDD for Example 6 (ABA framework from Example 4). Label − represents played blocked pieces, label ∗ complete played pieces, label “ unexpanded statements of the proponent, label ∧ fully expanded statements (non assumptions), and label ! represents opposing pieces. The dispute derivation ends with the opponent not being able to advance further in O-F-$\langle \mathscr{A} \rangle$ manner.

Step and move type	$\mathbb{B}$	$\gamma \cup \overline{\mathscr{C}} \setminus \mathbb{P}^*$	$\mathscr{D}$	$\mathscr{C}$	$\mathscr{I} \setminus \mathscr{D}$
0	$\{ {}^{``\$}p \}$	$\{p\}$	$\{\}$	$\{\}$	$\{a,b,c,d\}$
1 (P-B-i, $p \leftarrow q$)	$\{ {}^{\wedge\$}p;\ {}^{``\$}q;\ {}^{\$}p \leftarrow q \}$	$\{p\}$	$\{\}$	$\{\}$	$\{a,b,c,d\}$
2 (P-B-i, $q \leftarrow a$)	$\{ {}^{**\$}a;\ {}^{**\wedge\$}p;\ {}^{**\wedge\$}q;\ {}^{**\$}p \leftarrow q;\ {}^{**\$}q \leftarrow a \}$	$\{\}$	$\{a\}$	$\{\}$	$\{b,c,d\}$
3 (O-B-ii, $t \leftarrow b$)	$\{ {}^{**\$}a;\ {}^{*!}b;\ {}^{**\wedge\$}p;\ {}^{**\wedge\$}q;\ {}^{*!}t;\ {}^{**\$}p \leftarrow q;\ {}^{**\$}q \leftarrow a;\ {}^{*}t \leftarrow b \}$	$\{\}$	$\{a\}$	$\{\}$	$\{b,d\}$
4 (P-B-ii \ P-F-i, $r \leftarrow p$)	$\{ {}^{**\$}a;\ {}^{-*!}b;\ {}^{**\wedge\$}p;\ {}^{**\wedge\$}q;\ {}^{**\wedge\$}r;\ {}^{*!}t;\ {}^{**\$}p \leftarrow q;\ {}^{**\$}q \leftarrow a;\ {}^{-*}t \leftarrow b;\ {}^{**\$}r \leftarrow p \}$	$\{\}$	$\{a\}$	$\{b\}$	$\{c,d\}$
5 (P-F-ii, c)	$\{ {}^{**\$}a;\ {}^{-*!}b;\ {}^{**\$}c;\ {}^{**\wedge\$}p;\ {}^{**\wedge\$}q;\ {}^{**\wedge\$}r;\ {}^{*!}t;\ {}^{**\$}p \leftarrow q;\ {}^{**\$}q \leftarrow a;\ {}^{-*}t \leftarrow b;\ {}^{**\$}r \leftarrow p \}$	$\{\}$	$\{a,c\}$	$\{b,d\}$	$\{\}$

Example 6. Consider again the ABA framework from Examples 4 and 5. A DC + TC (and DS + TS) RlFlexDD following the DC + TC FlexDD from Table 9 is in Table 12.

Based on the results for FlexDDs from Sect. 3.3 and the fact that RlFlexDDs essentially implement FlexDDs we obtain the following results for RlFlexDDs:

Corollary 7. *$\{DAB, DABF, DC, DS, DF\} + \{TA, TC, TS\}$ RlFlexDDs are sound for the admissible, complete, and stable semantics respectively.*

Corollary 8. *If $\mathscr{L}$ is finite $\{DAB, DABF, DC, DS, DF\} + TA$ RlFlexDDs are complete for credulous acceptance w.r.t. the admissible semantics. Moreover, $\{DC, DF\} + TC$ RlFlexDDs are complete for the complete semantics and $\{DS, DF\} + TS$ RlFlexDDs are complete for the stable semantics. Finally, $\{DC, DS, DF\} + TA$ RlFlexDDs are complete for the admissible semantics.*

5 Implementation

We have implemented an interactive reasoner, `aba-dd-rule-based`, for all variants of RlFlexDDs considered in this work. At the moment the system is conceived mainly for didactic and research purposes. The code is freely available[2]. The system allows for choosing a combination of dispute advancement type and termination criteria and then guiding the user through an RlFlexDD of that nature (advancement types and termination criteria can also be changed on the fly). At each step the user can choose which move to make from the list of allowed moves provided by the system. See Fig. 1 for a screenshot of the interface for step 4 of the RlFlexDD from Table 12. Limited automatisation is also possible in that the user can choose that the system move forward a

[2] https://github.com/gorczyca/aba-dd-rule-based.

