Putting Perspective into OWL [sic]:

Complexity-Neutral Standpoint Reasoning for Ontology Languages via Monodic S5 over Counting Two-Variable First-Order Logic

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Inria
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Motivation

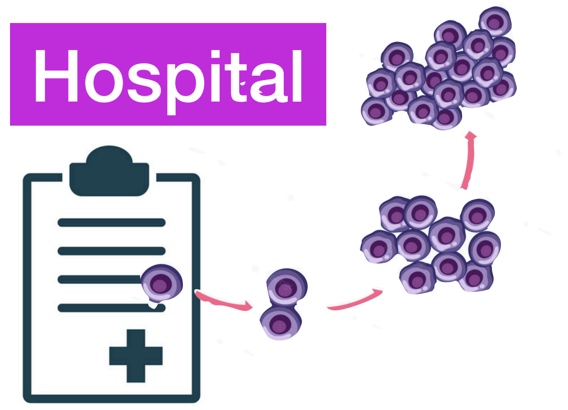
Multiperspective Reasoning

Motivation

Non-trivial combinations of the huge diversity of knowledge sources available
Knowledge sources embed the perspectives of their creators!

SNOMED CT

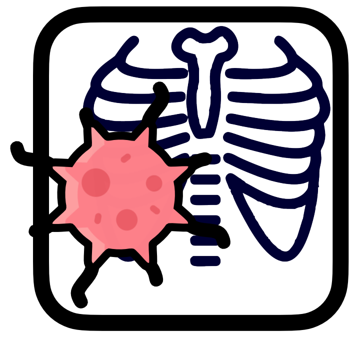
Hospital



WIKIDATA

yago select knowledge

Private Lab



DBpedia

Diverse Knowledge Sources

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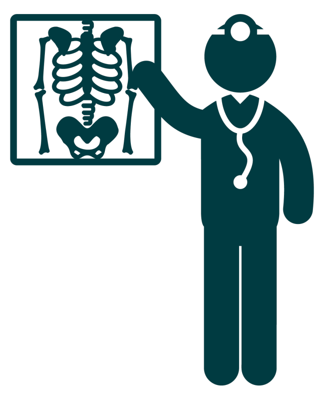

Tumor(t)
hasTissue(p, t)

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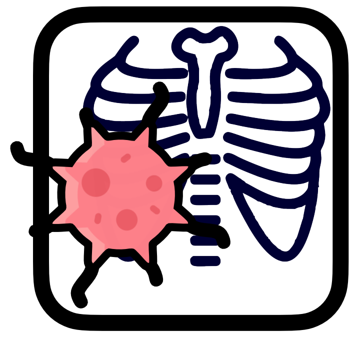

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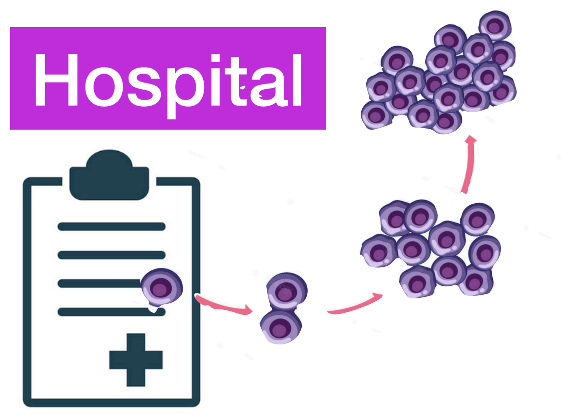

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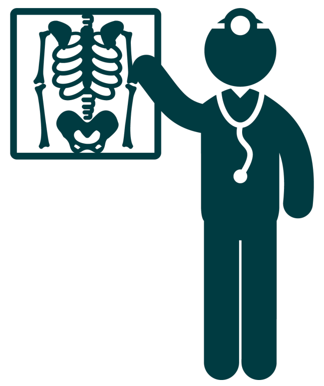

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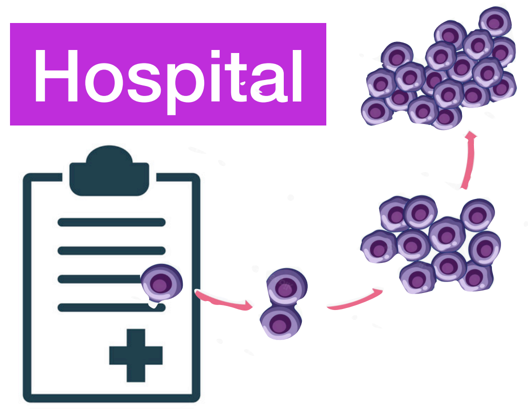

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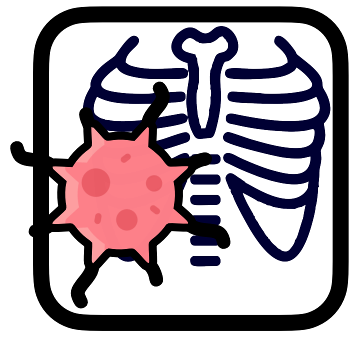

HighRisk(p)

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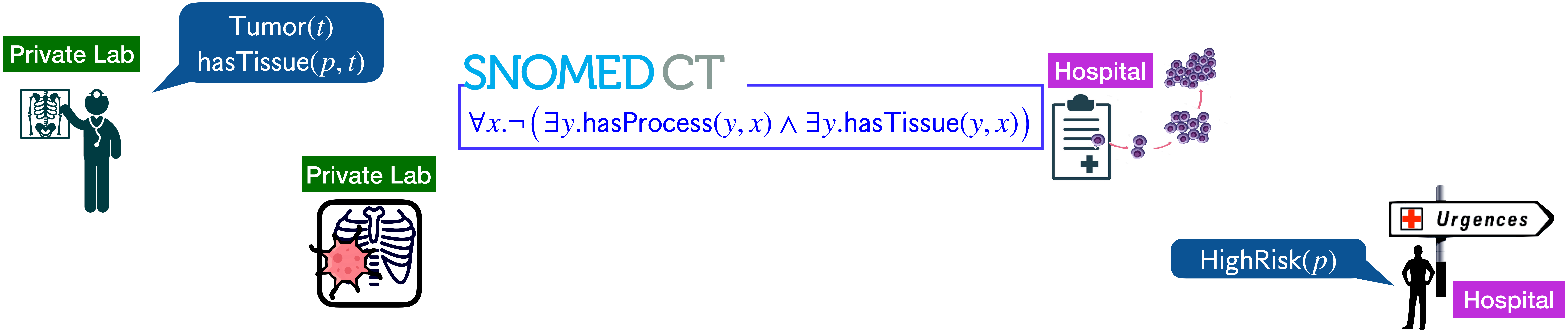

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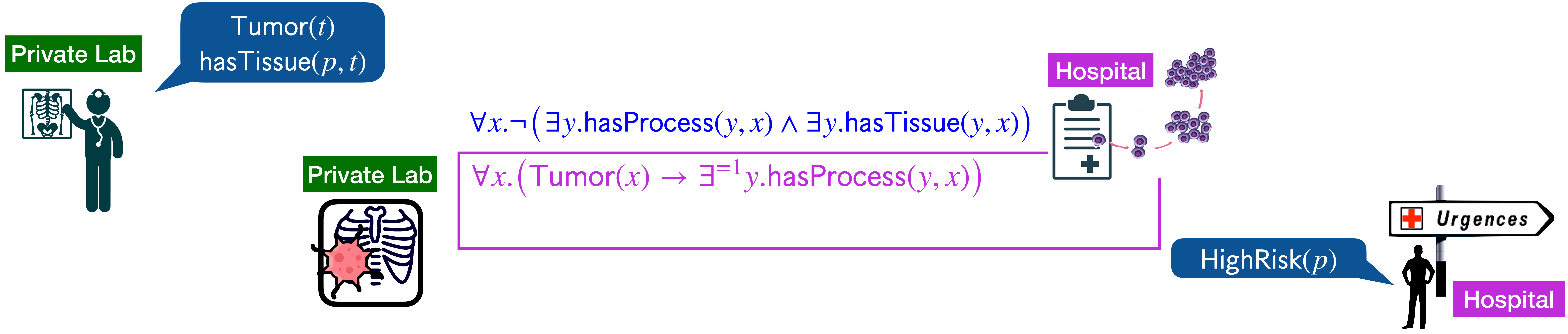

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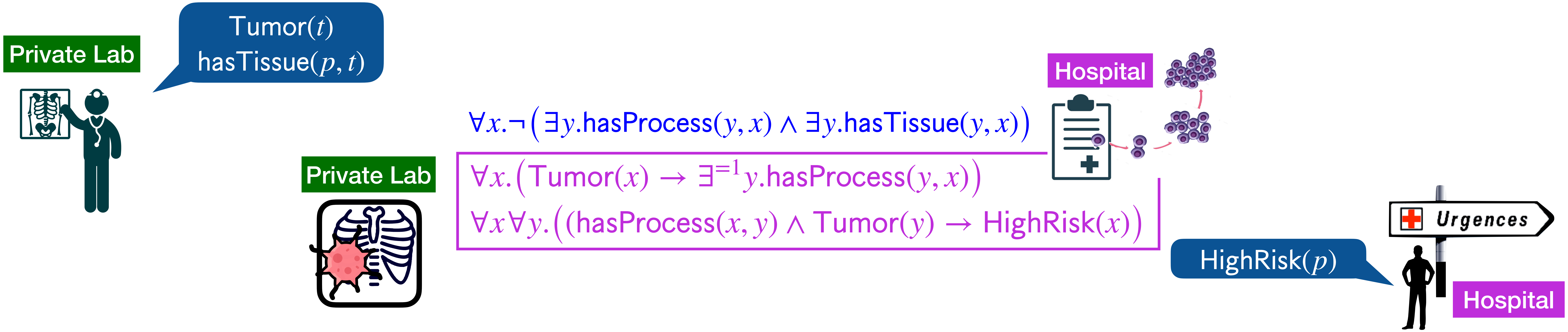

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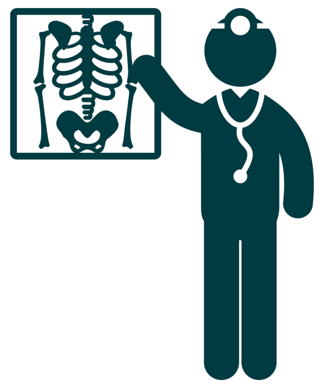

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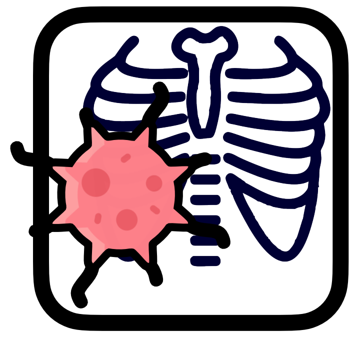

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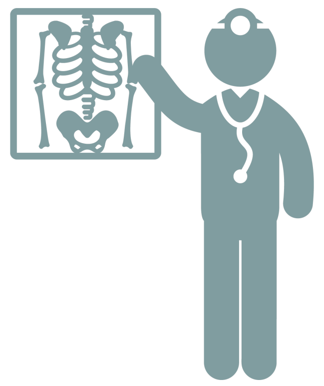

Knowledge Fusion

$$\forall x.\neg\big(\exists y.\mathsf{hasProcess}(y,x) \land \exists y.\mathsf{hasTissue}(y,x)\big)$$

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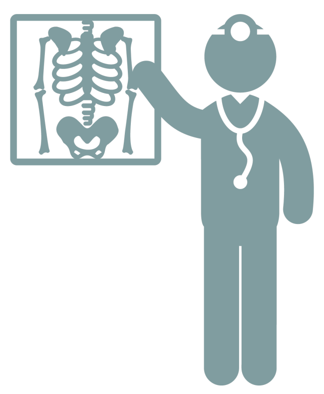

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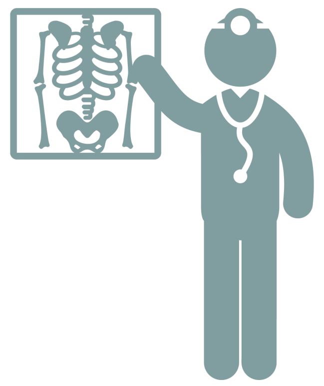

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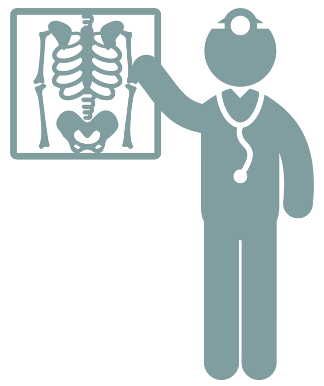

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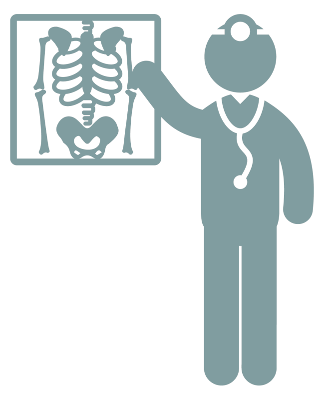

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Multiperspective Knowledge Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

➡ **Multimodal logic** characterised by simplified Kripke semantics

➡ Knowledge relative to “points of view” (standpoints)

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$\Box_e$ Unequivocal according to e

$\Diamond_e$ Conceivable according to e

CONSISTENT

$\Box_* \forall x.\neg\big(\exists y.\mathrm{hasProcess}(y,x) \wedge \exists y.\mathrm{hasTissue}(y,x)\big)$

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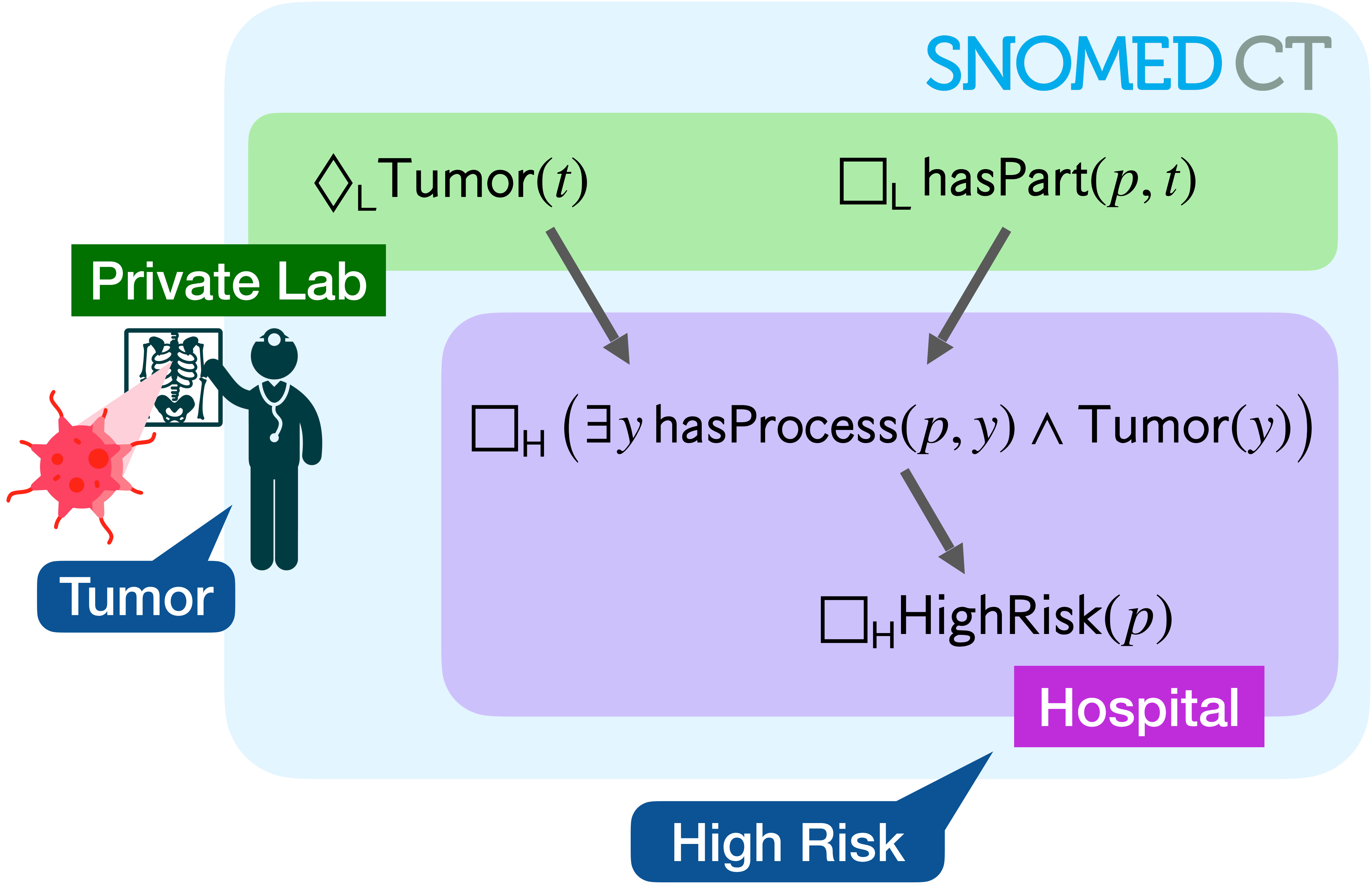

Contents

- ➡ Motivation for the framework (DONE)
- ➡ First Order Standpoint Logic and Monodic Standpoint C^2 (THE LANGUAGE)
- ➡ Transformations (PREPROCESSING)
- ➡ Satisfiability in Monodic Standpoint C^2 (MAIN TECHNICAL RESULT)
- ➡ Application to Ontology Languages (OWL)
- ➡ Nominals Cause Trouble
- ➡ Final Observations and Conclusion

First-Order Standpoint Logic
(Syntax and Semantics)

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

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Syntax of $\mathbb{S}$

Signature $\langle \mathbf{P}, \mathbf{C}, \mathbf{S} \rangle$ of predicates, constants and standpoints.

First-Order Standpoint Logic: Syntax

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Signature $\langle \mathbf{P}, \mathbf{C}, \mathbf{S} \rangle$ of predicates, constants and standpoints.

FOSL formulas:

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid t_1 \doteq t_2 \mid \neg\phi \mid \phi \land \psi \mid \exists^{\triangleleft n} x.\phi \mid \lozenge_{\mathsf{e}}\phi$$

- $\lozenge_{\mathsf{e}}\phi \longrightarrow$ "To *e*, it is **conceivable** that ϕ"

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counting quantifiers with $\triangleleft \in \{\leq, =, \geq\}$,

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$\Diamond_{\mathrm{e}}\neg\phi \equiv \neg\Box_{\mathrm{e}}\,\phi$ (dual)

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Standpoint expressions:

$$\mathrm{e}_1, \mathrm{e}_2 ::= * \mid s \mid \mathrm{e}_1 \cup \mathrm{e}_2 \mid \mathrm{e}_1 \cap \mathrm{e}_2 \mid \mathrm{e}_1 \backslash \mathrm{e}_2$$

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A formula is:

- **Monodic** : at most one free variable in each subformula of the form $\Diamond_{\mathrm{e}} \phi$ (or $\Box_{\mathrm{e}} \phi$)

$\forall x.\mathrm{Tissue}(x) \rightarrow \Box_{*} \mathrm{Tissue}(x)$ ✅

$\forall x \forall y.\mathrm{hasTissue}(x, y) \rightarrow \Box_{*} \mathrm{hasTissue}(x, y)$ ❌

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- C^2 : only 2 variables, all predicates of arity ≤ 2

  $\Box_{*}\, \forall x \forall y.\mathsf{hasTissue}(x, y) \rightarrow \exists x.\mathsf{hasCell}(y, x)$ ✅

  $\Box_{*}\, \forall x \forall y.\mathsf{hasTissue}(x, y) \rightarrow \exists z.\mathsf{hasOrgan}(x, z) \wedge \mathsf{from}(y, z)$ ❌

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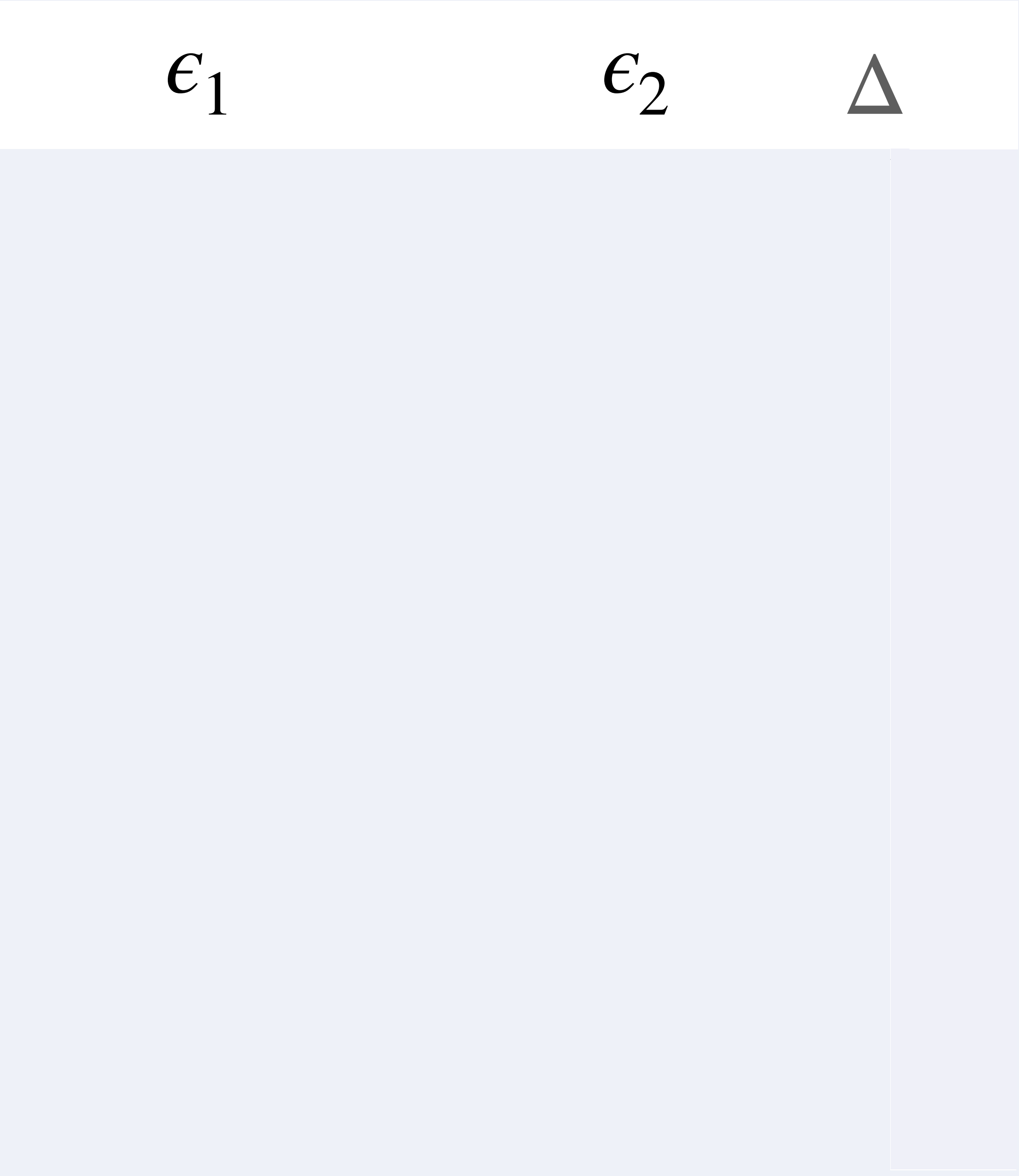

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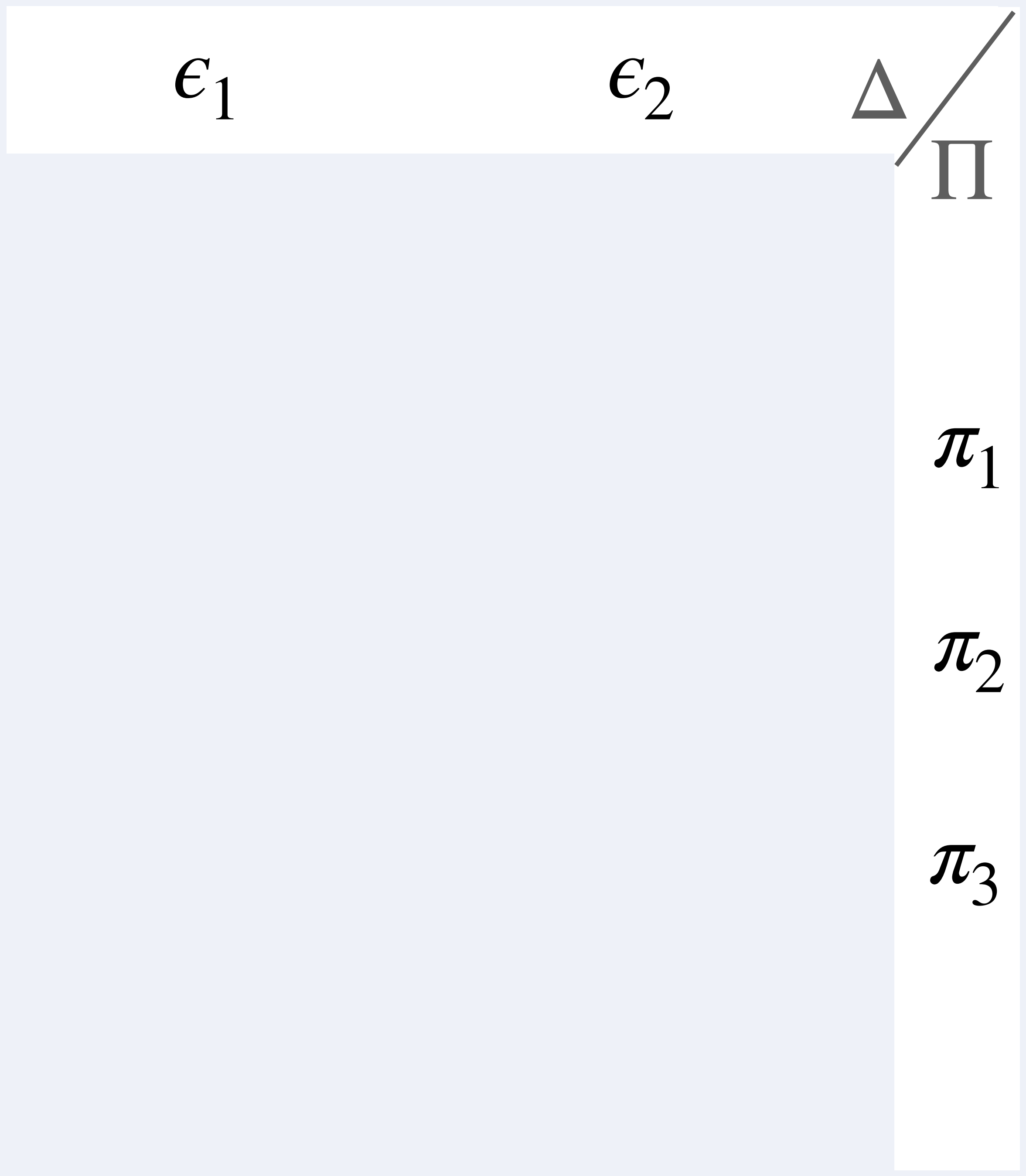

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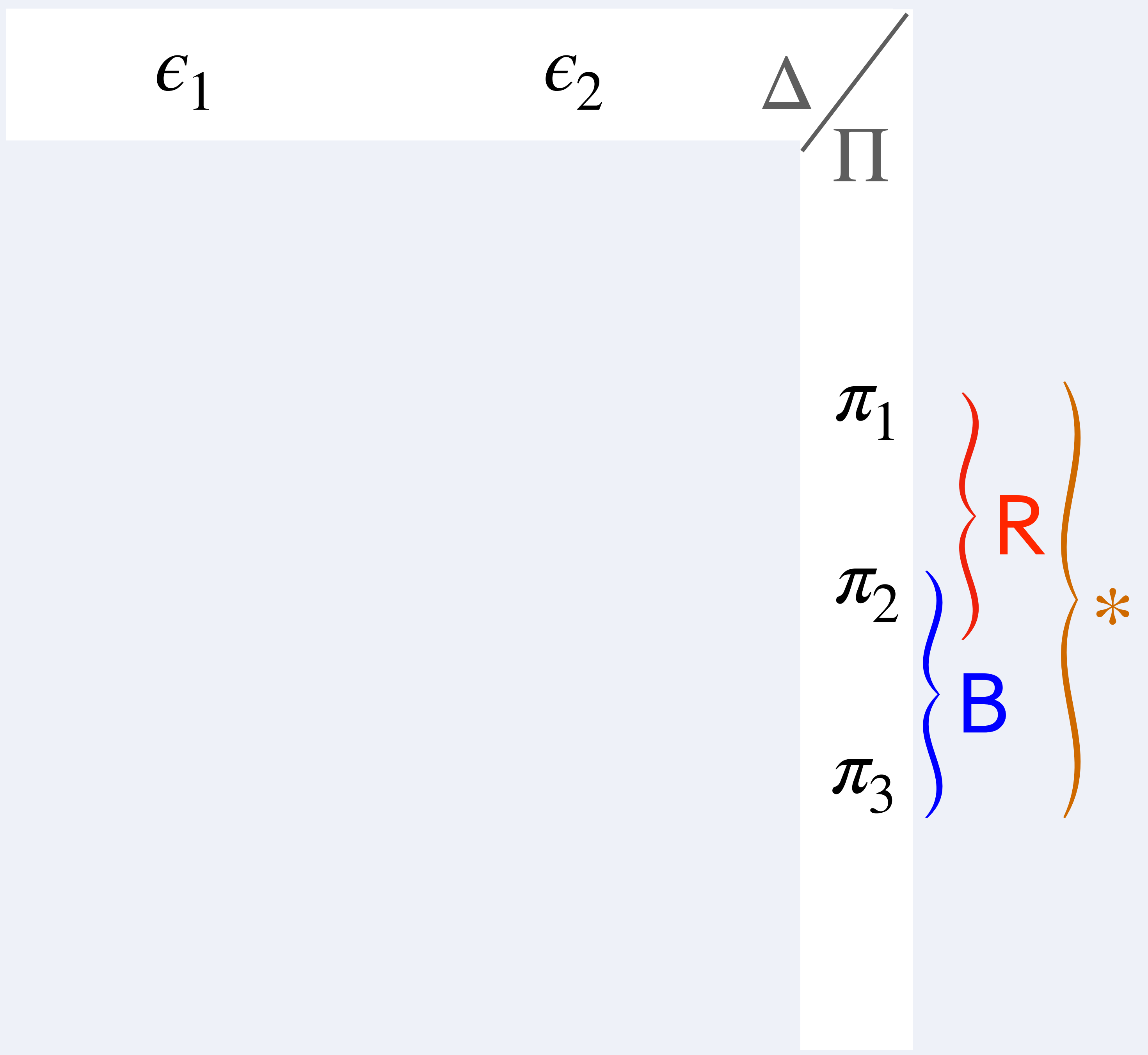

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- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$ standpoint
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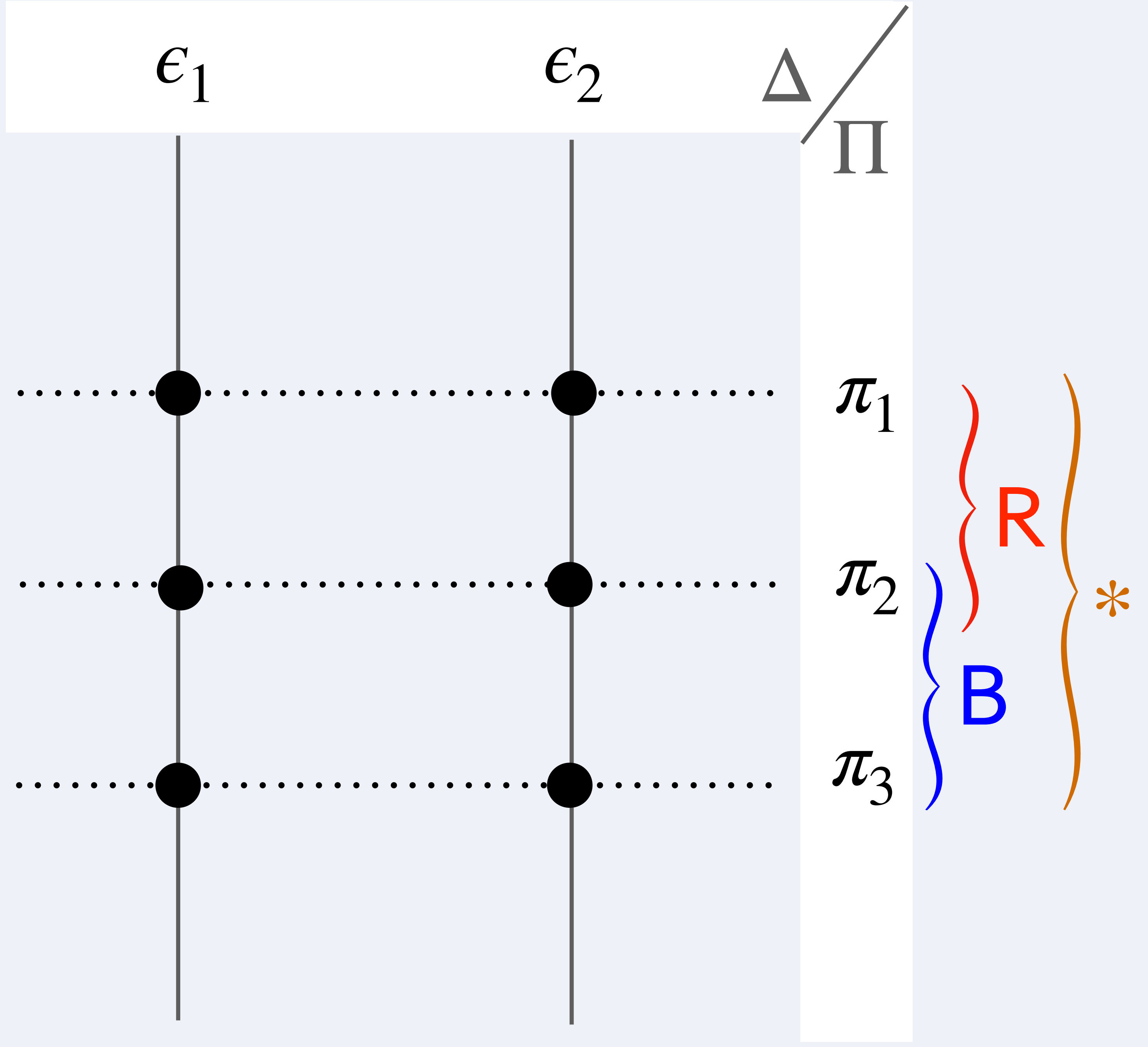
- $\gamma(\pi_1) = \{p \mapsto \emptyset, \ldots\}$ interpretation assignment to worlds



First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

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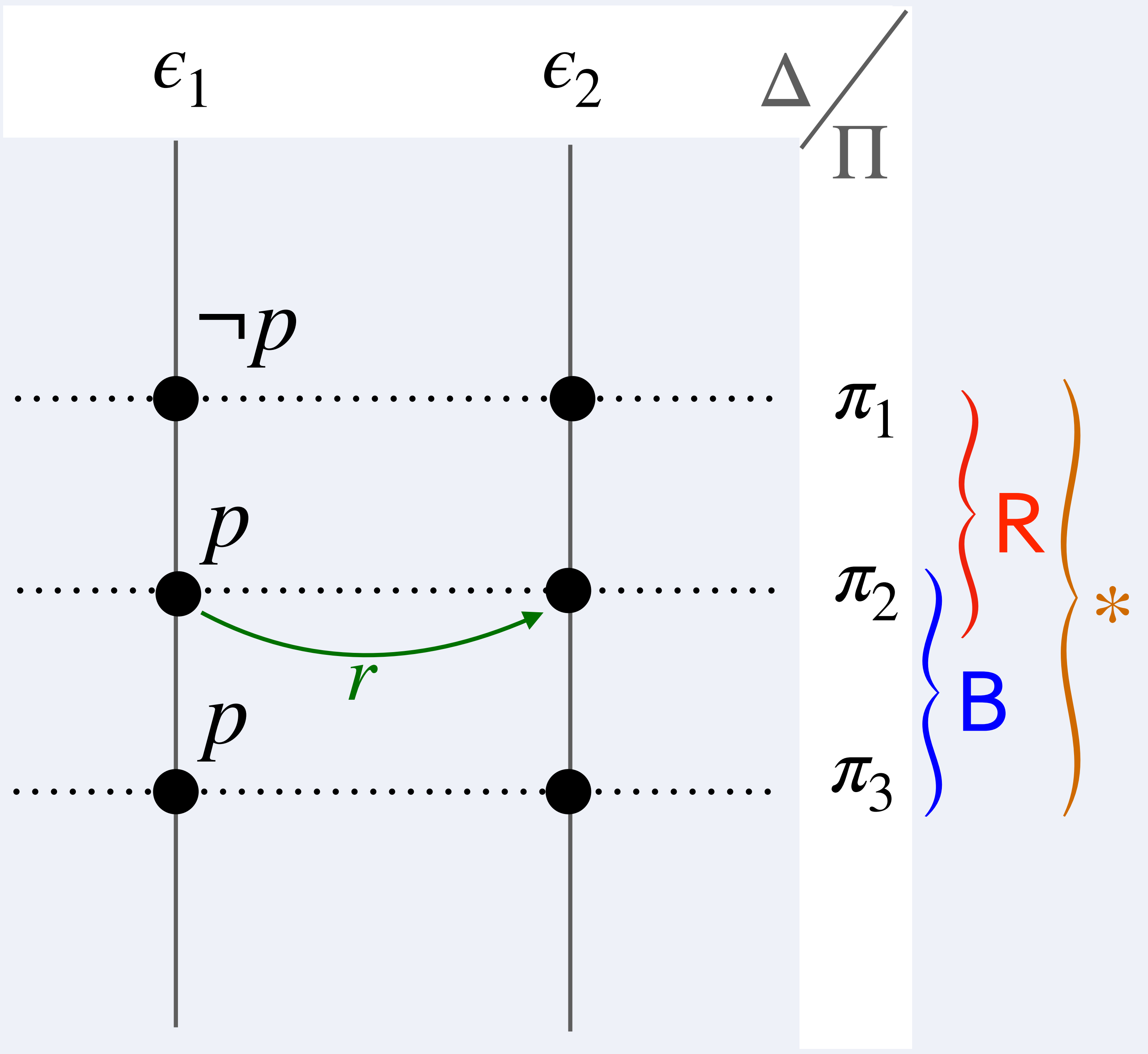

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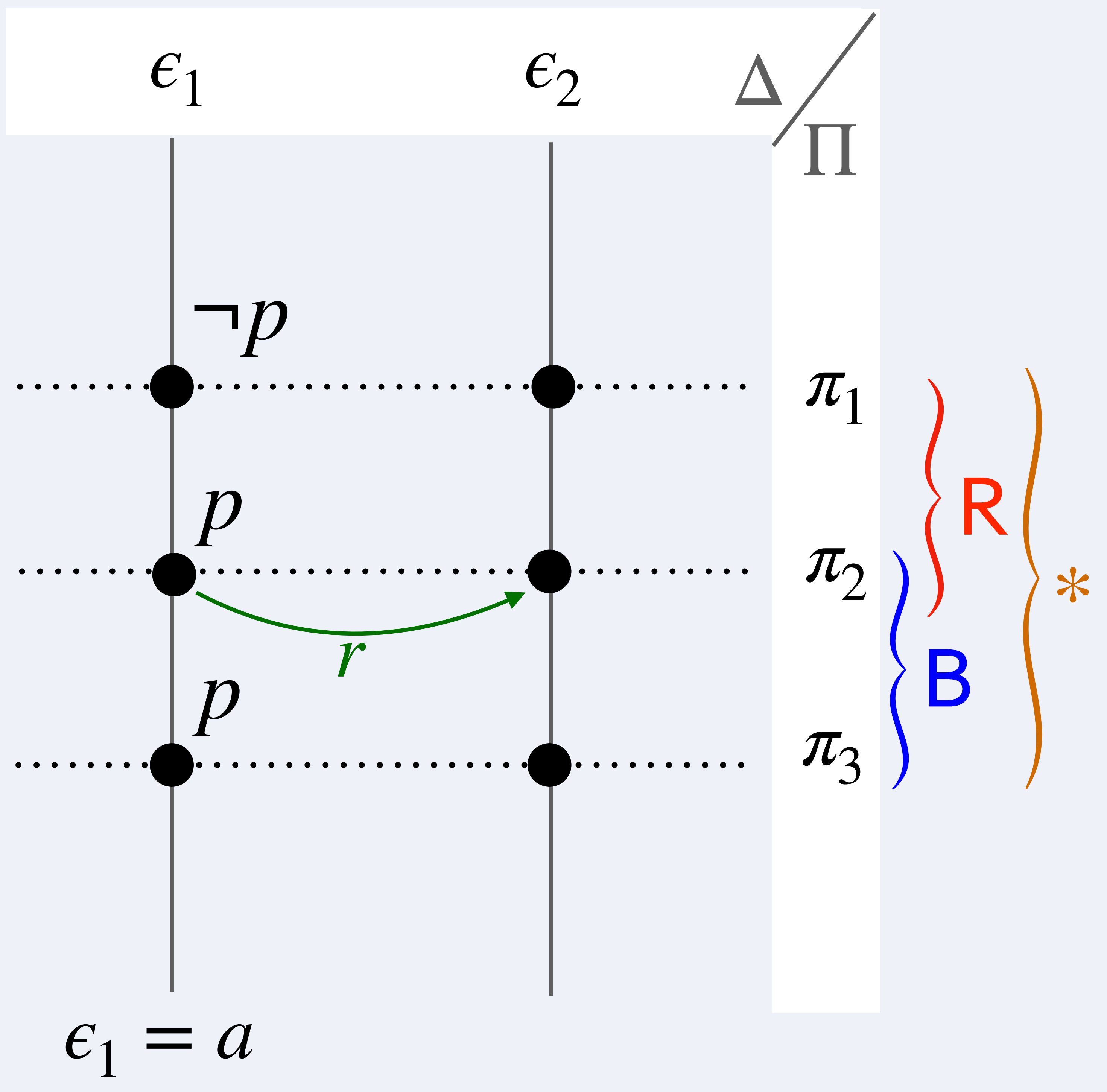

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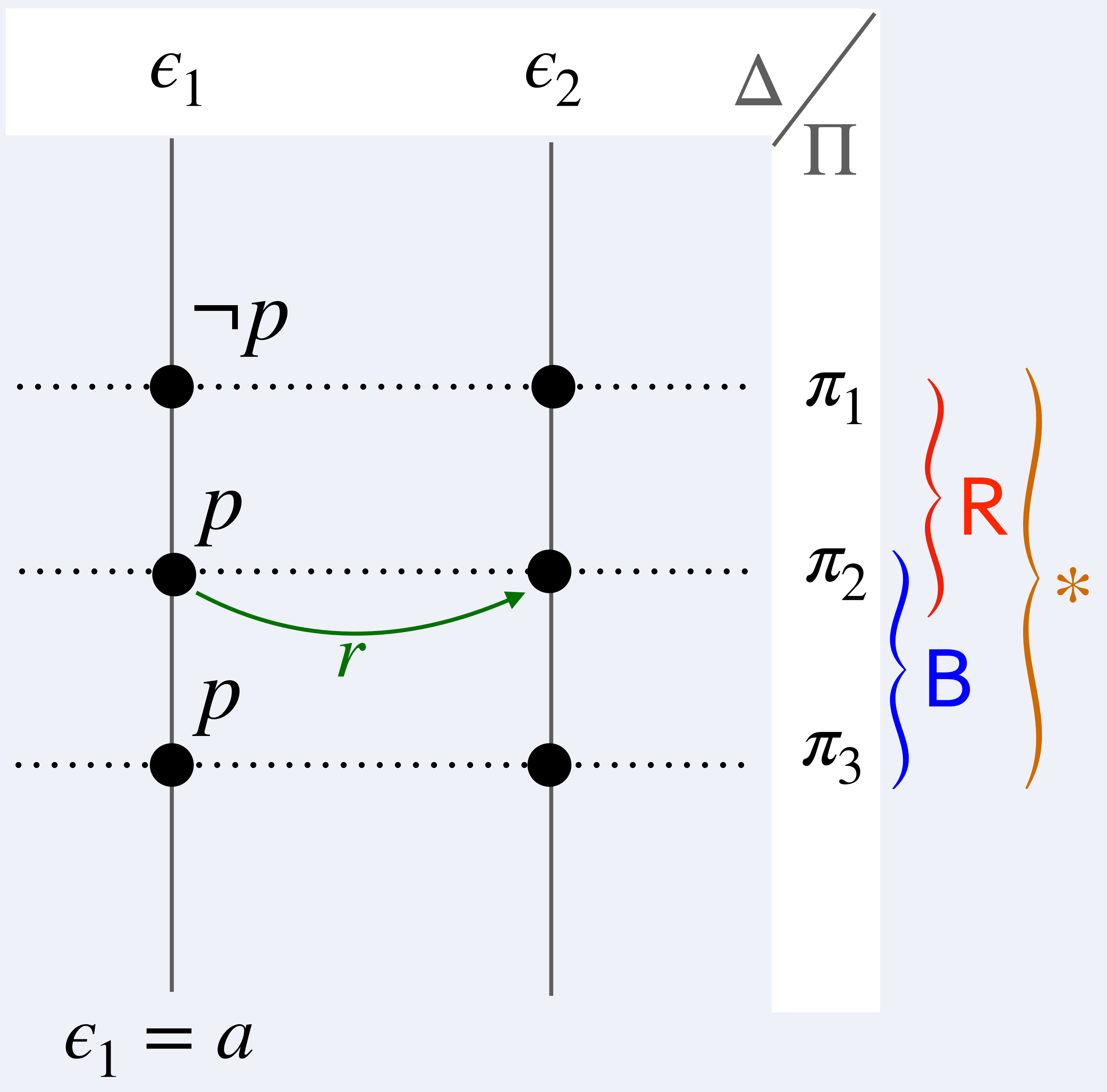

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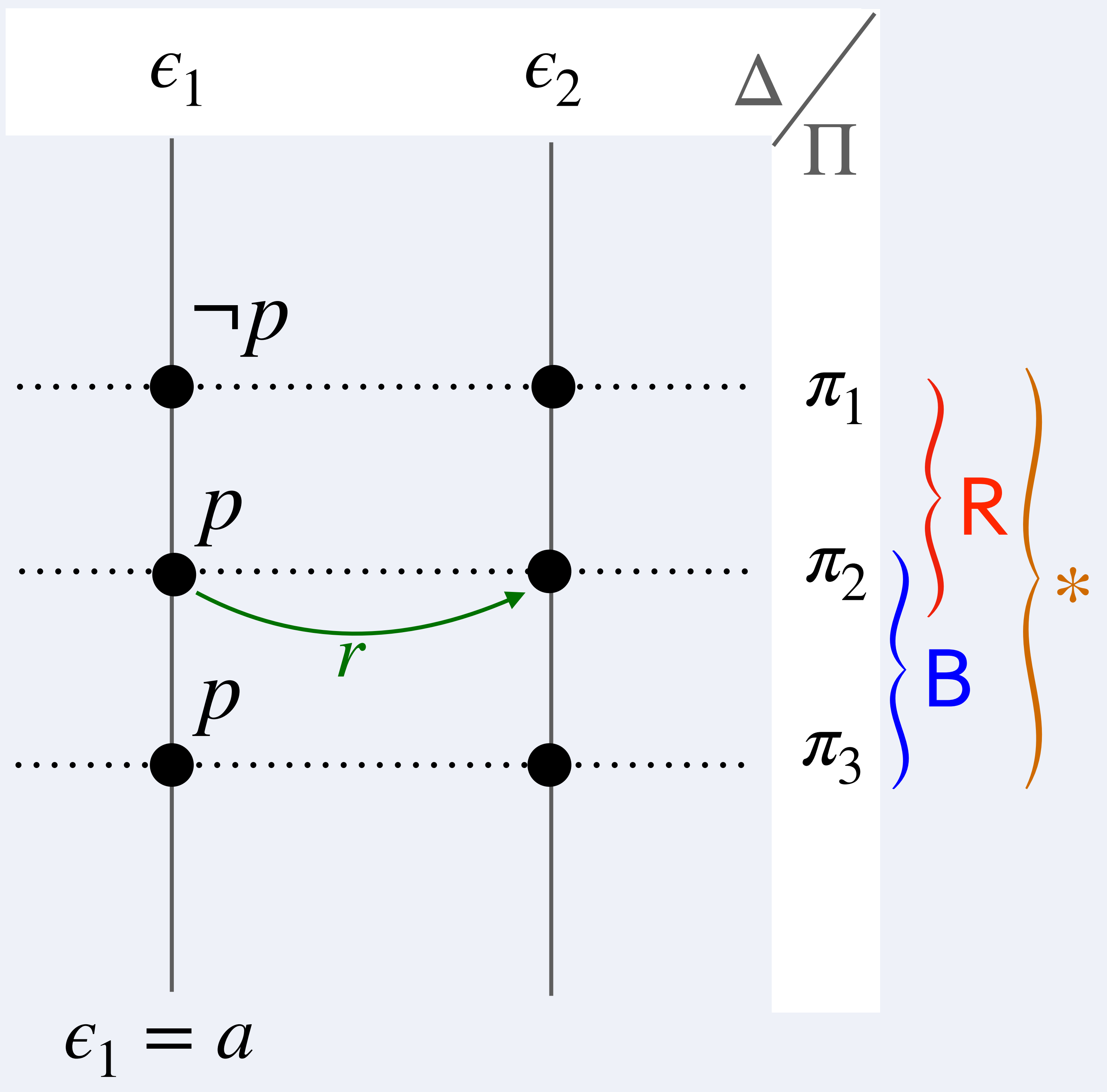

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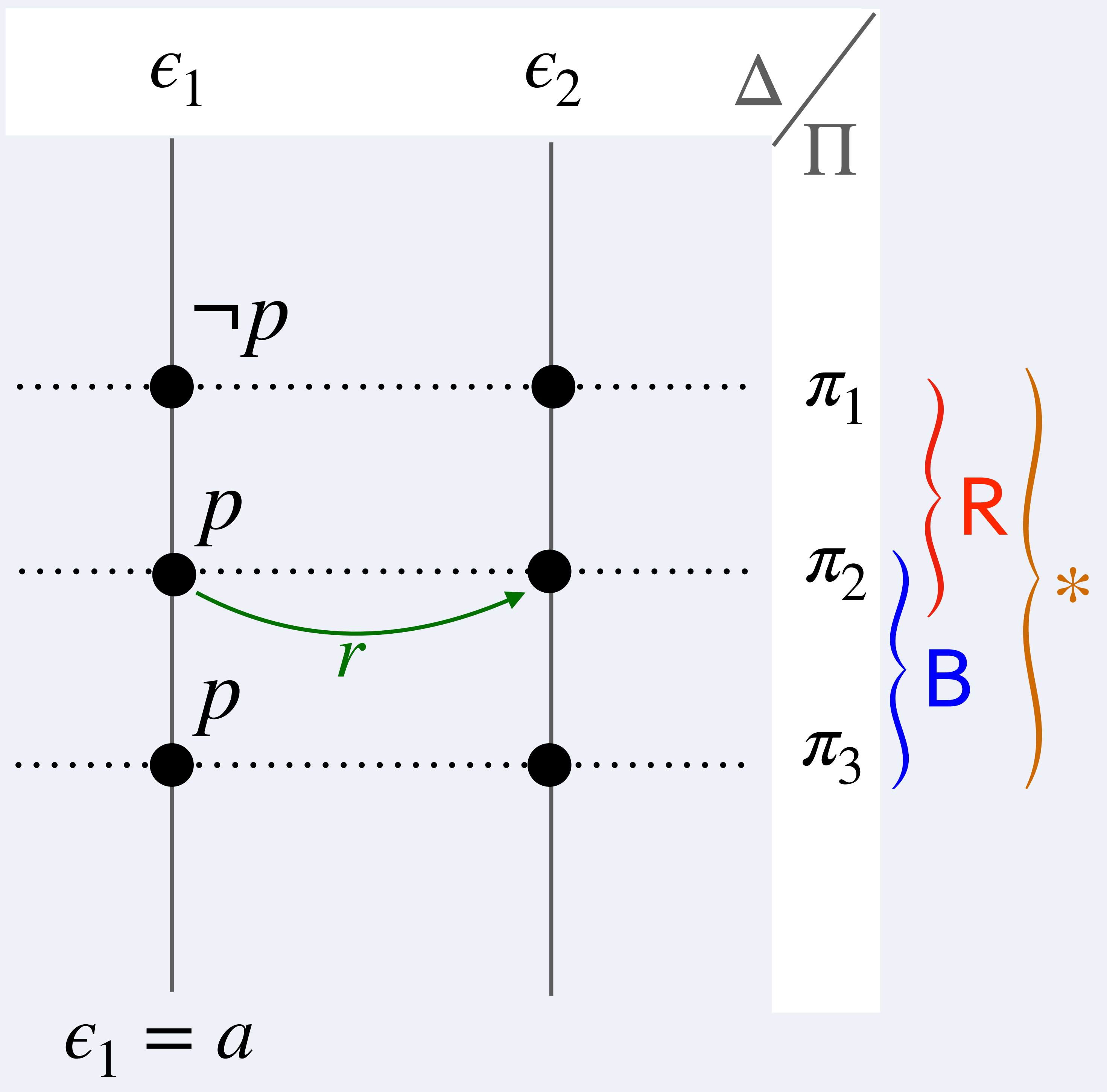

Transformations

Stepwise Translation

monodic C^2 FOSL → <u>nullary-</u> and <u>constant-free</u> <u>S5</u> monodic C^2 FOSL

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$$\exists x.\Box_{\mathsf{L}}\left(\mathsf{Tumor}(x) \wedge \mathsf{hasTissue}(John, x)\right)$$

⬇

$$\exists x.\Box_{*}\left(\mathsf{L} \rightarrow (\mathsf{Tumor}(x) \wedge \mathsf{hasTissue}(John, x))\right)$$

S5: no standpoint expressions except *

Simulate standpoint expressions by marking worlds with nullary predicates.