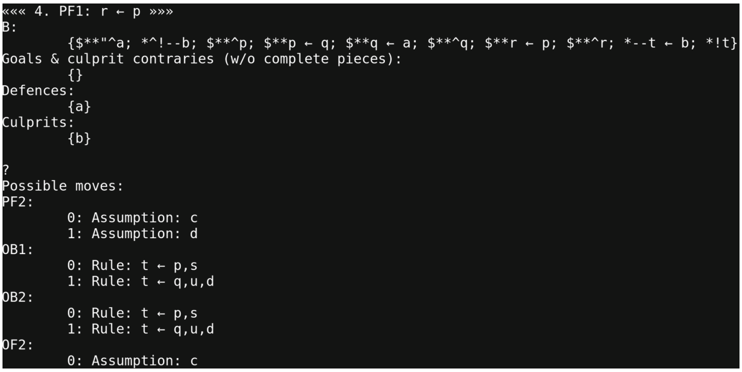

Fig. 1. Interface of `aba-dd-rule-based` at step 4 of the RlFlexDD from Table 12

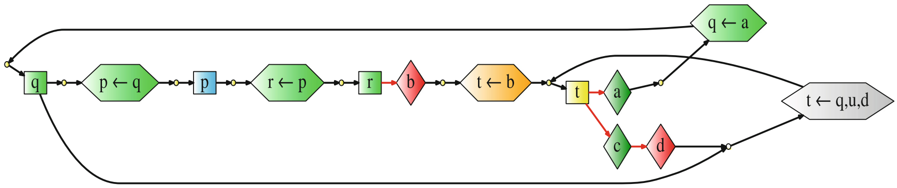

Fig. 2. Graphical output of `aba-dd-rule-based` at step 5 (end state) of the RlFlexDD from Table 12. Here green nodes represents the proponents pieces. The goal is in blue. Yellow is for the opponents pieces, while those in orange are blocked. Red is for culprits. Grey nodes represent remaining rules which are blocked but whose heads have been made use of in the dispute. Black arrows represent support, while red ones denote attacks. (Color figure online)

random number of steps (of some type) and the system can also backtrack a number of steps. The system can also produce a graphical representation of the statements and rules put forward during the dispute until that point. See Fig. 2 for the graphical output of `aba-dd-rule-based` at step 5 (end state) of the RlFlexDD from Table 12. We refer to the webpage for many other features of `aba-dd-rule-based` as well as larger and more realistic examples on which to experiment with RlFlexDDs.

6 Conclusions and Future Work

We have defined a variant of dispute derivations which allows for forward in addition to backward reasoning and thus for computing admissible, complete, and stable assumption sets in addition to reasoning about credulous acceptance of statements. We have given an abstract argument-based definition of such dispute derivations which we have derived from similarly abstract representations of flexible variants of dispute derivations from [10, 33]. We have then provided a more implementation focused rule-based definition. For this version we also implemented an interactive system. Ultimately, we have generalised graph-based dispute derivations from [10] on two fronts: incorporating forward reasoning, as well as in that both the proponents and opponents arguments are represented in a shared graph rather than only the proponents arguments being represented in a graph.

While the most immediate benefits of dispute derivations lie in the domain of interactive reasoning, investigating to what extent some of the variants of dispute derivations we have defined in this work can be turned into fully automated reasoning procedures (even if only to support interactive reasoning) is of interest. In particular, DABF + TA is an obvious candidate for obtaining more efficient procedures for credulous reasoning. Also, with forward reasoning giving at least argument-based flexible dispute derivations for non-flat ABA as well as for sceptical acceptance of statements should be in reach. We also would like to improve our interactive system, in particular for making the whole interface graphical and also for allowing switching between argument-based and rule-based views.

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