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monodic C^2 FOSL → nullary- and constant-free S5 monodic C^2 FOSL

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↓

$$\exists x.\square_{*} \big(\mathsf{L} \to (\mathsf{Tumor}(x) \land \mathsf{hasTissue}(John, x))\big)$$

↓

$$\exists x.\square_{*} \big(\forall x.(\mathsf{L}'(x)) \to (\mathsf{Tumor}(x) \land \mathsf{hasTissue}(John, x))\big)$$

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$$\exists x.\Box_{*} \left(\forall x.(\mathsf{L}'(x)) \rightarrow \left(\mathsf{Tumor}(x) \wedge \exists y.(\mathsf{hasTissue}(y, x) \wedge \mathsf{John}(y))\right)\right)$$
$$\wedge\ \exists^{=1} x\,.\,\mathsf{John}(x) \wedge \exists^{=1} x\,.\,\Box_{*}\,\mathsf{John}(x)$$

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Easy: Simulate nullary predicates by immediately quantified unary ones.

constant-free: no constants

Simulate constants by unary predicates (axiomatising uniqueness and rigidity).

Satisfiability in Monodic Standpoint C^2

Context

➡ C^2 is NExpTime-complete

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- ➡ for other logics' standpoint extensions, we often relied on existence of models with few worlds ("small model property")
- ➡ yet, here we can enforce infinitely many worlds:

$$\Box_* \forall x. \Big(\big(\mathsf{First}(x) \leftrightarrow \neg \exists y\, \mathsf{Next}(y,x)\big) \wedge \exists^{\leq 1} y\, \big(\mathsf{Next}(y,x)\big) \wedge \exists^{=1} y \big(\mathsf{Next}(x,y)\big) \Big)$$

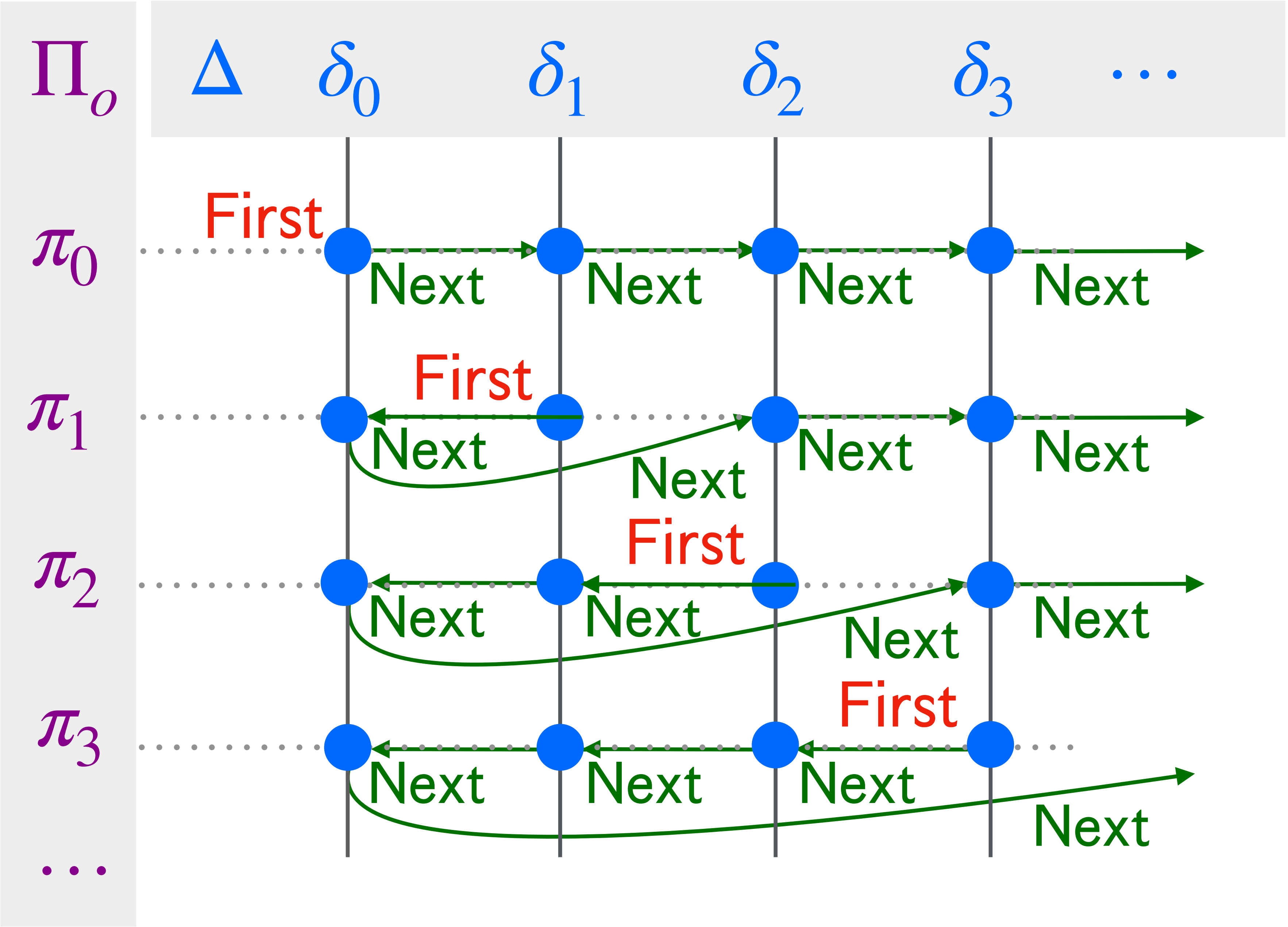
infinite domain

$$\forall x. \Diamond_* \big(\mathsf{First}(x)\big)$$

every entity is first in some world

$$\Box_* \exists^{=1} x. \big(\mathsf{First}(x)\big)$$

only one first entity per world



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(nullary- and constant-free S5)

monodic C^2 FOSL formula

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FOSL
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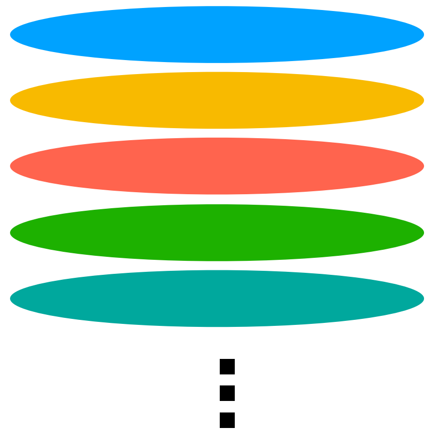
satisfied in ↓

FOSL Structure

(possibly infinite, irregular)



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satisfied in ← iff → satisfied in

FOSL Structure	FOSL Structure
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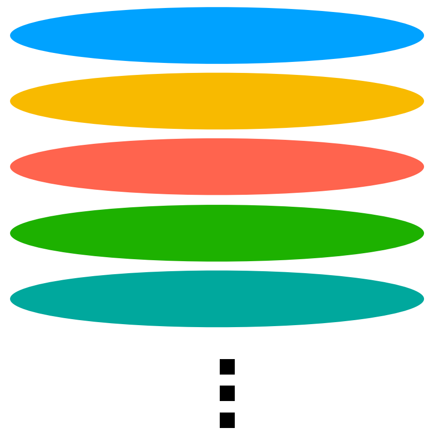

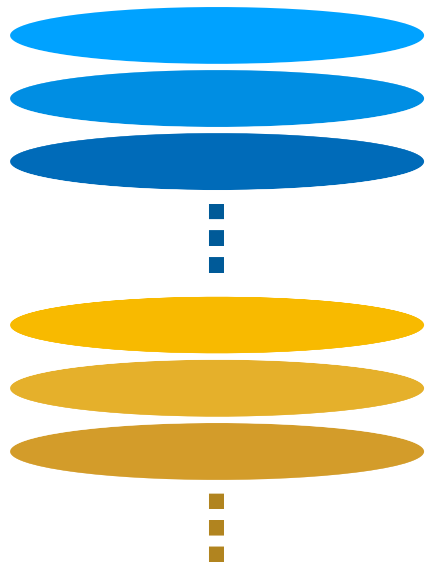

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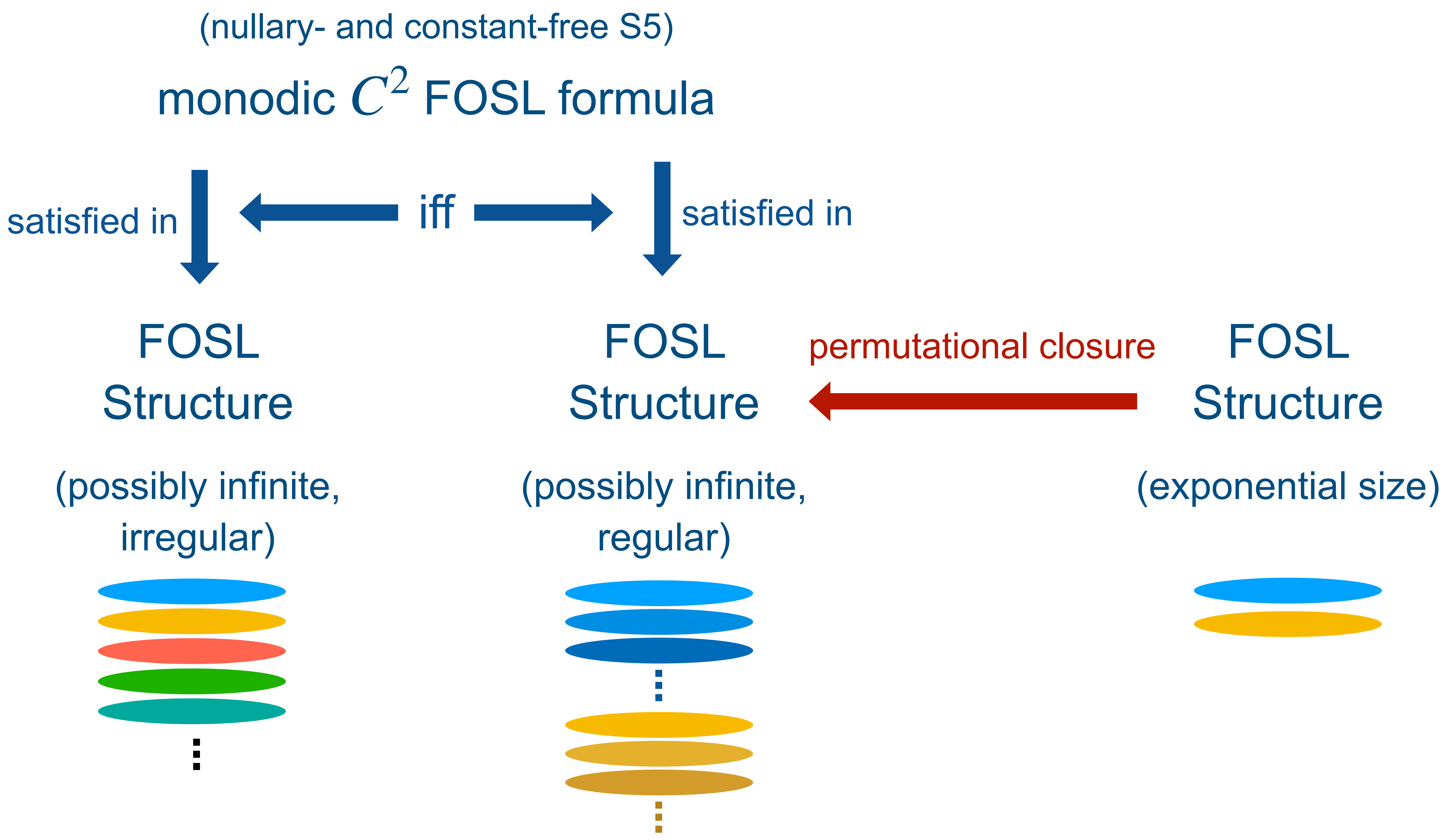

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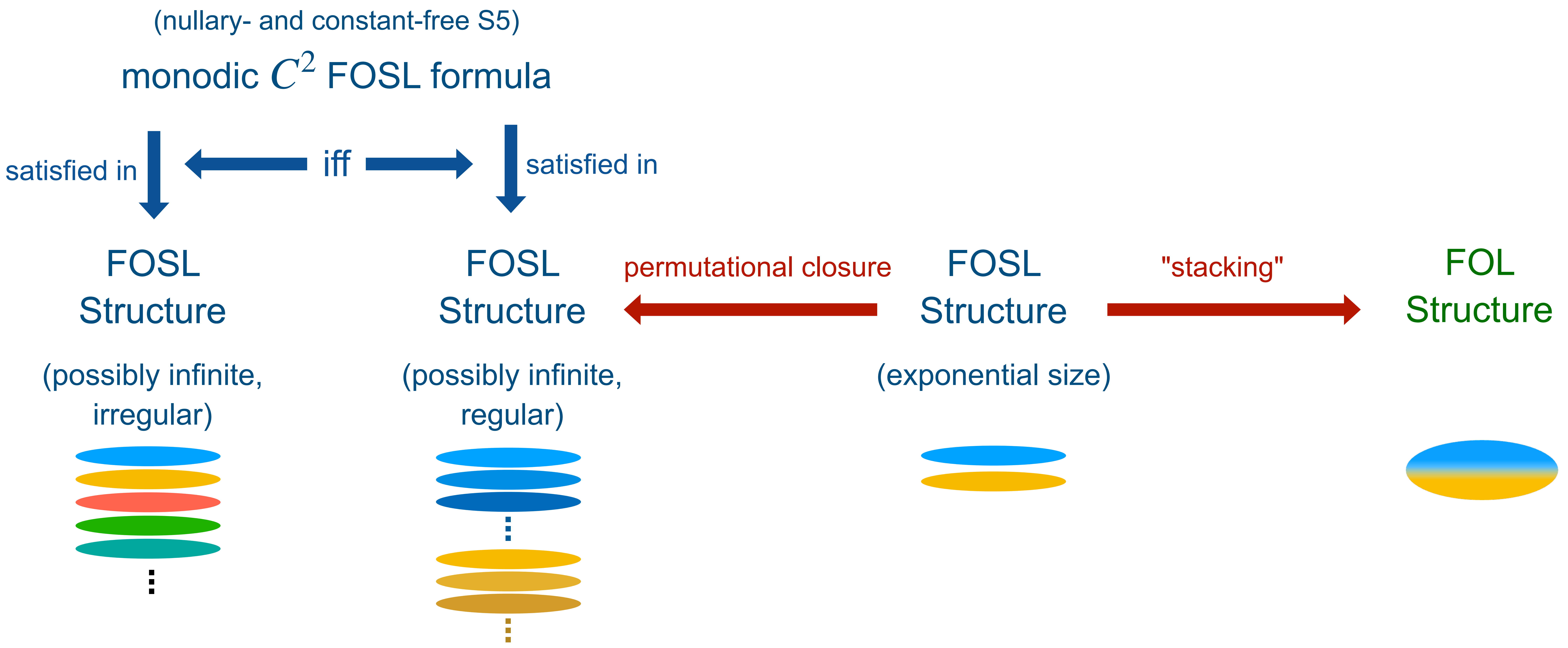

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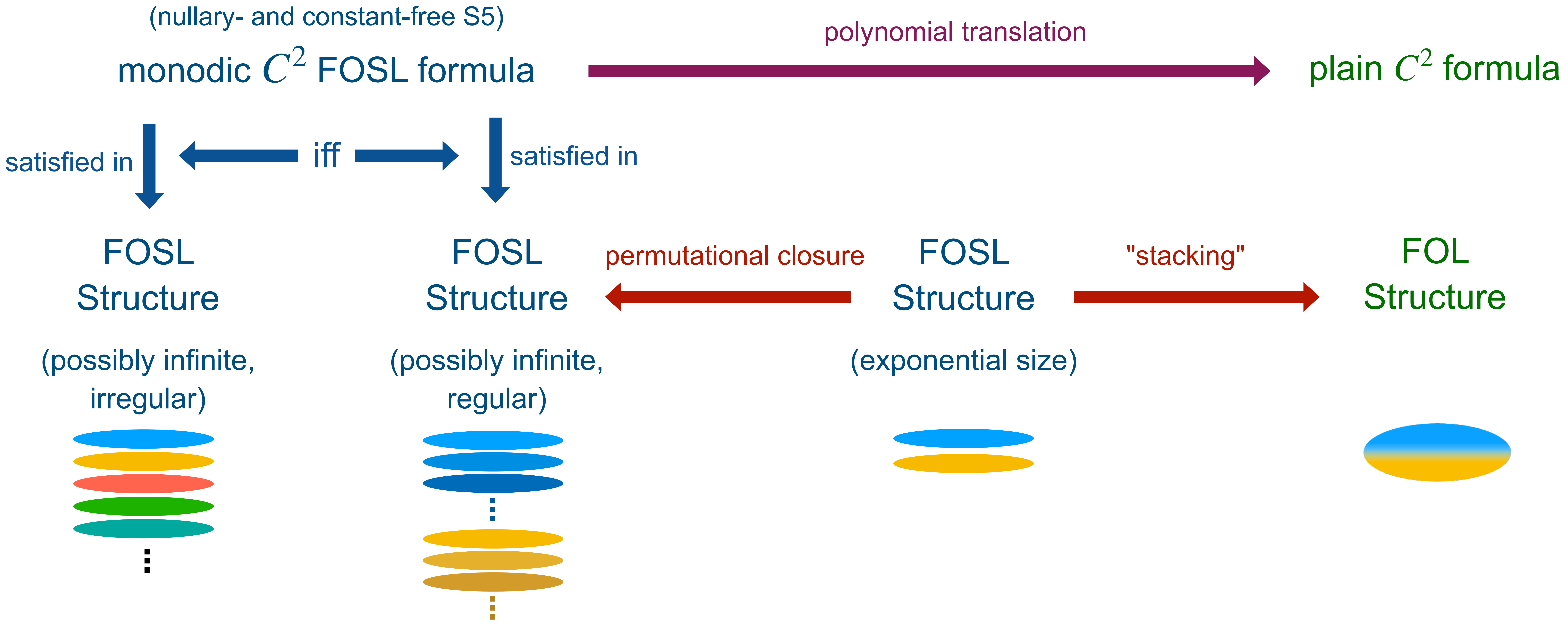

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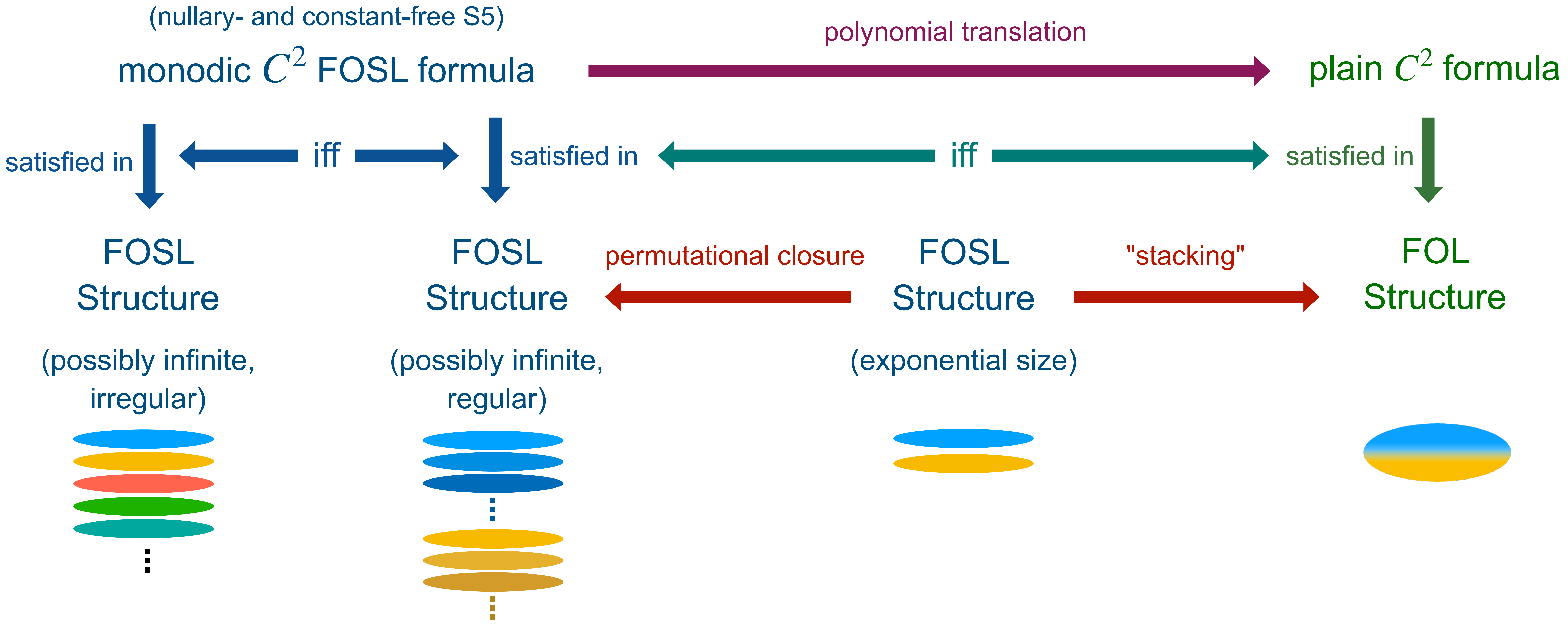

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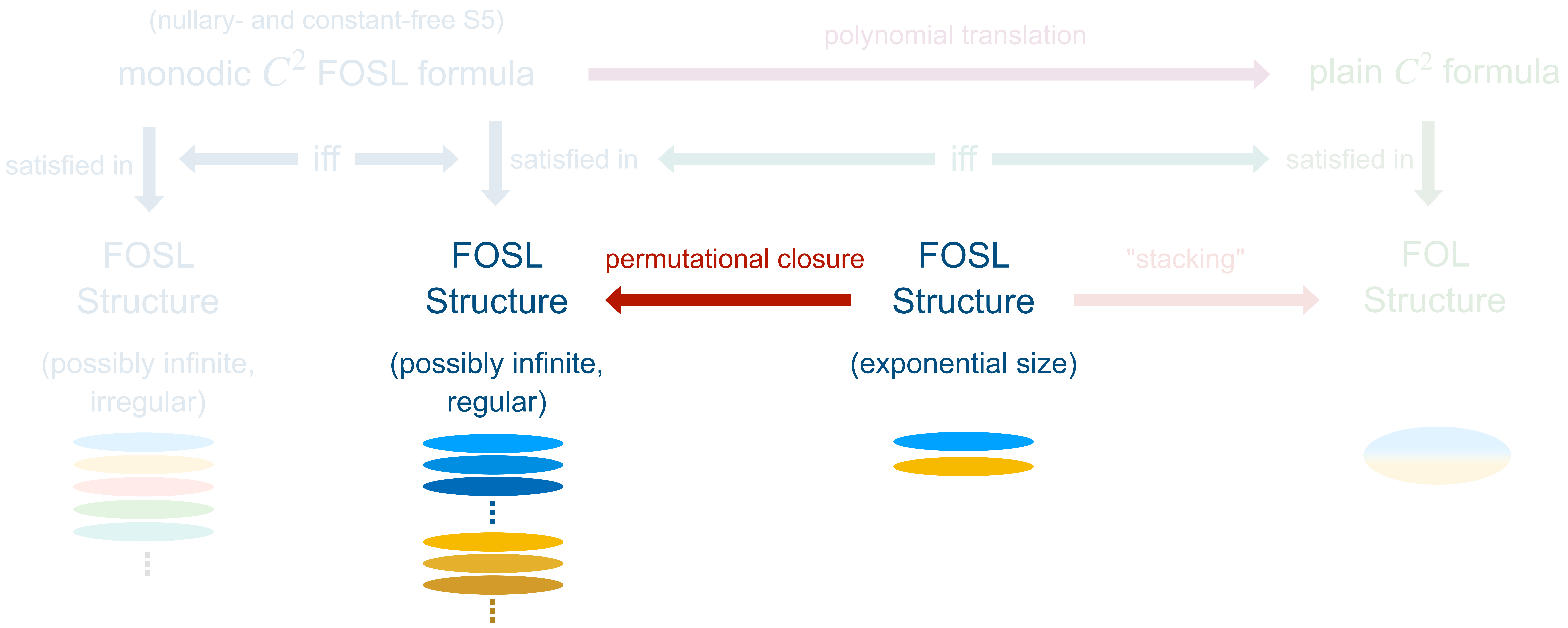
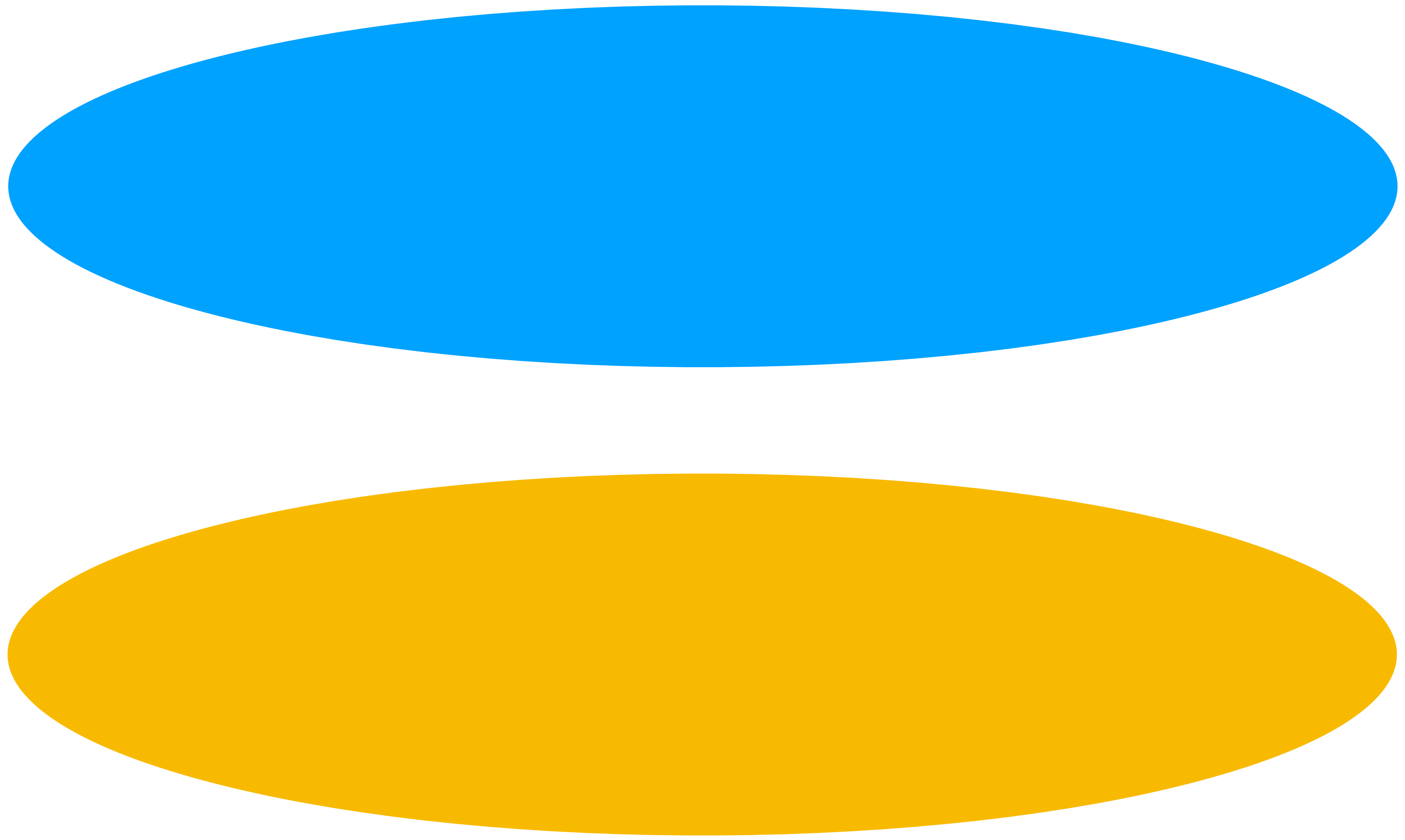


Permutational Closure



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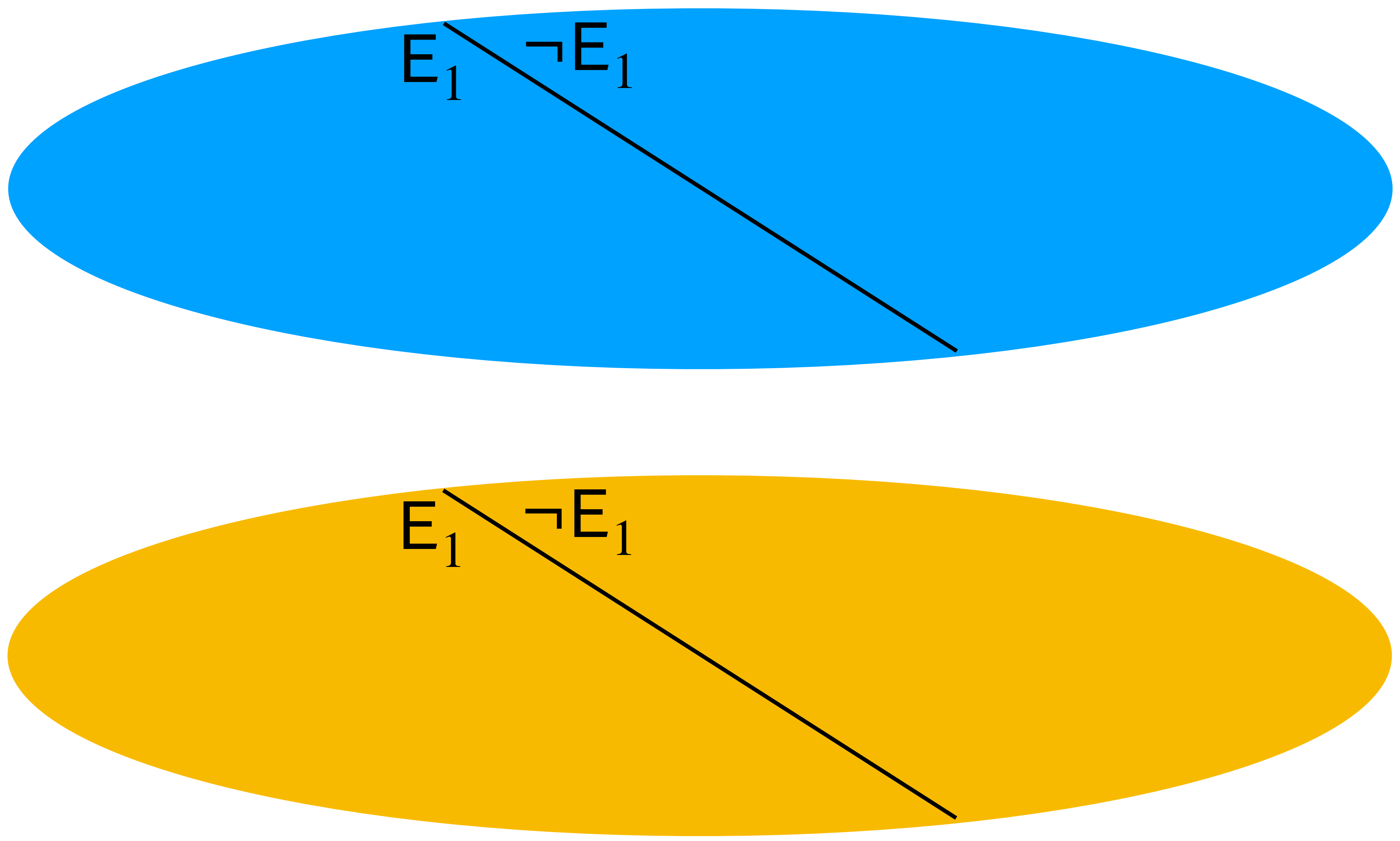

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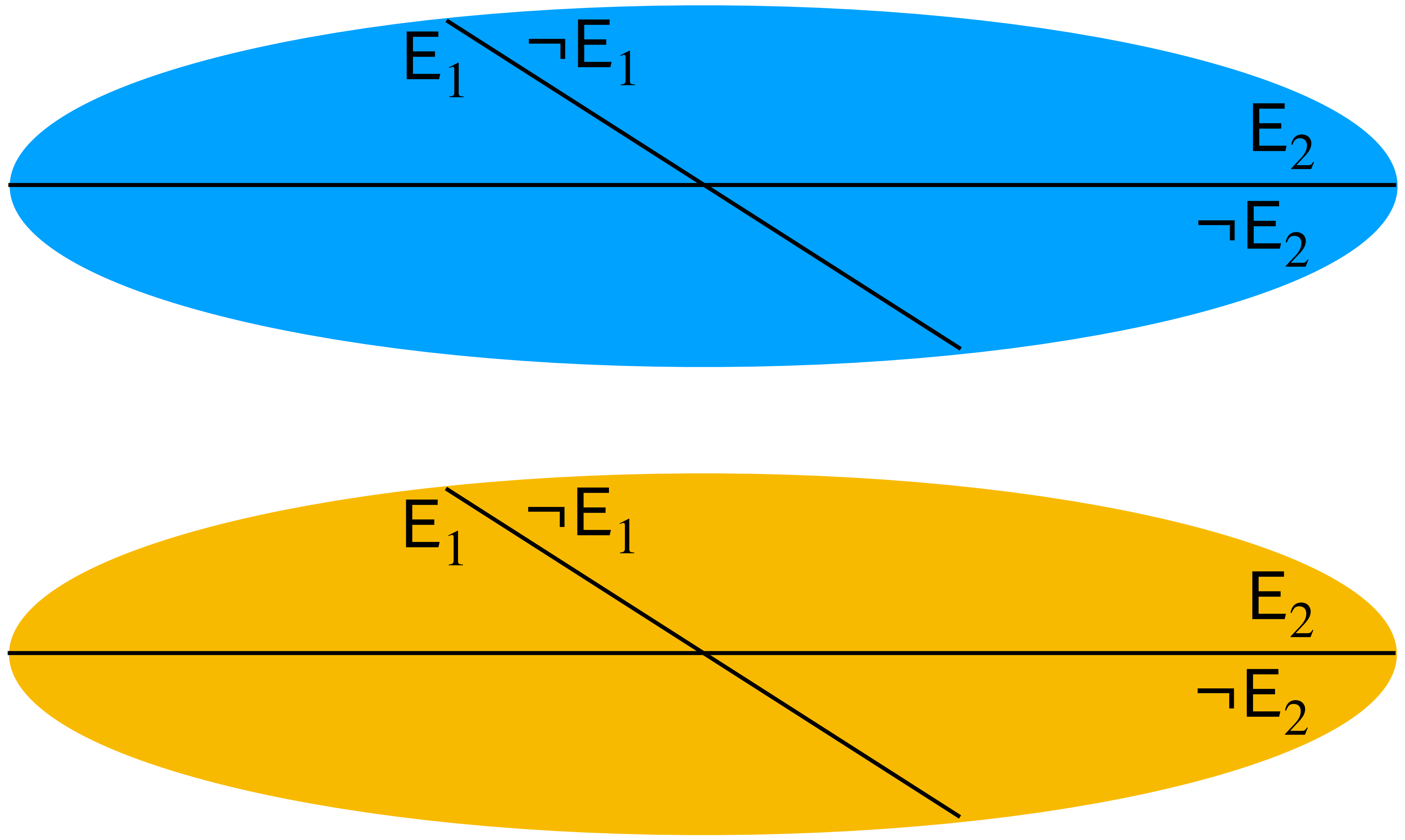

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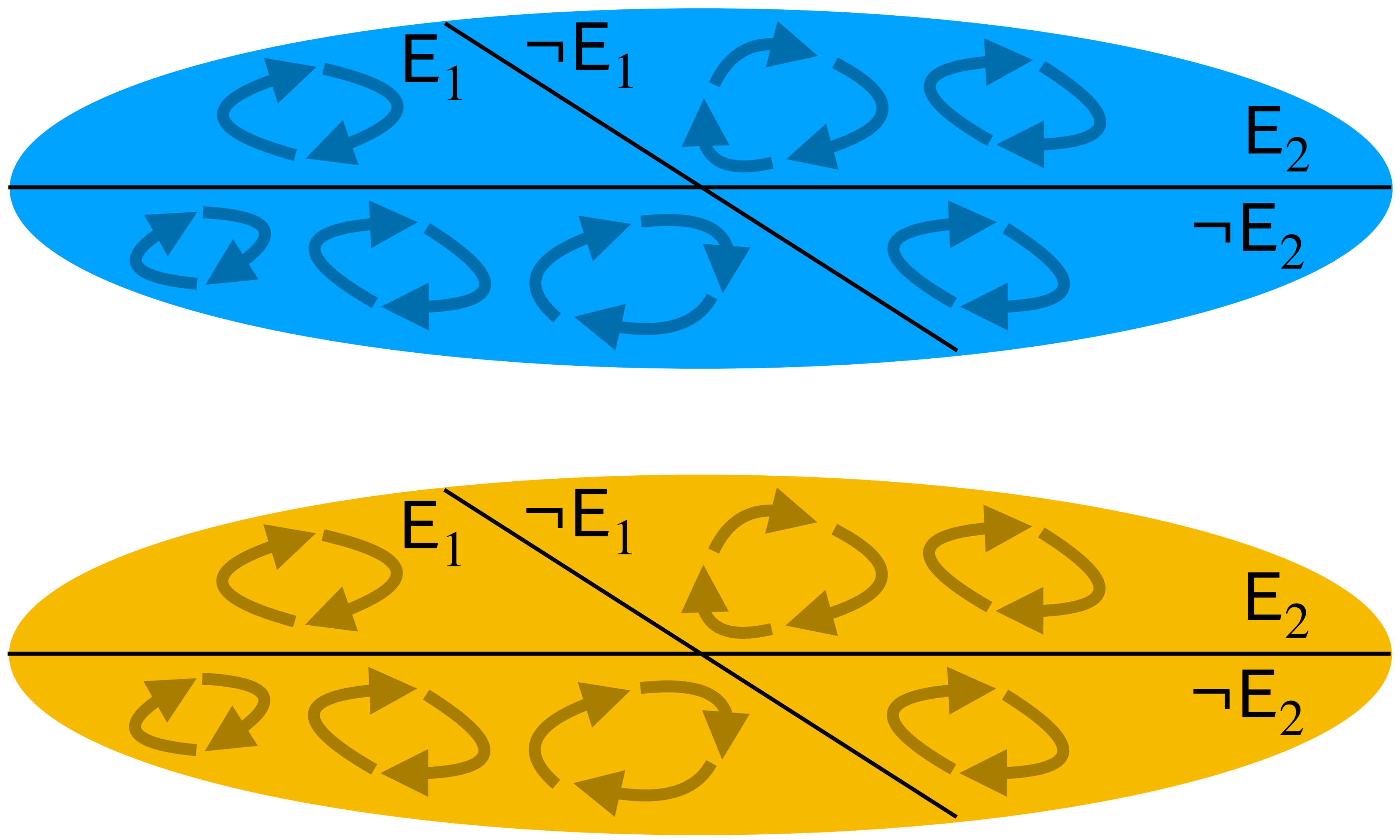

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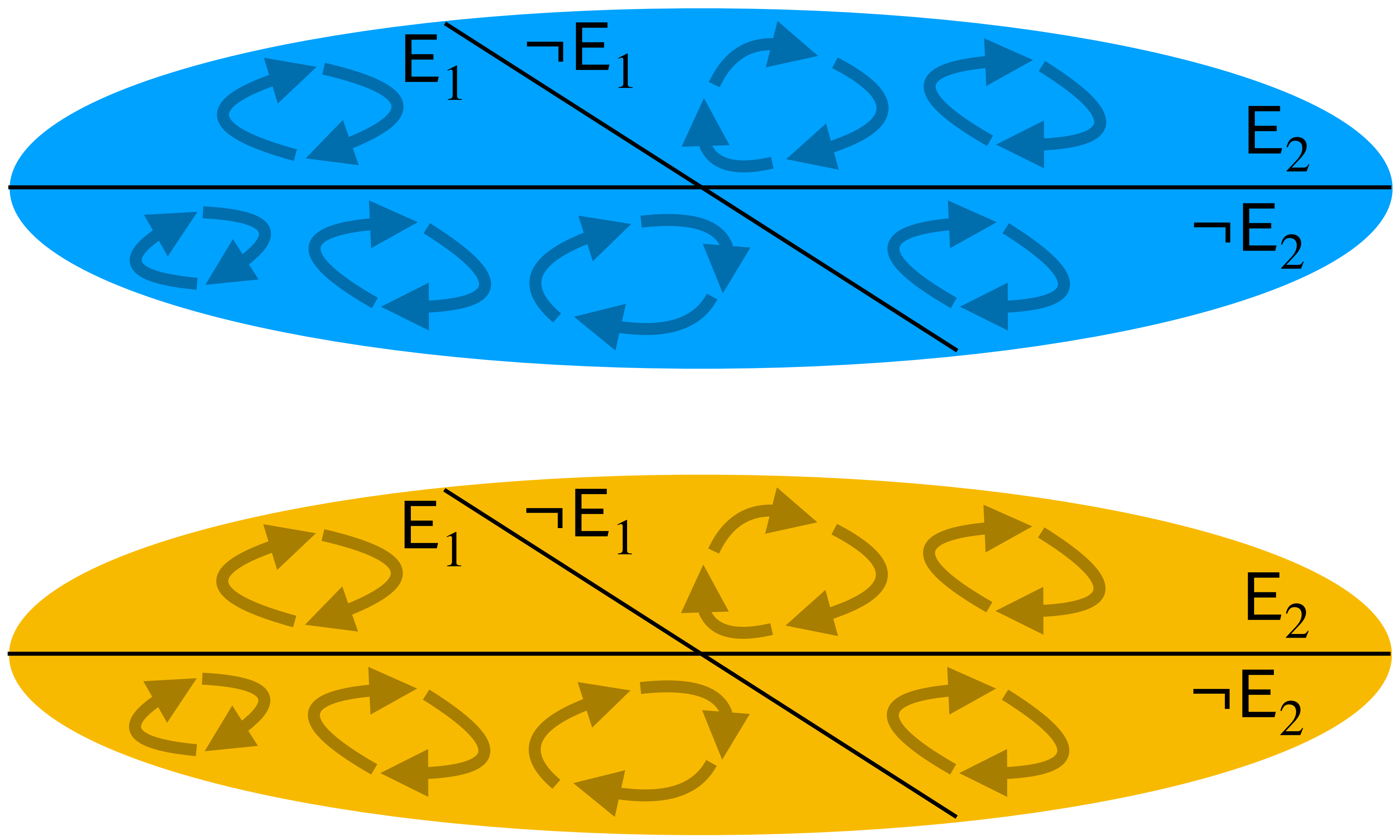

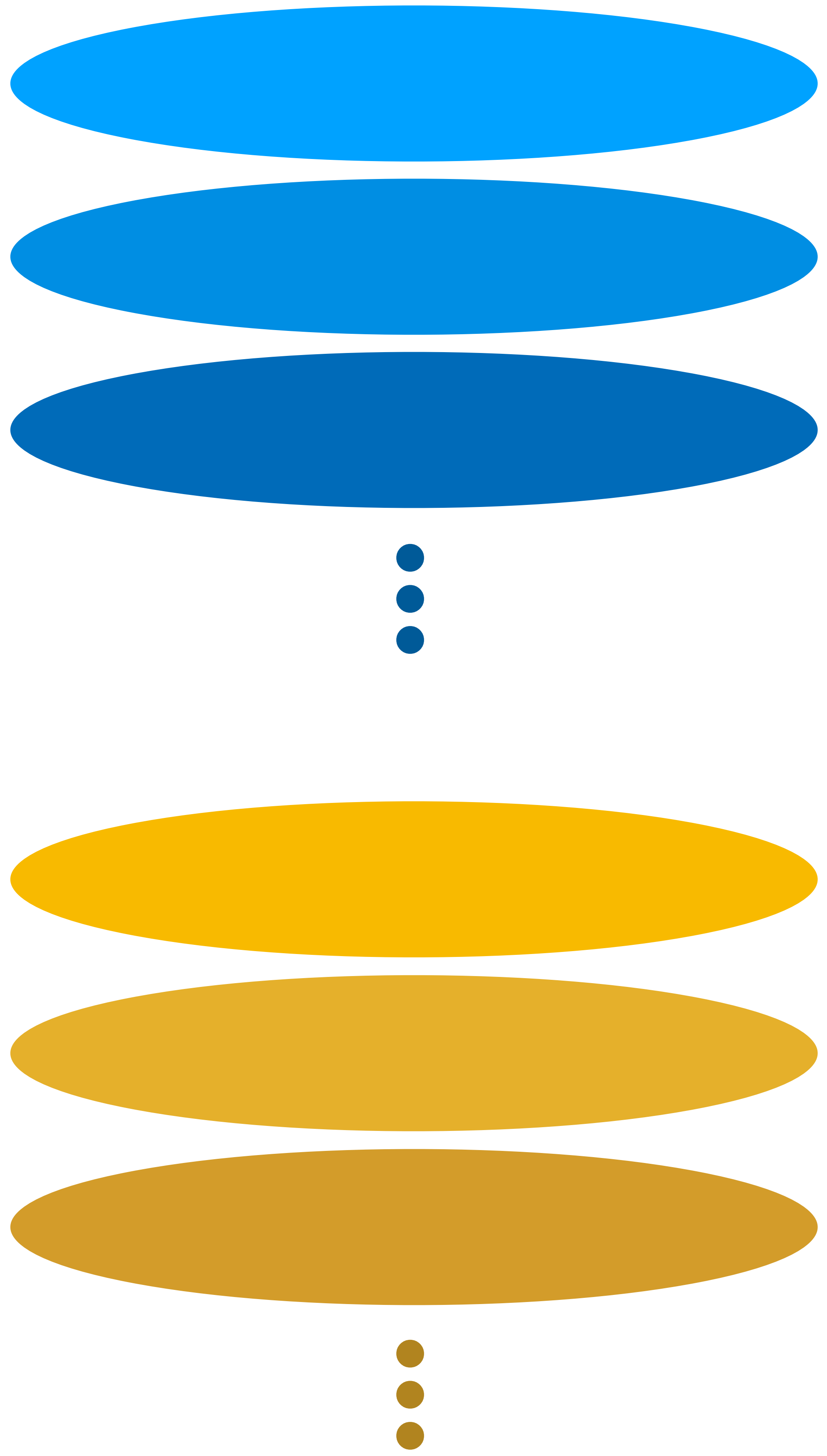

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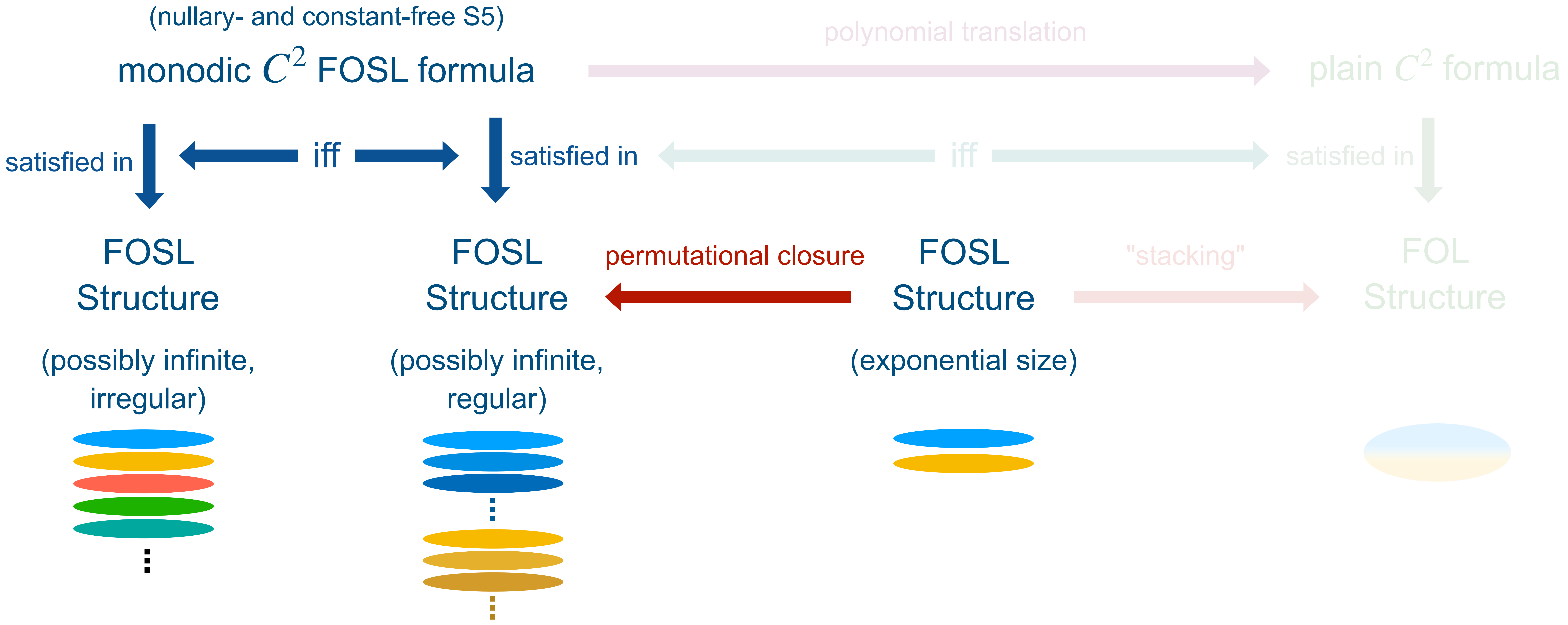

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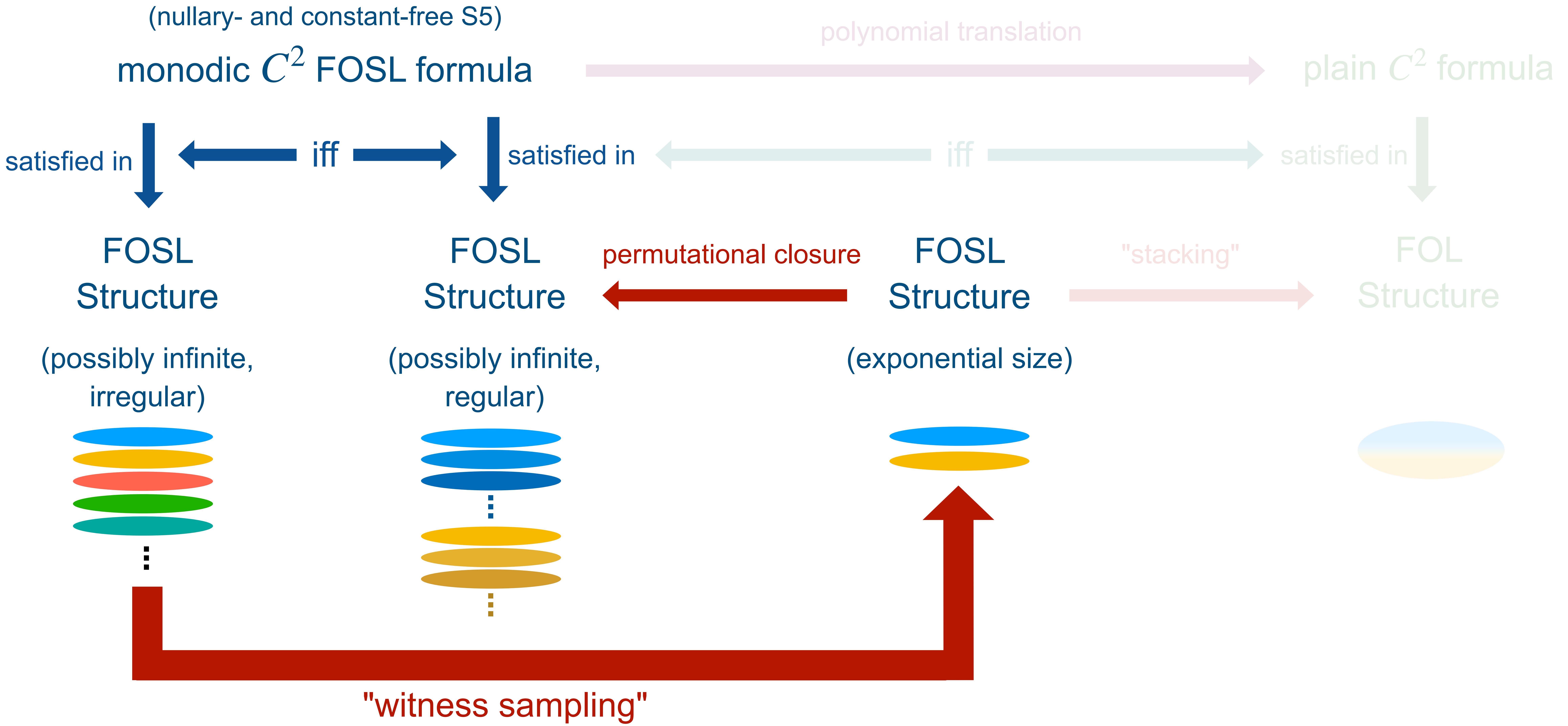

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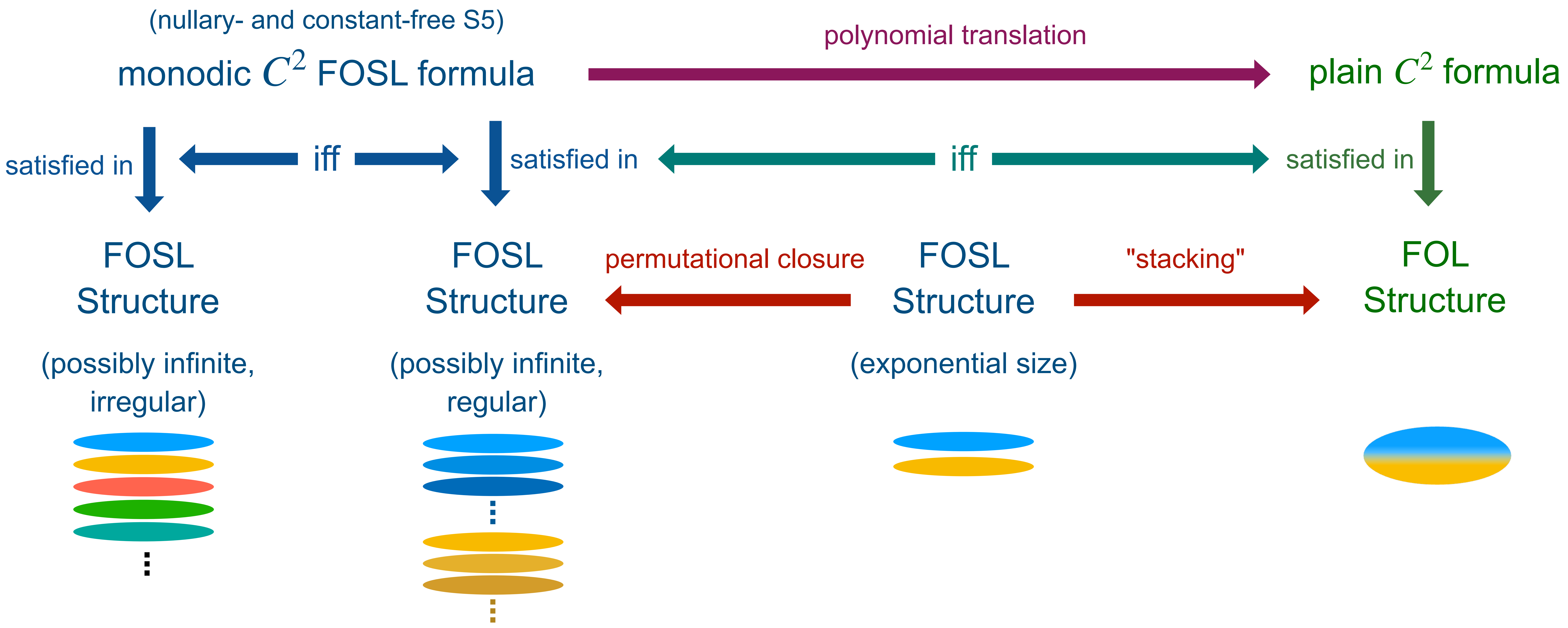

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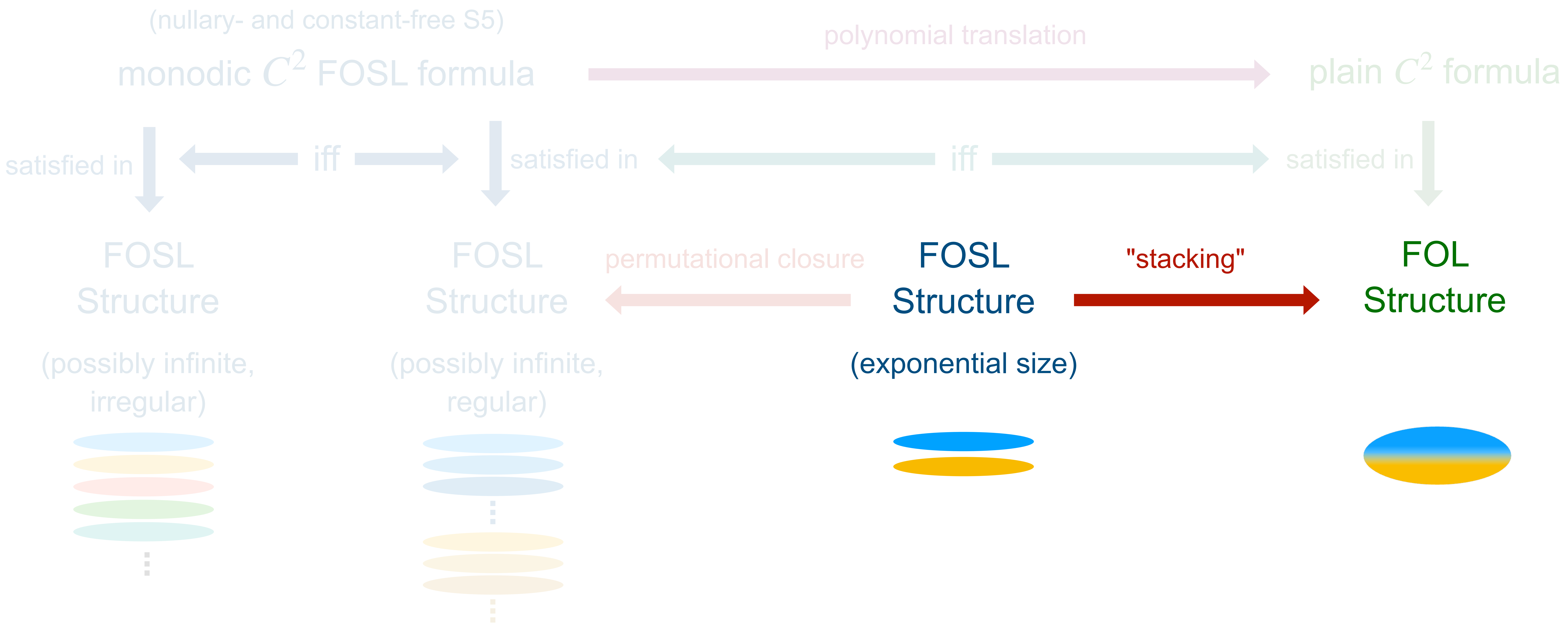
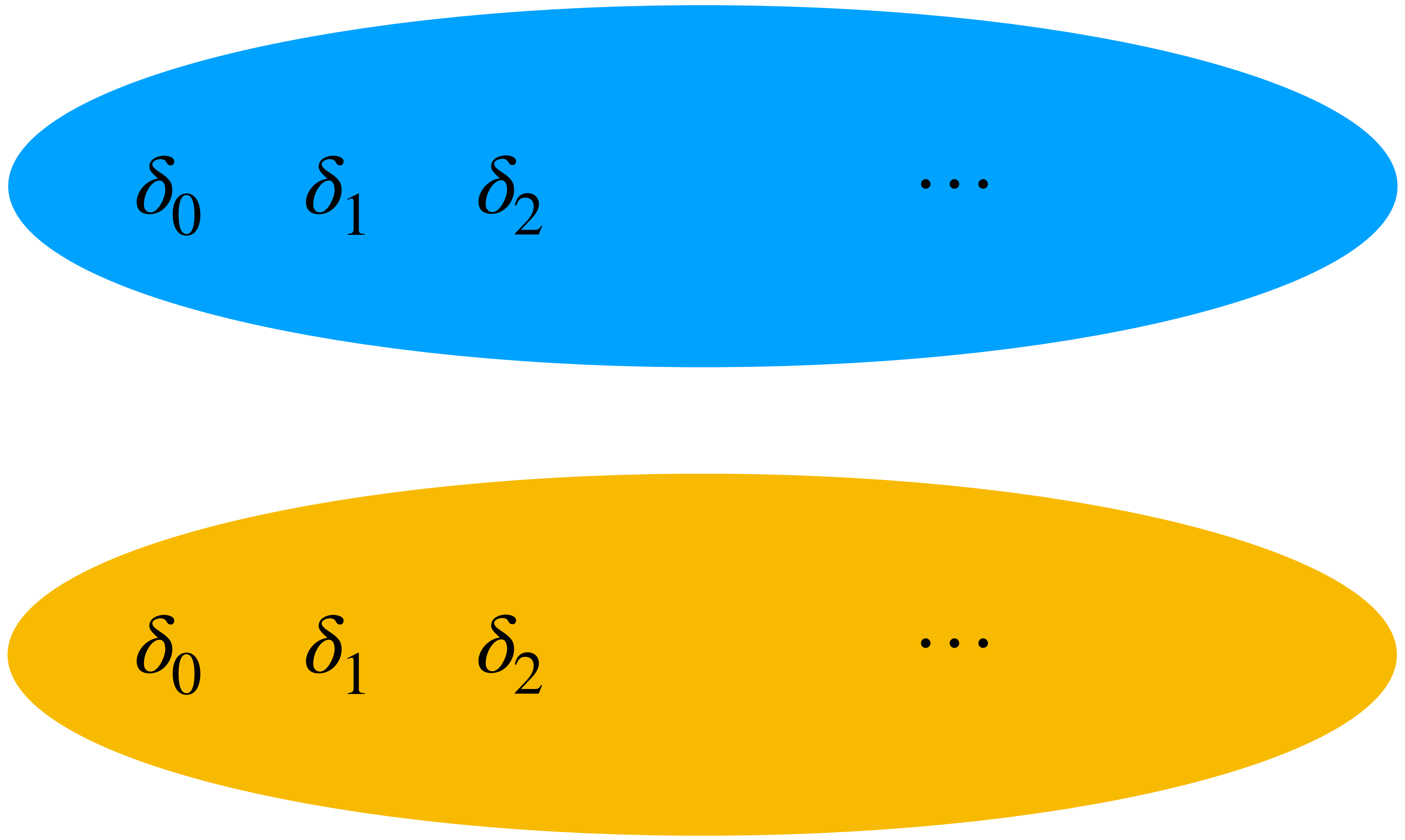


Stacking Worlds



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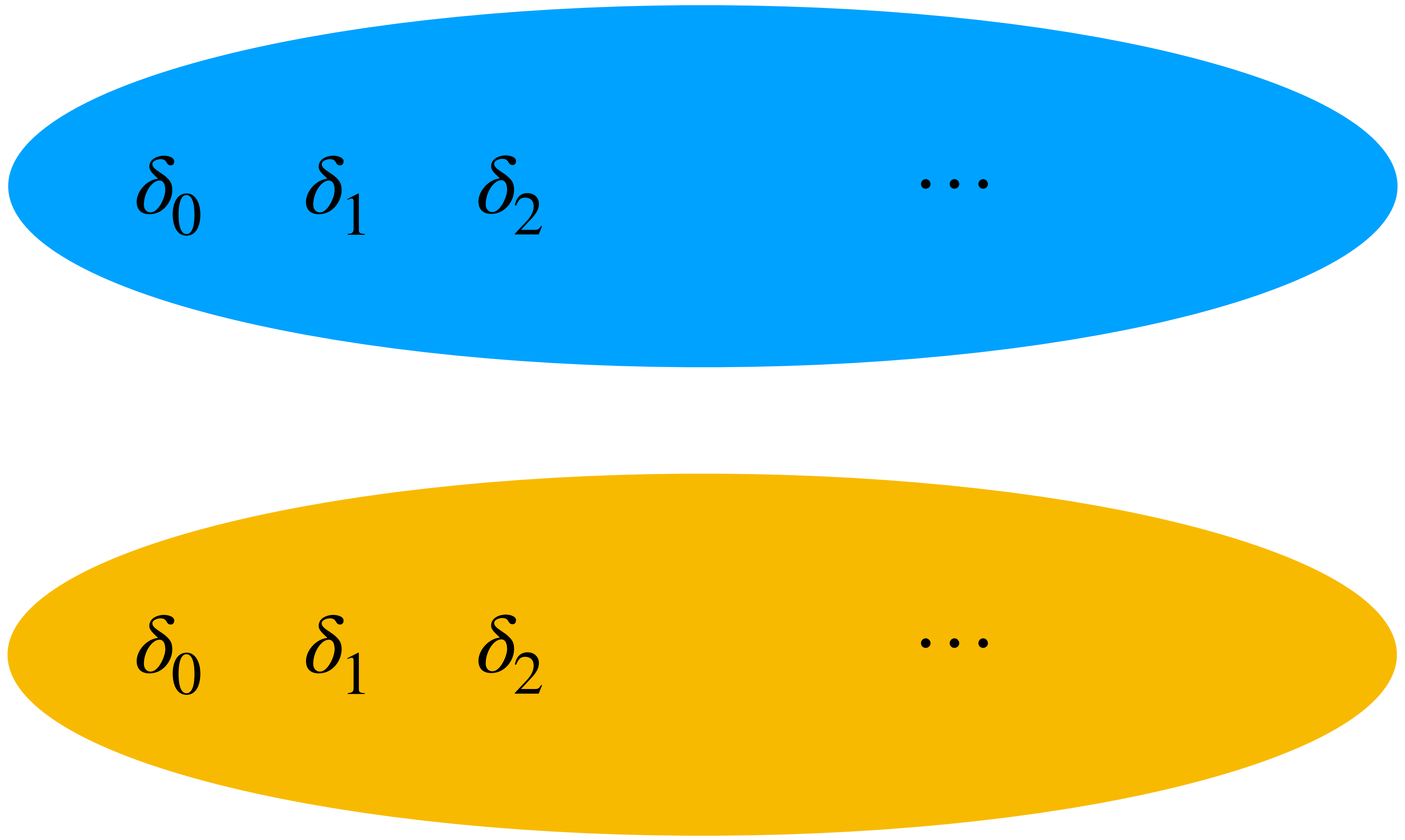

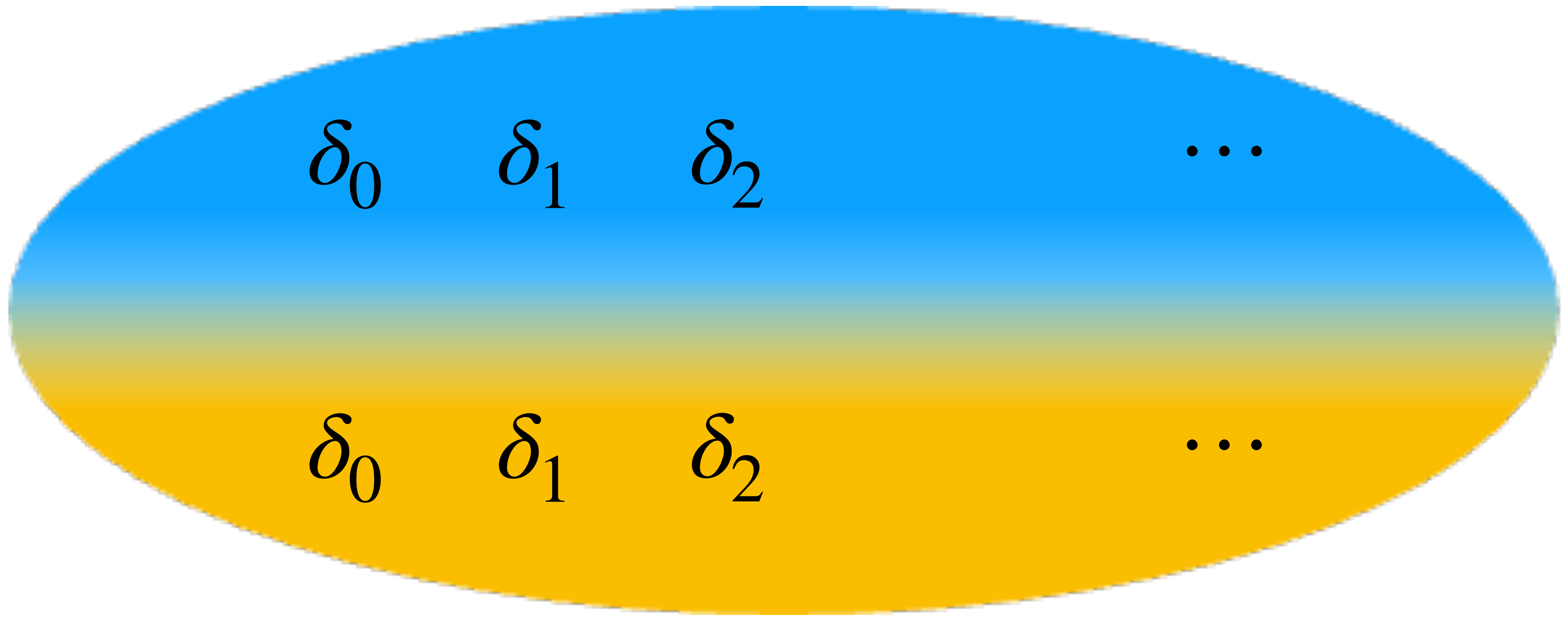

- Disjoint union

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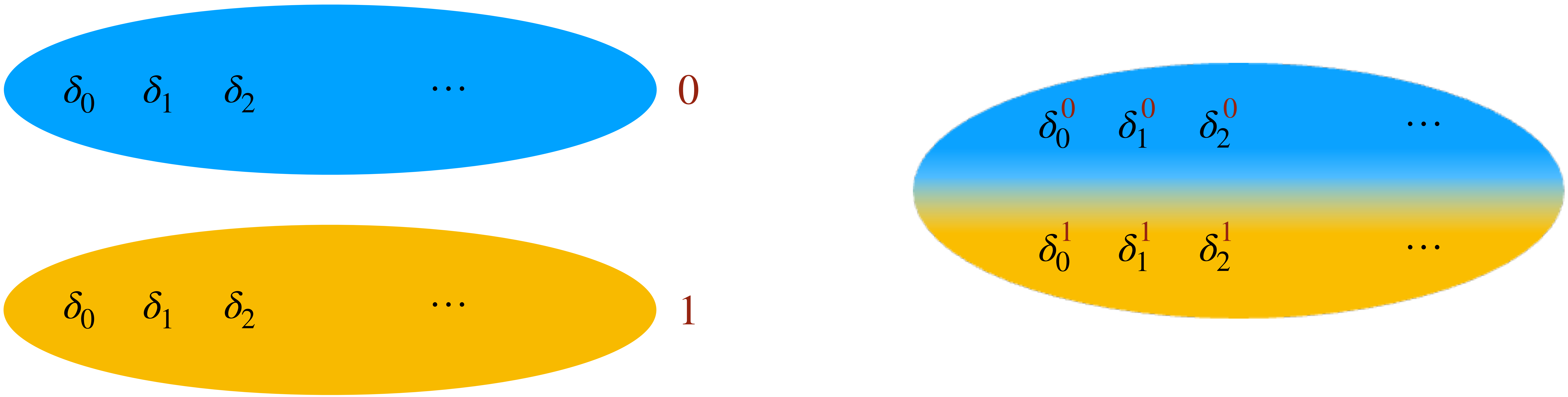

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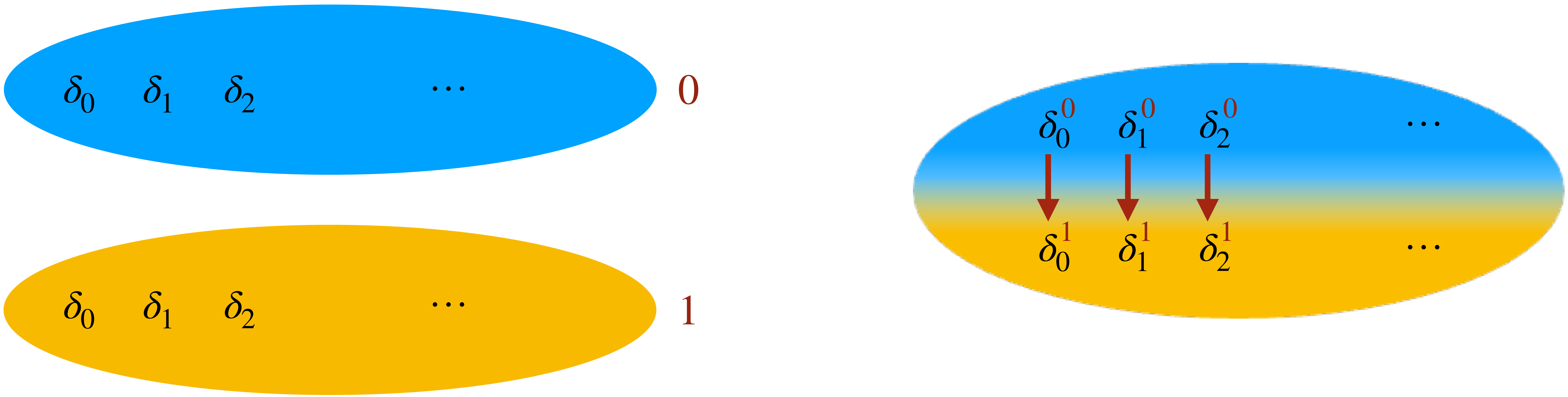

- Disjoint union
- Bookkeeping 1: indicate originating worlds by assigning numbers (bit-encoding by unary predicates L_0,L_1,...)
- Bookkeeping 2: connect corresponding elements with consecutive numbers by binary predicate F
- N.B.: Being "well-stacked" interpretation (i.e. the result of such a stacking) can be characterized in C^2

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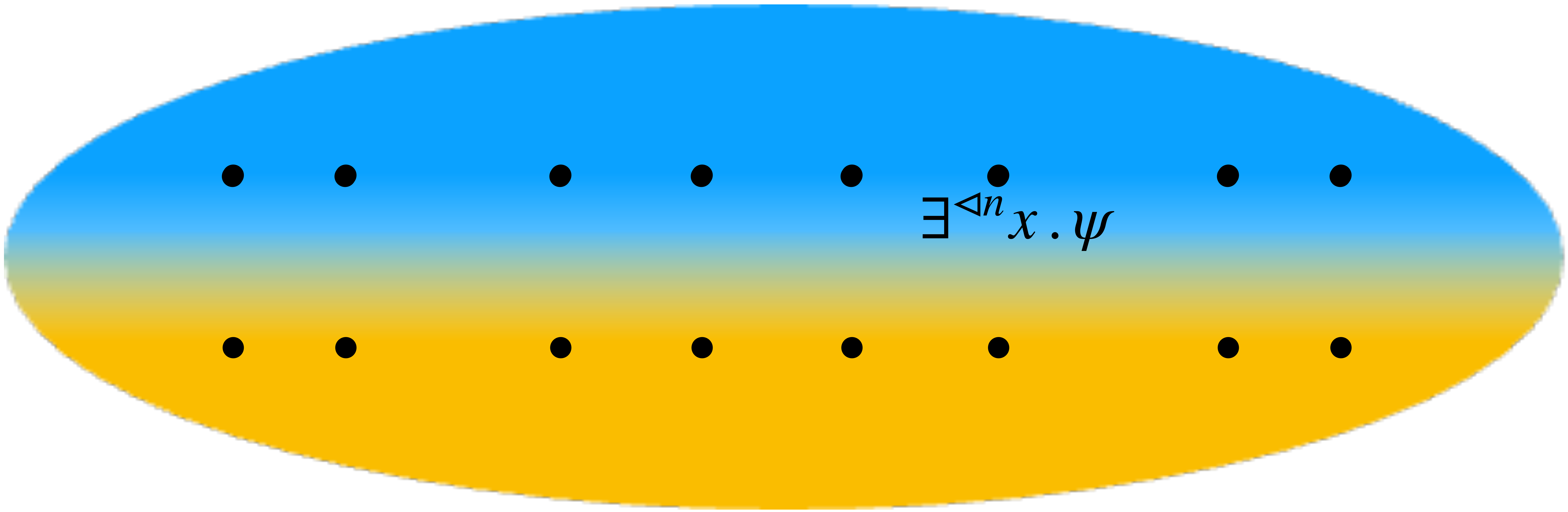

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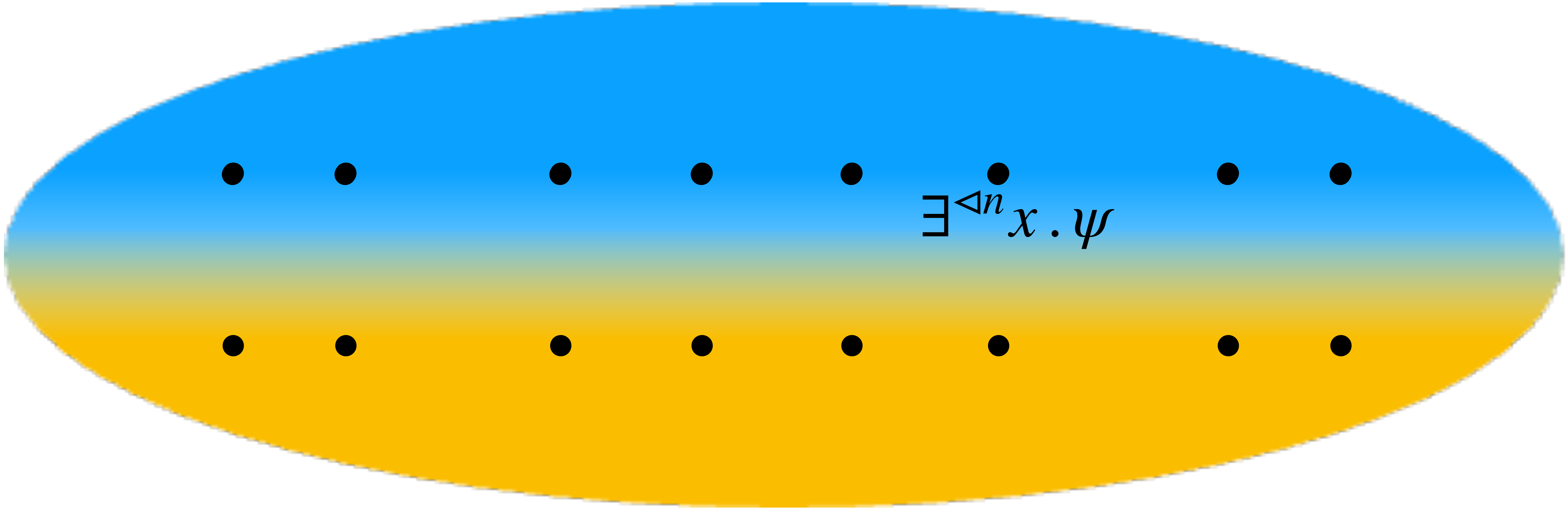

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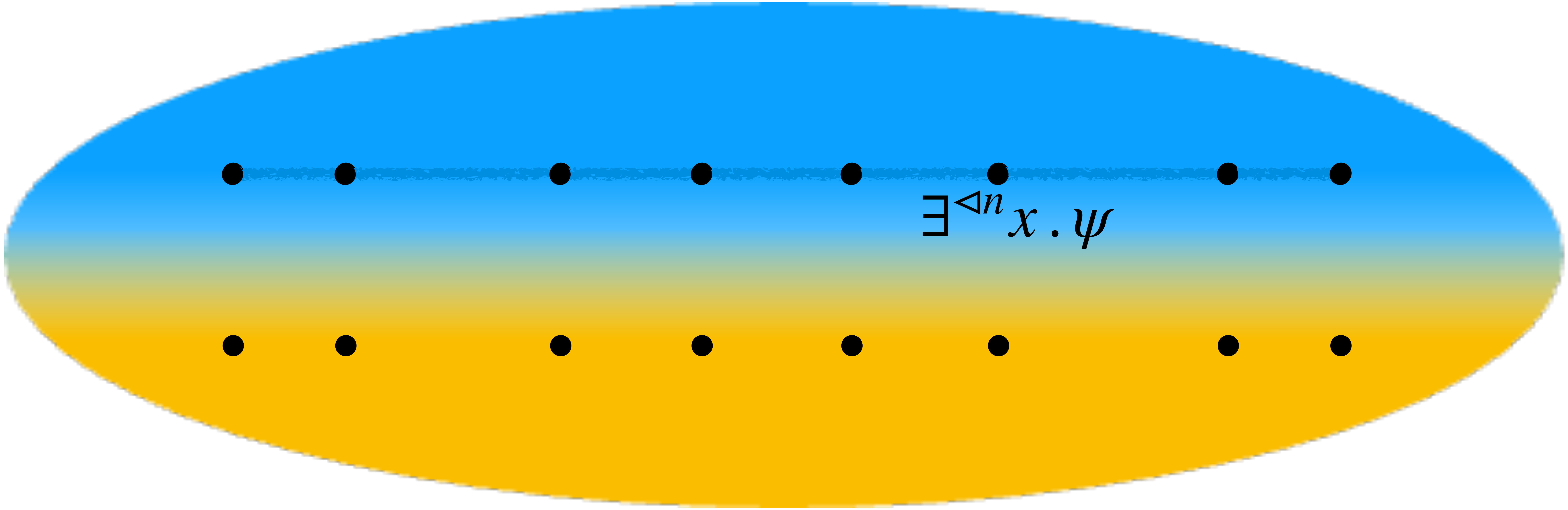

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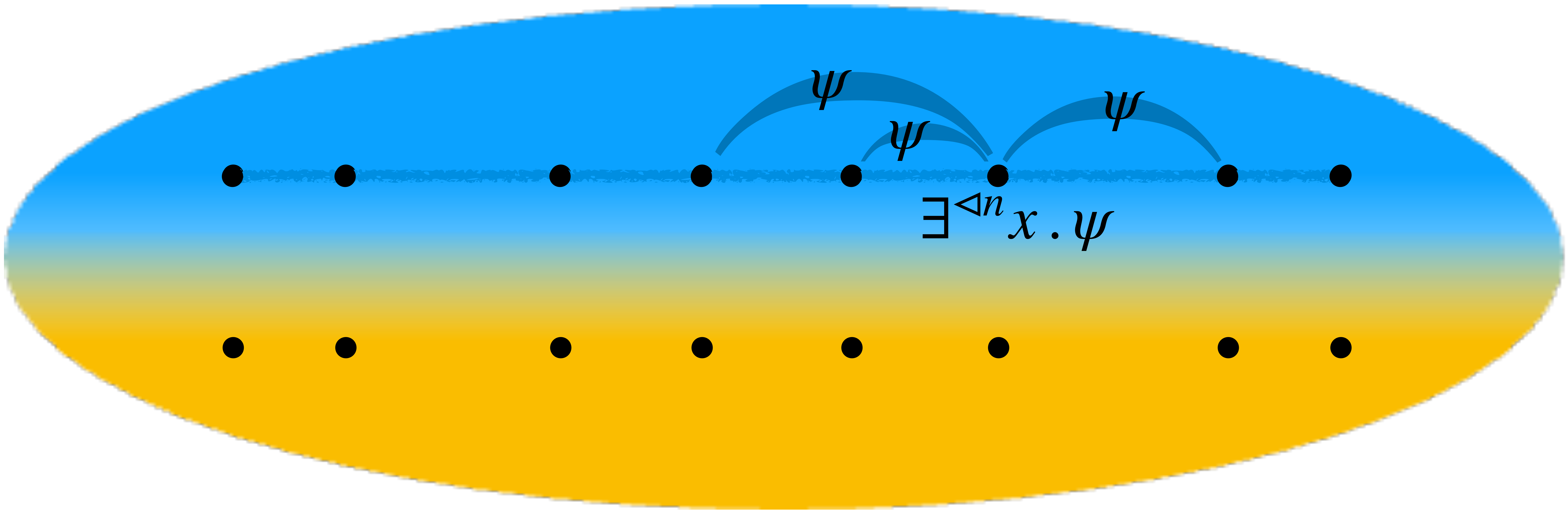

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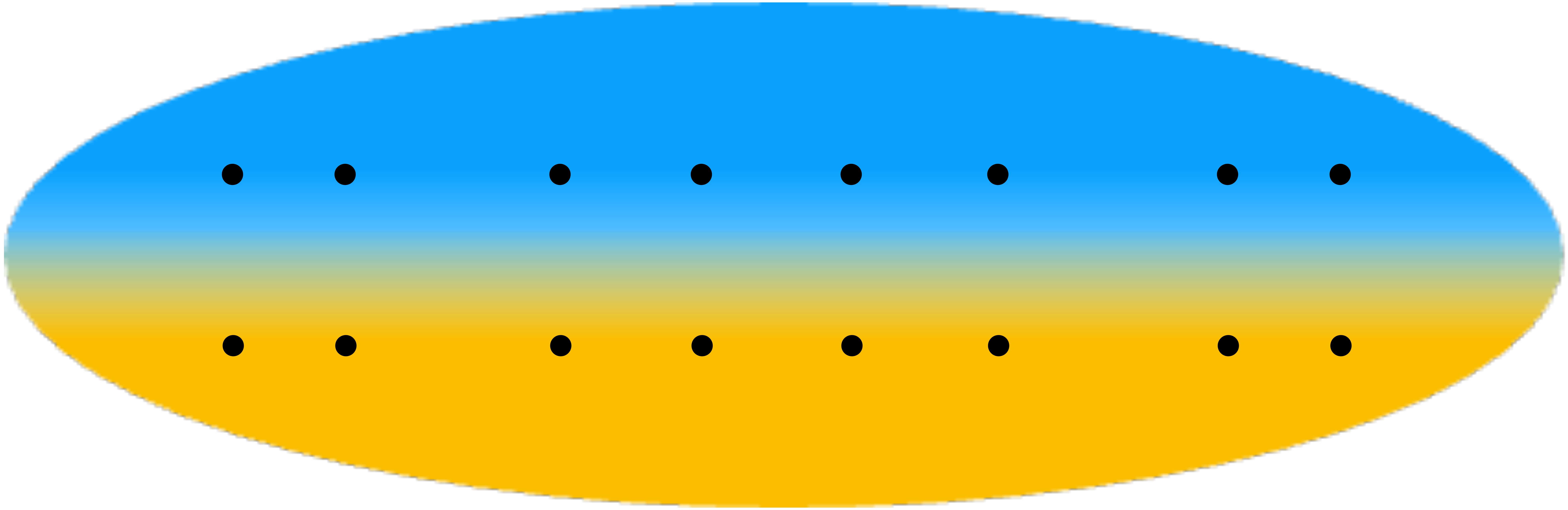

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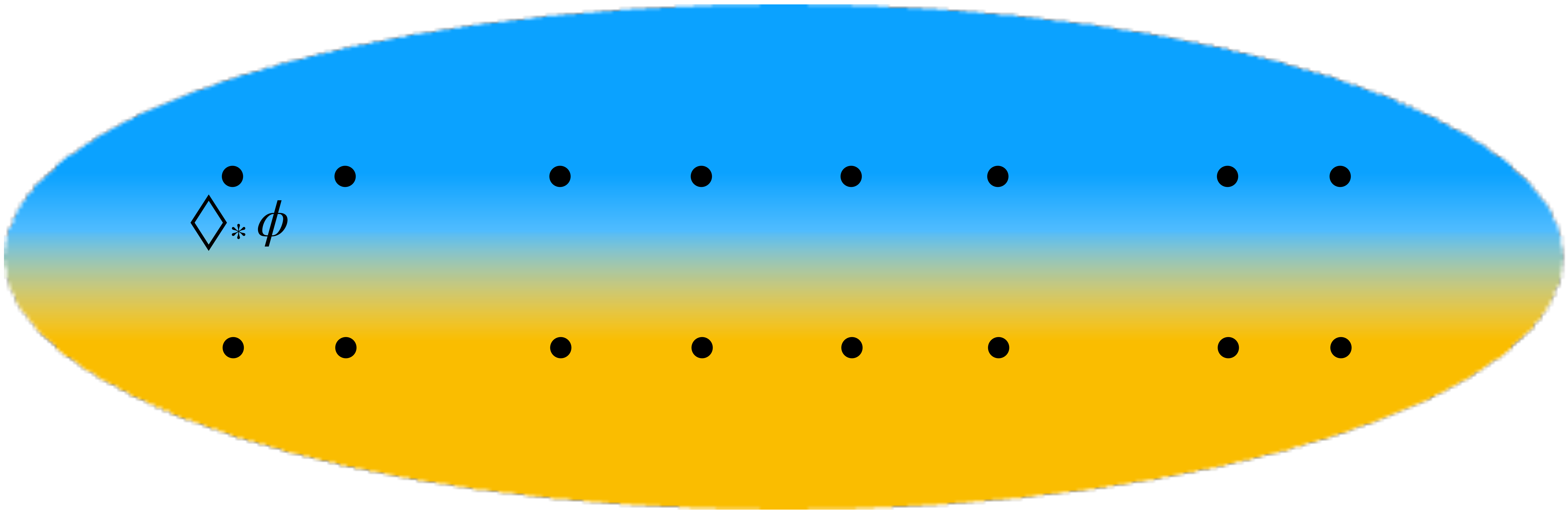

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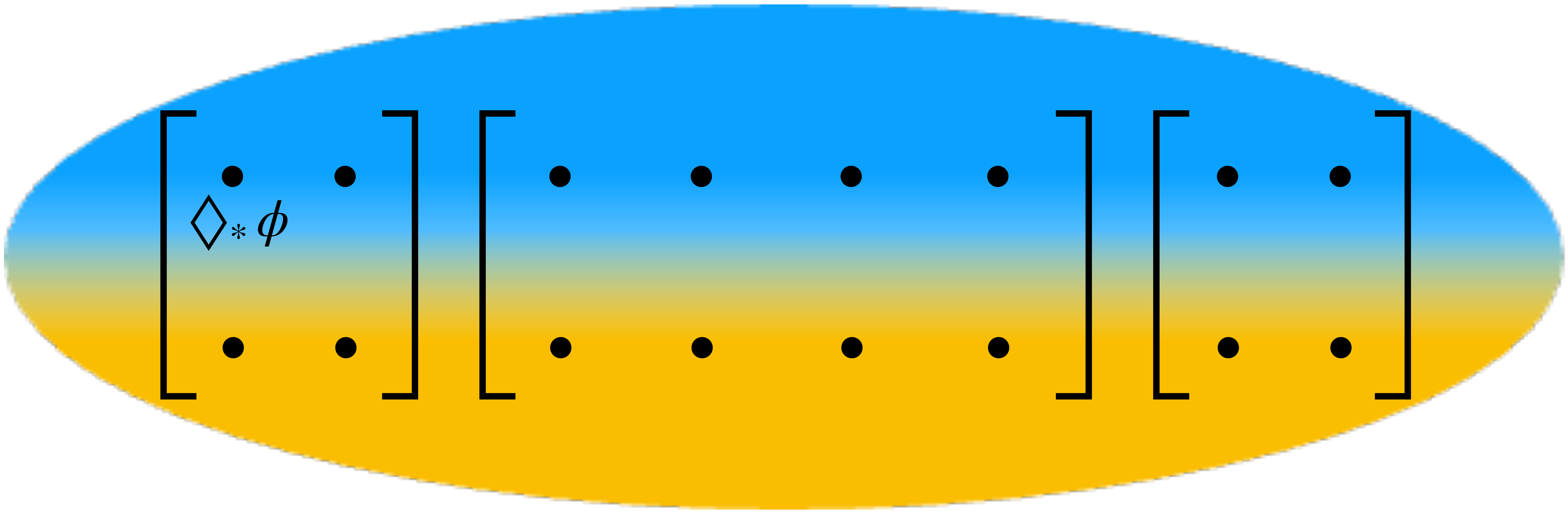

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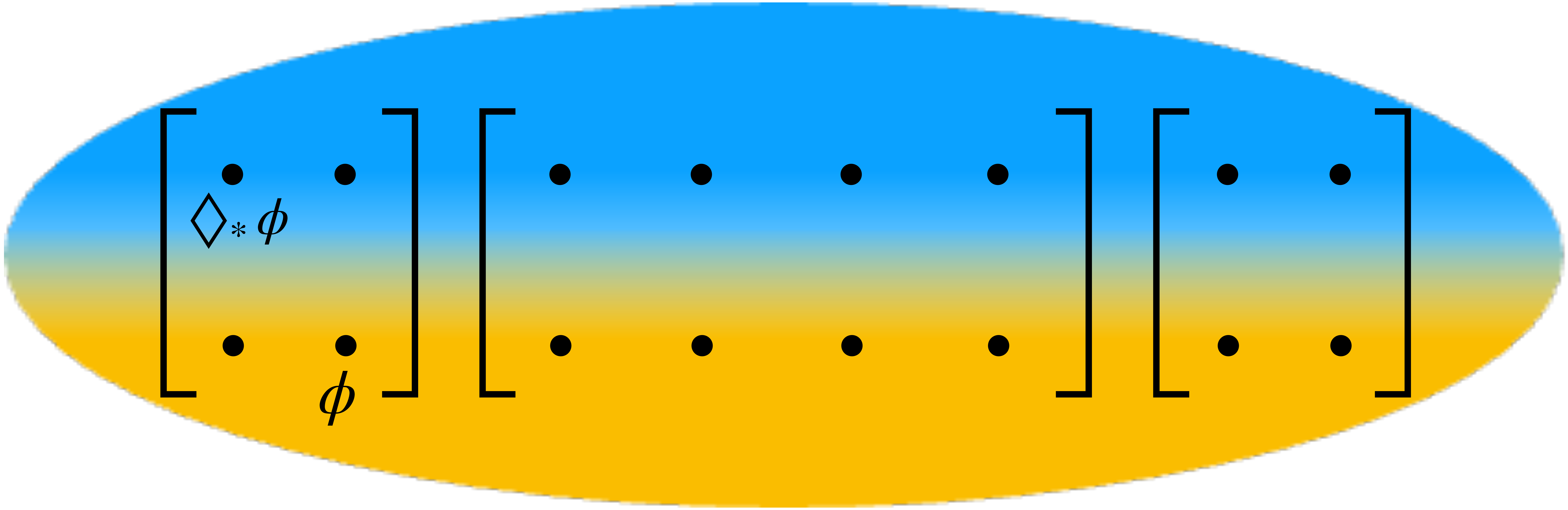

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Corollary: *Satisfiability in monodic standpoint* C^2 *is* $\textsc{NExpTime}$*-complete*

Application to Ontology Languages

Application to Description Logics and Ontology Languages

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(Easy: Essentially just C^2 in diguise)

➡ We show how to accommodate (possibly standpoint-annotated) role chain axioms.

We obtain monodic standpoint $\mathcal{SHOIQB}_\mathcal{S}$ and $\mathcal{SROIQB}_\mathcal{S}$

subsuming **OWL 1** and **OWL 2**

Application to Description Logics and Ontology Languages

Adding monodic standpoints to popular expressive DLs does not increase their computational complexity:

➡ We can add monodic standpoints to the description logic $\mathcal{ALCOIQB}^{\mathsf{Self}}$

(Easy: Essentially just C^2 in diguise)

➡ We show how to accommodate (possibly standpoint-annotated) role chain axioms.

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Theorem: *Checking satisfiability of monodic standpoint $\mathcal{SHOIQB}_\mathcal{S}$ sentences is $\textsc{NExpTime}$-complete.*

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➡ Lifting monodicity gentliest (1 binary rigid predicate) causes undecidability even for $\mathcal{ALCOIF}$

(tiling of infinite grid),

while monodic standpoint $\mathcal{ALCIF}$ with rigid predicates (and more) is known to be decidable.

Conclusions

Recap:

- ➡ Managing perspectives is interesting in knowledge integration scenarios.
- ➡ Reasoning with monodic Standpoint C^2 has the same complexity as with plain C^2.
- ➡ This warrants complexity-neutral extensions of Ontology Languages by monodic standpoints.
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Future Work:

➡ Implementation of translations and integration with existing reasoners

➡ Lifting the monodicity restriction

➡ Towards conceptual modelling with standpoints for knowledge integration challenges

Bonus Material

Permutational Representatives

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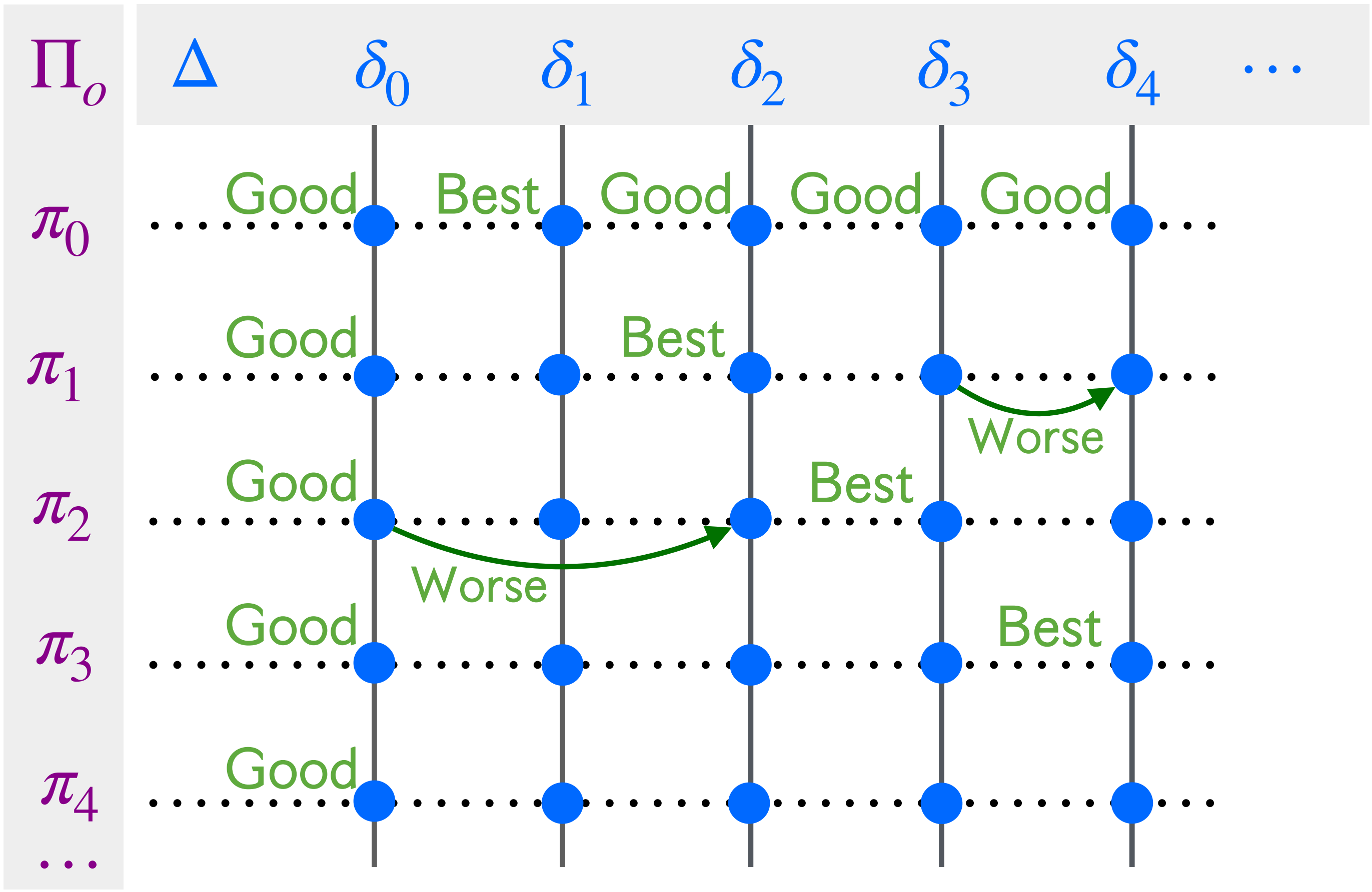

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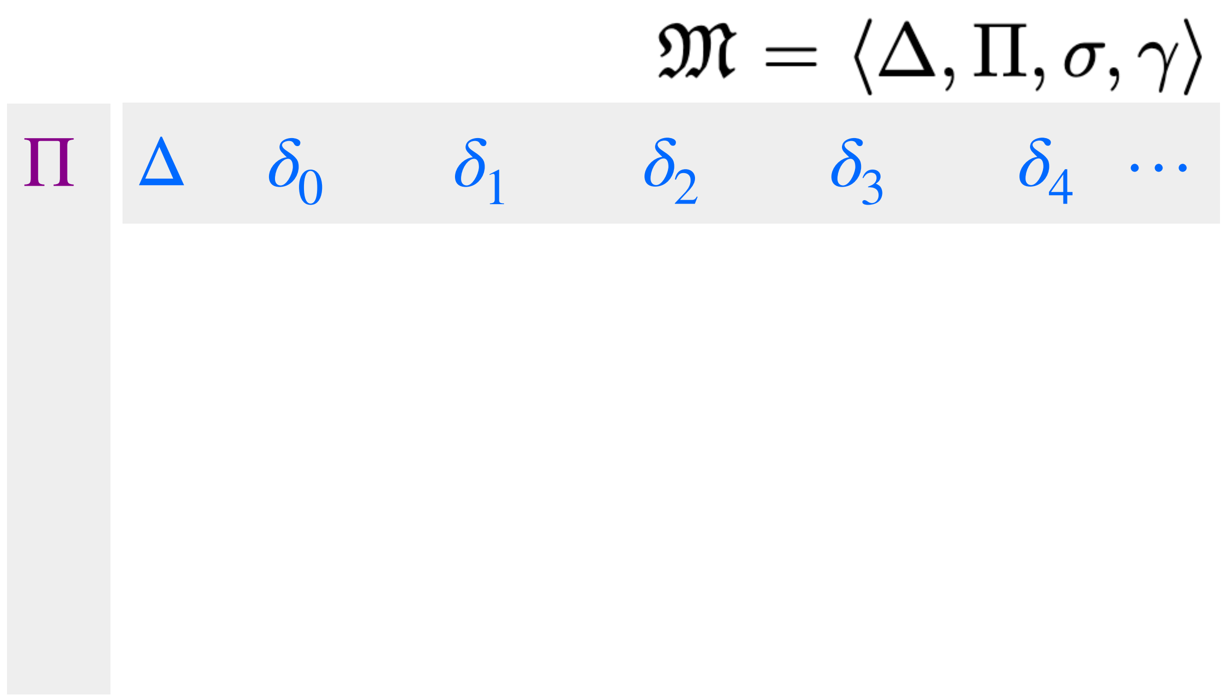

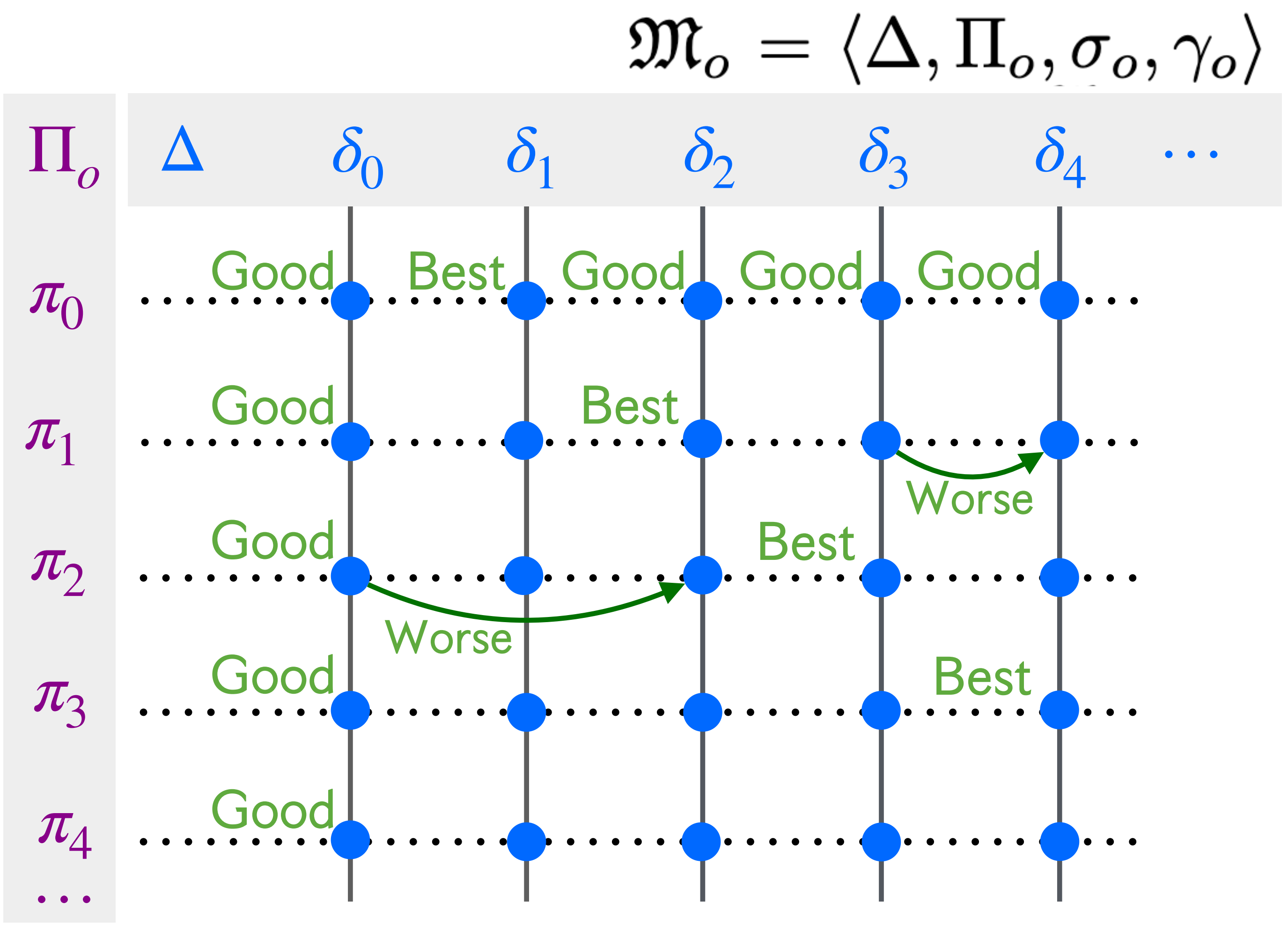

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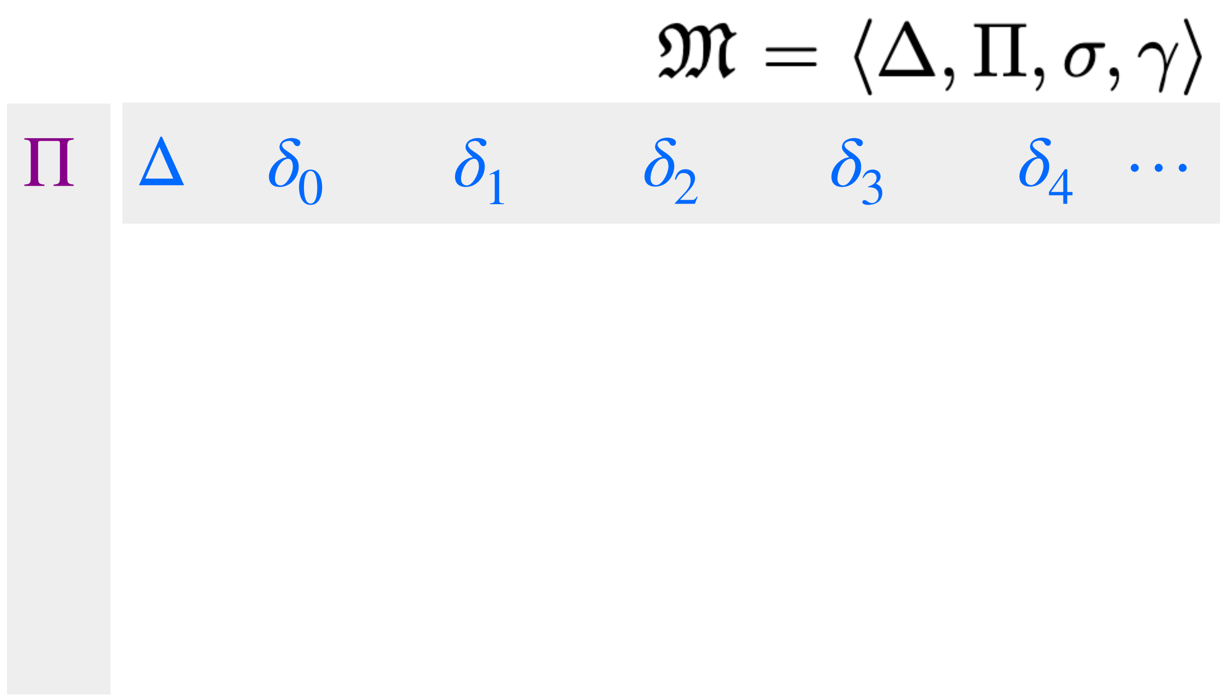

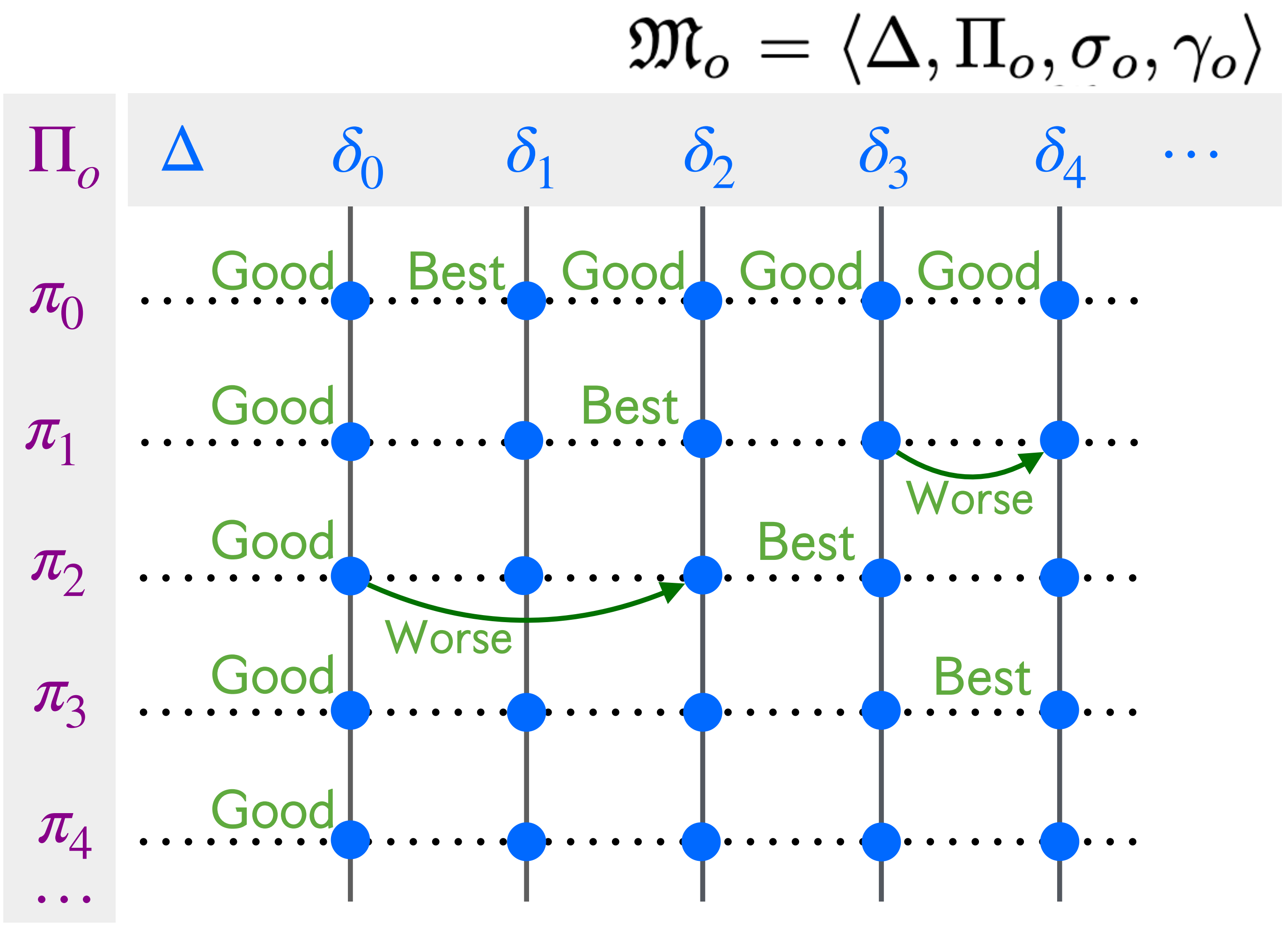

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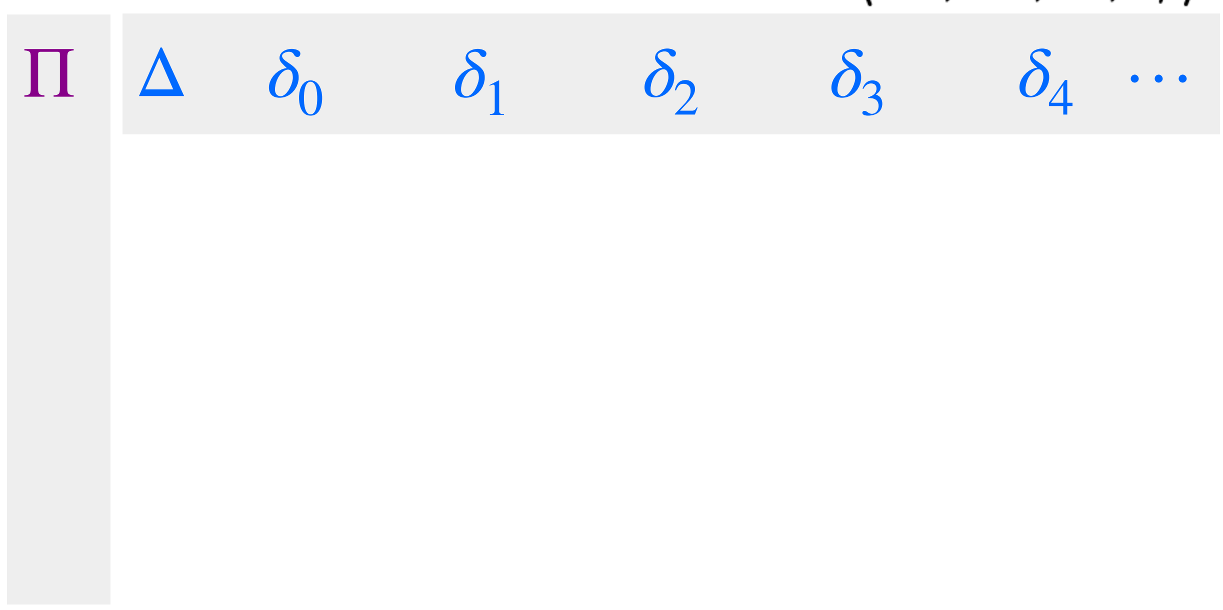

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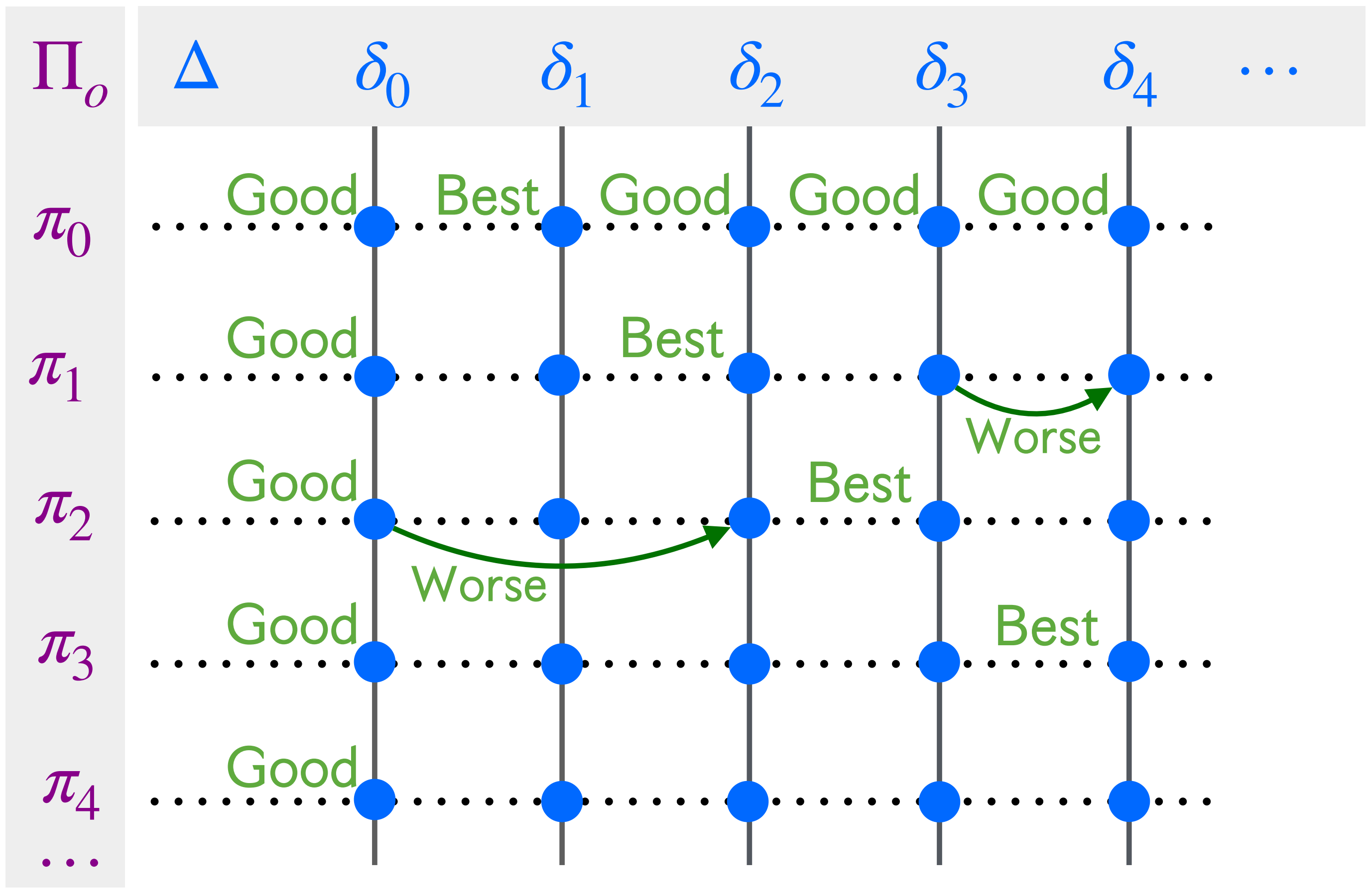

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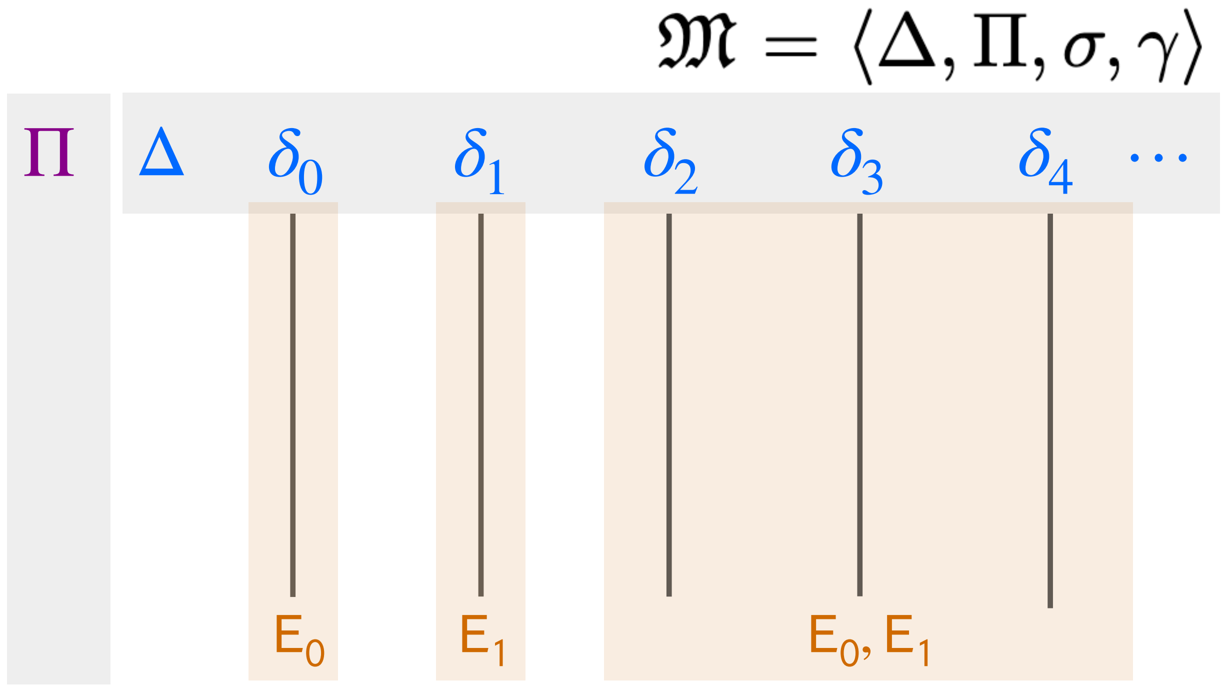

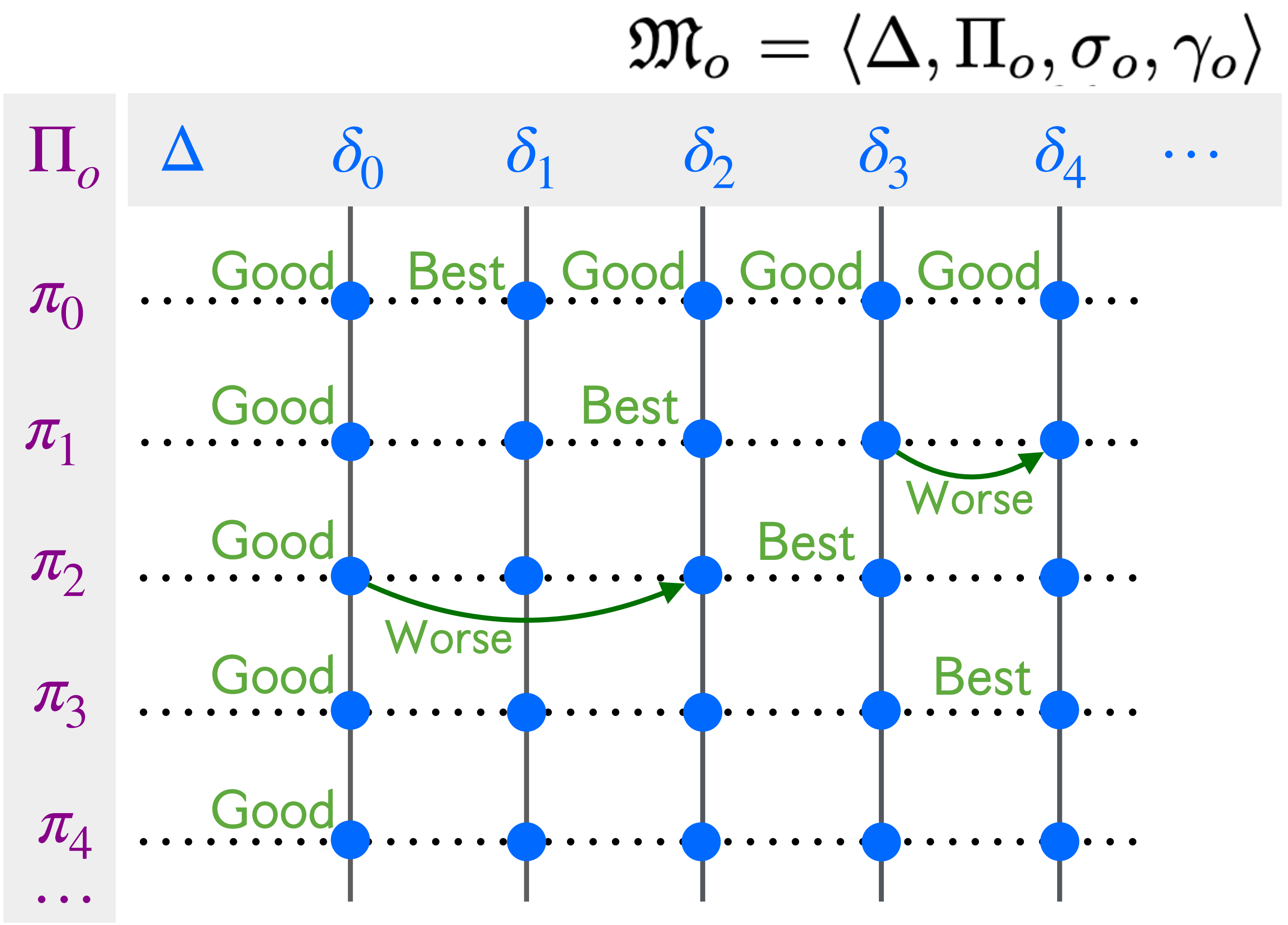

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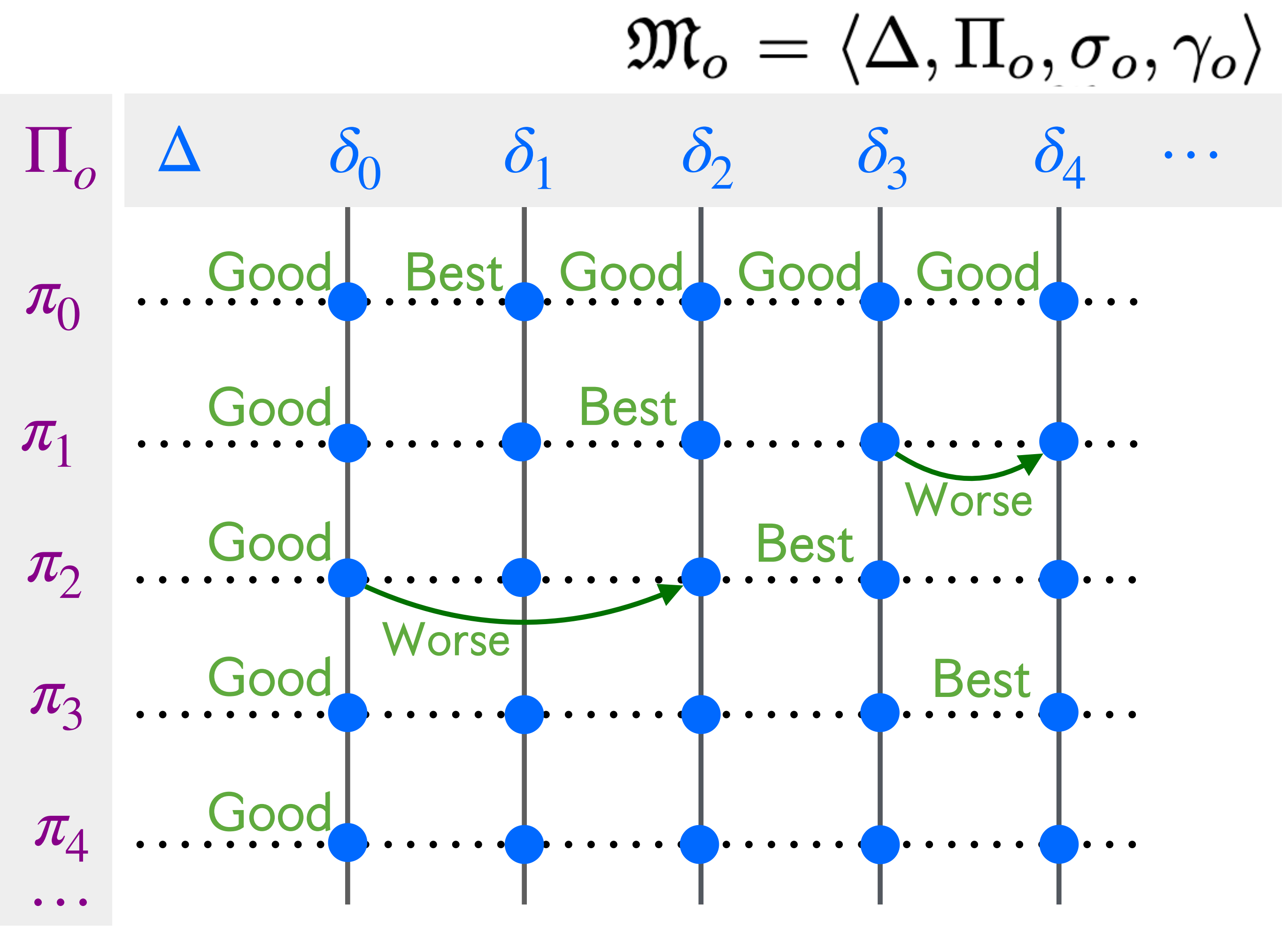

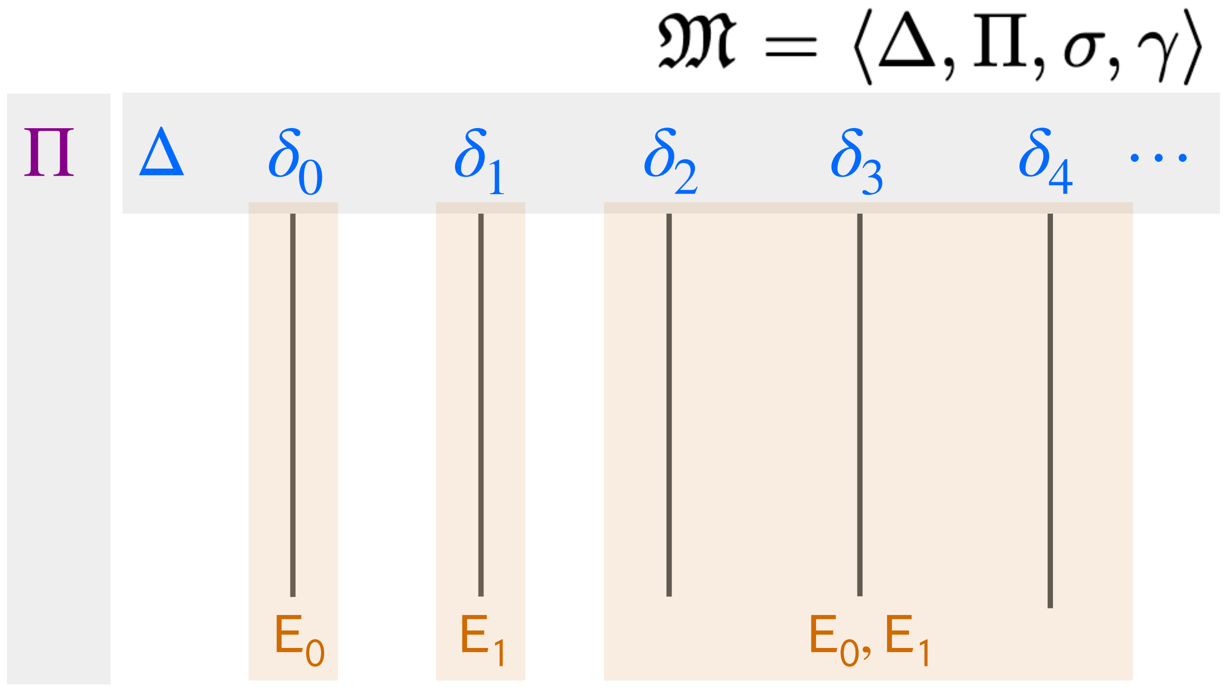

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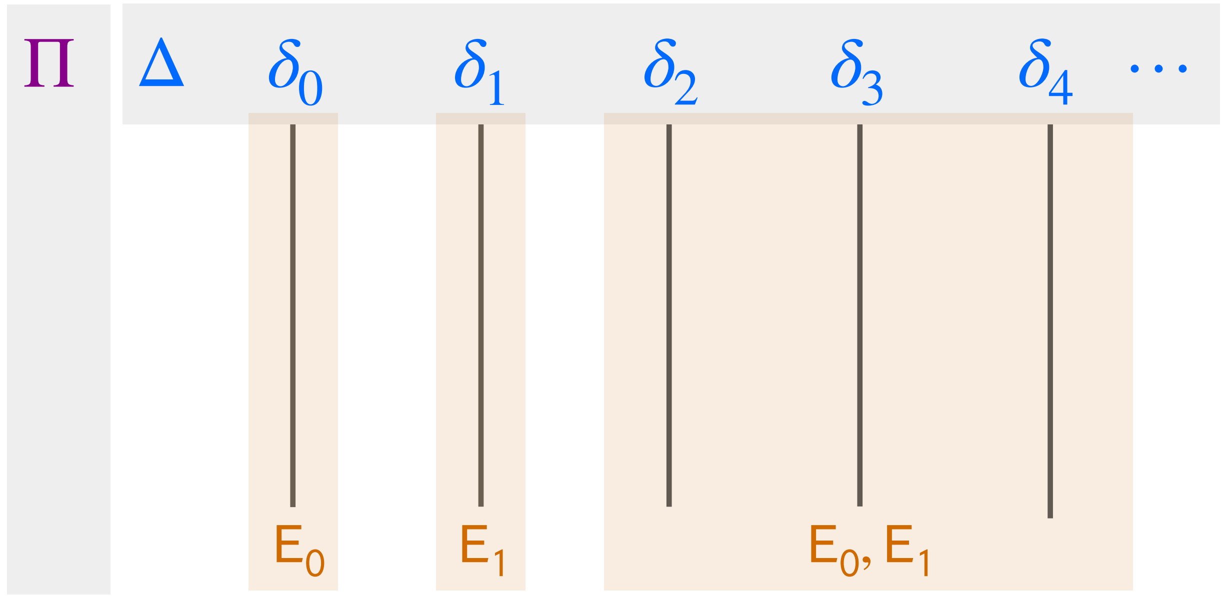

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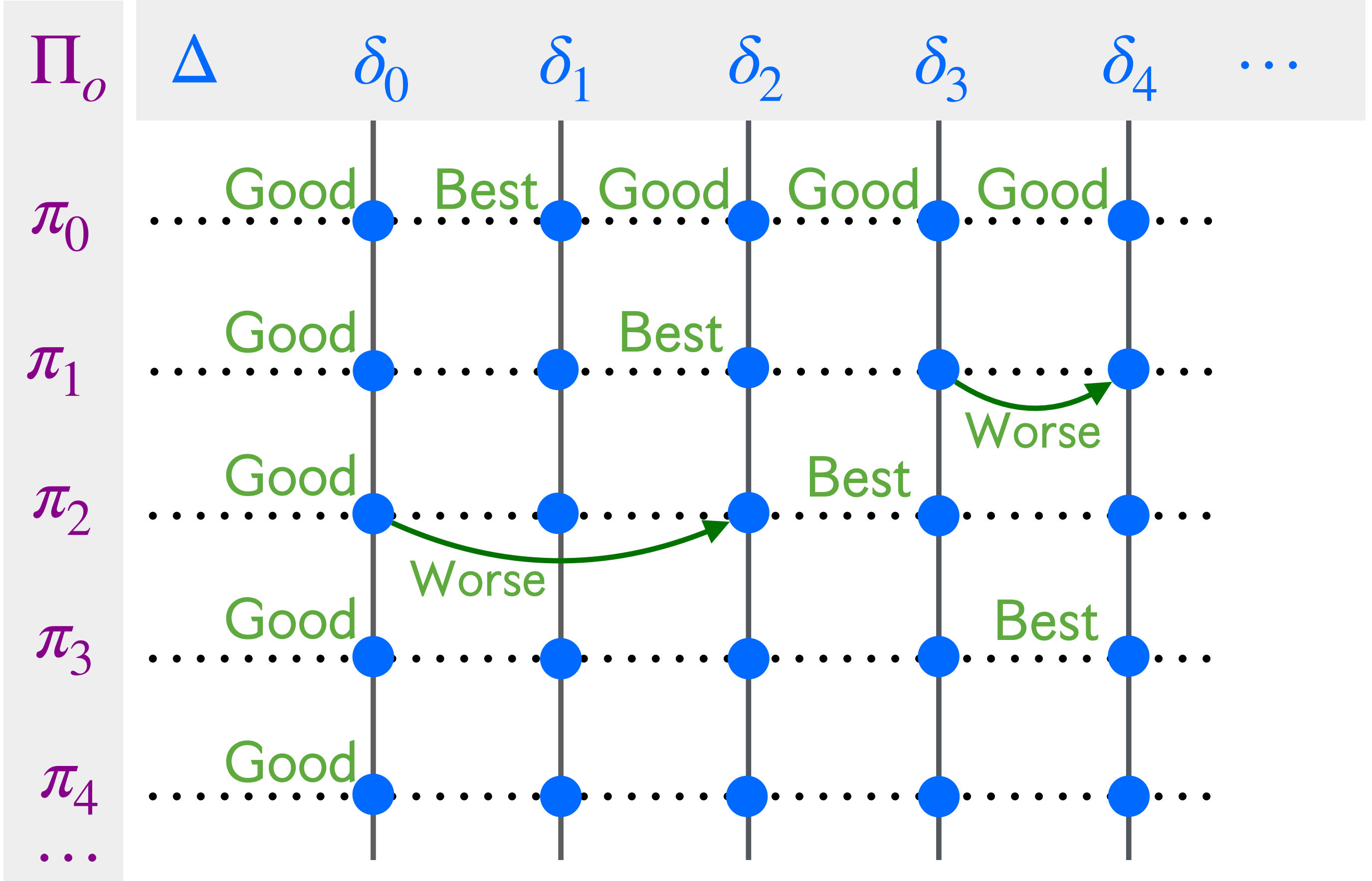

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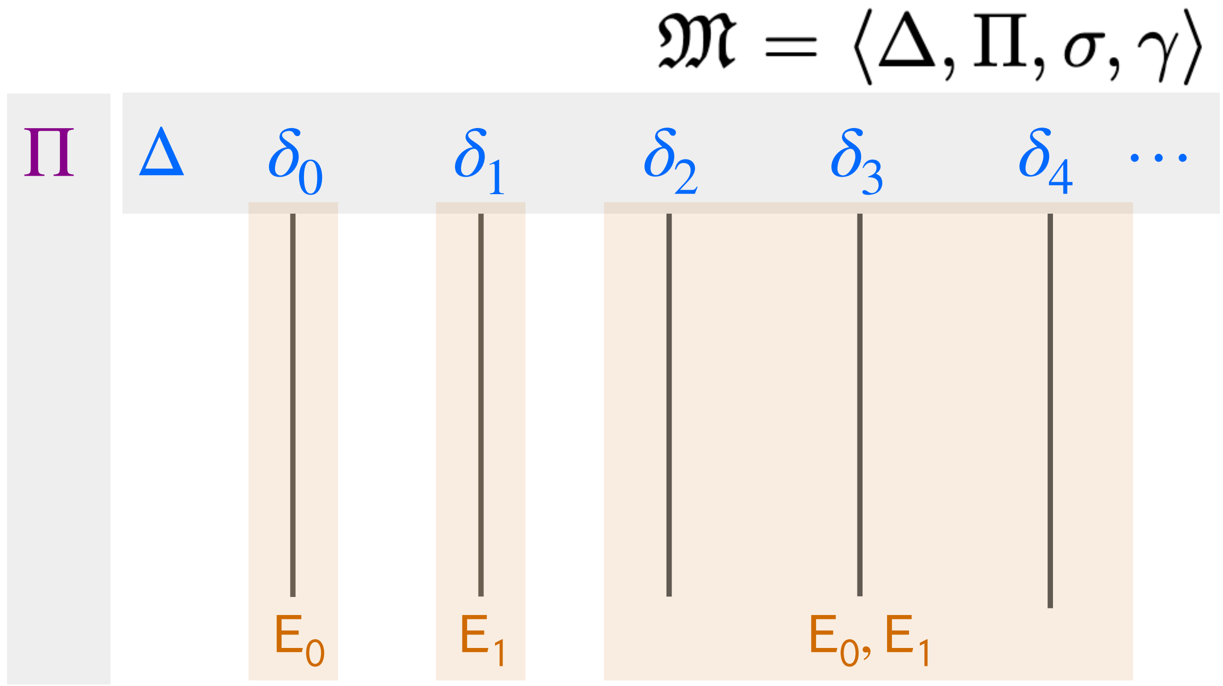

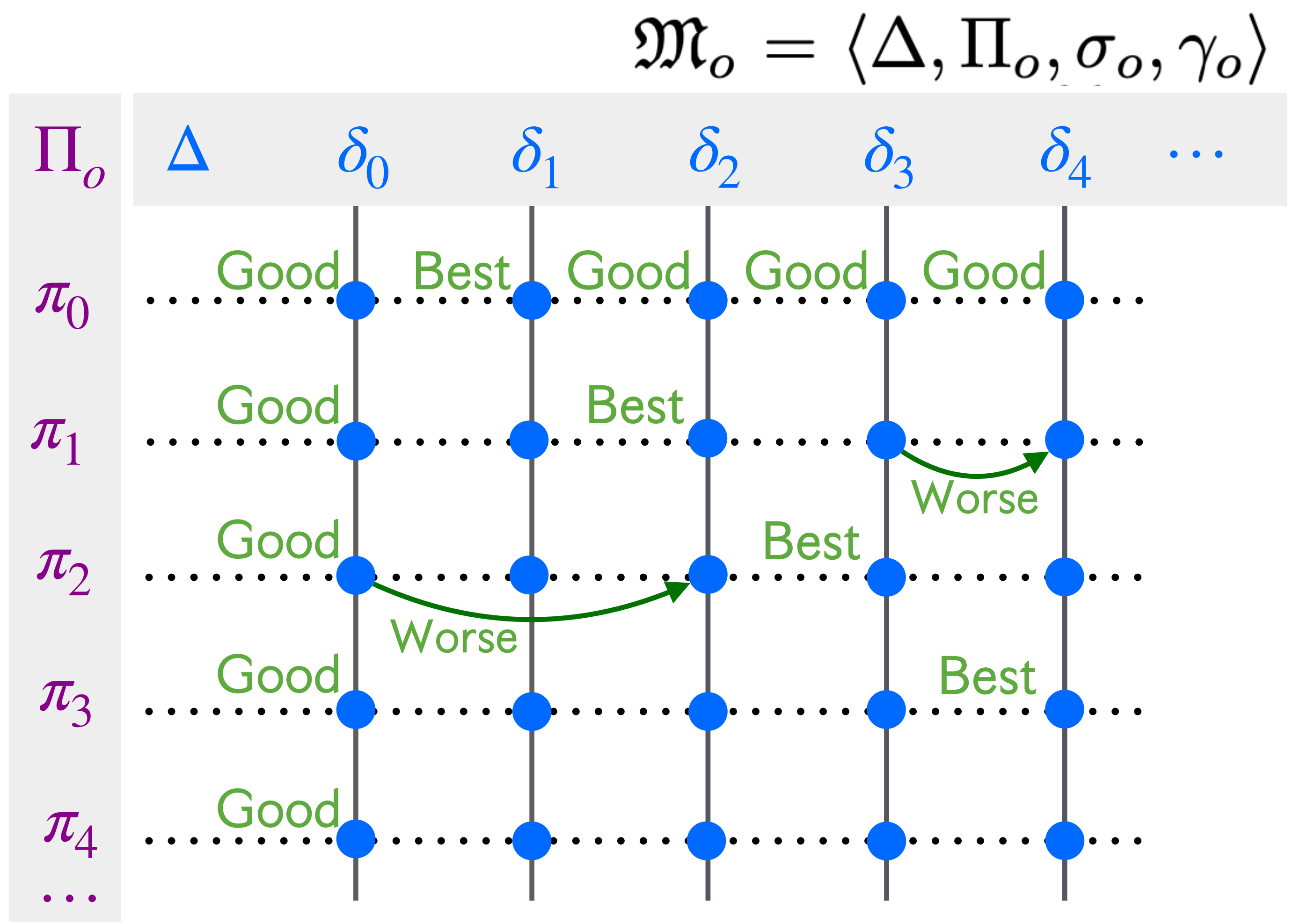

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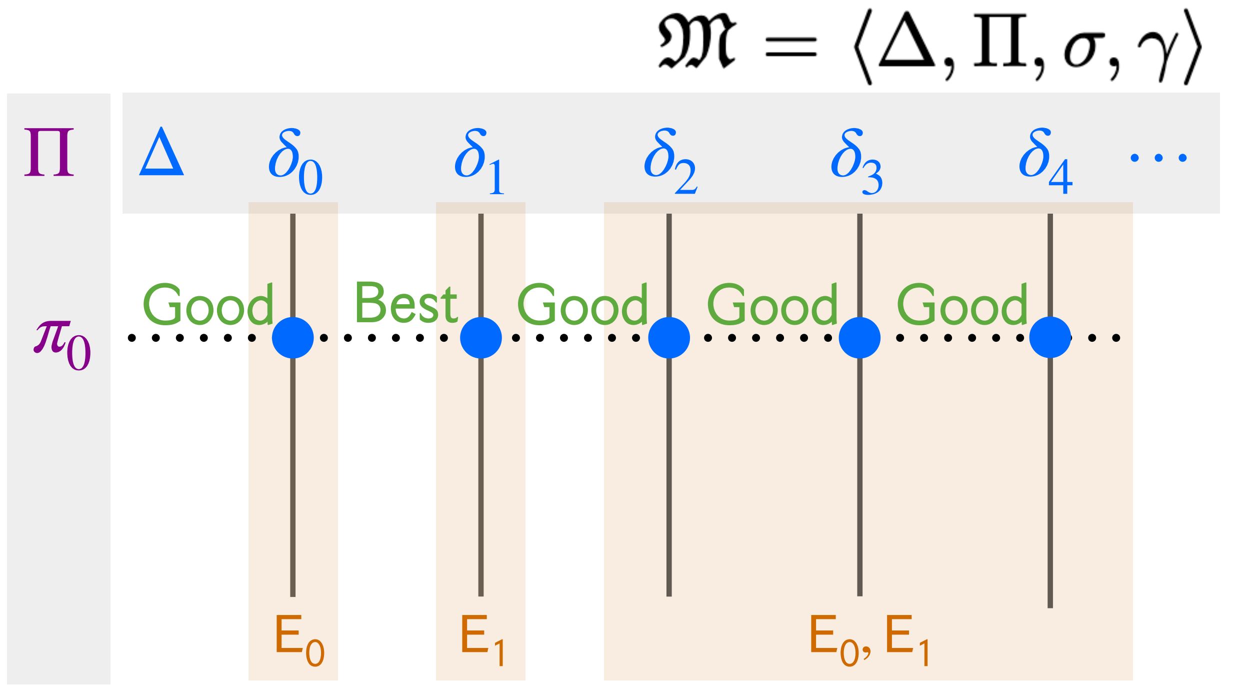

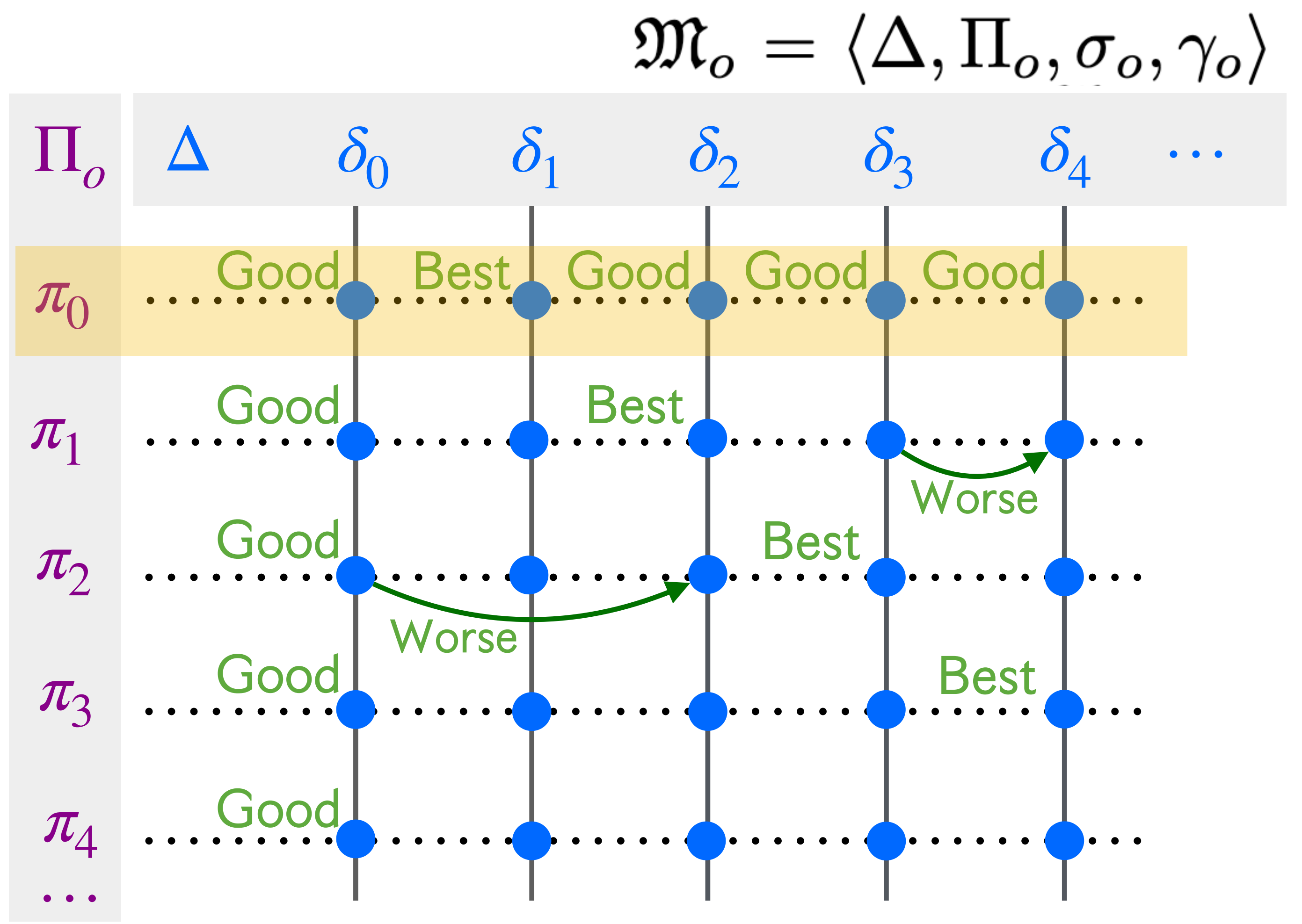

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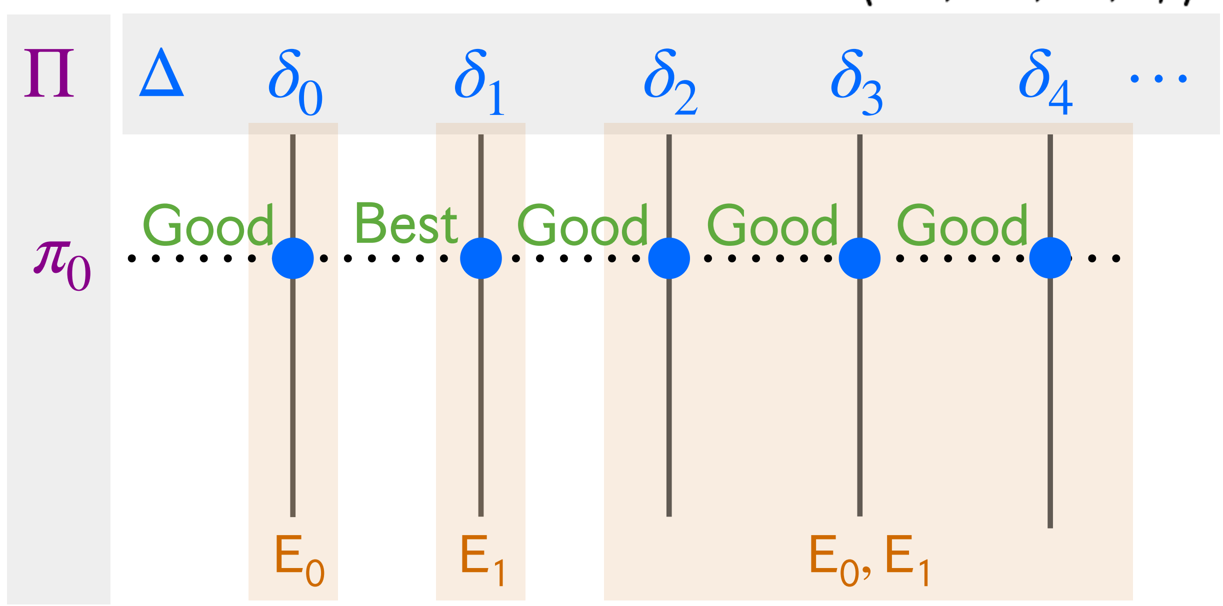

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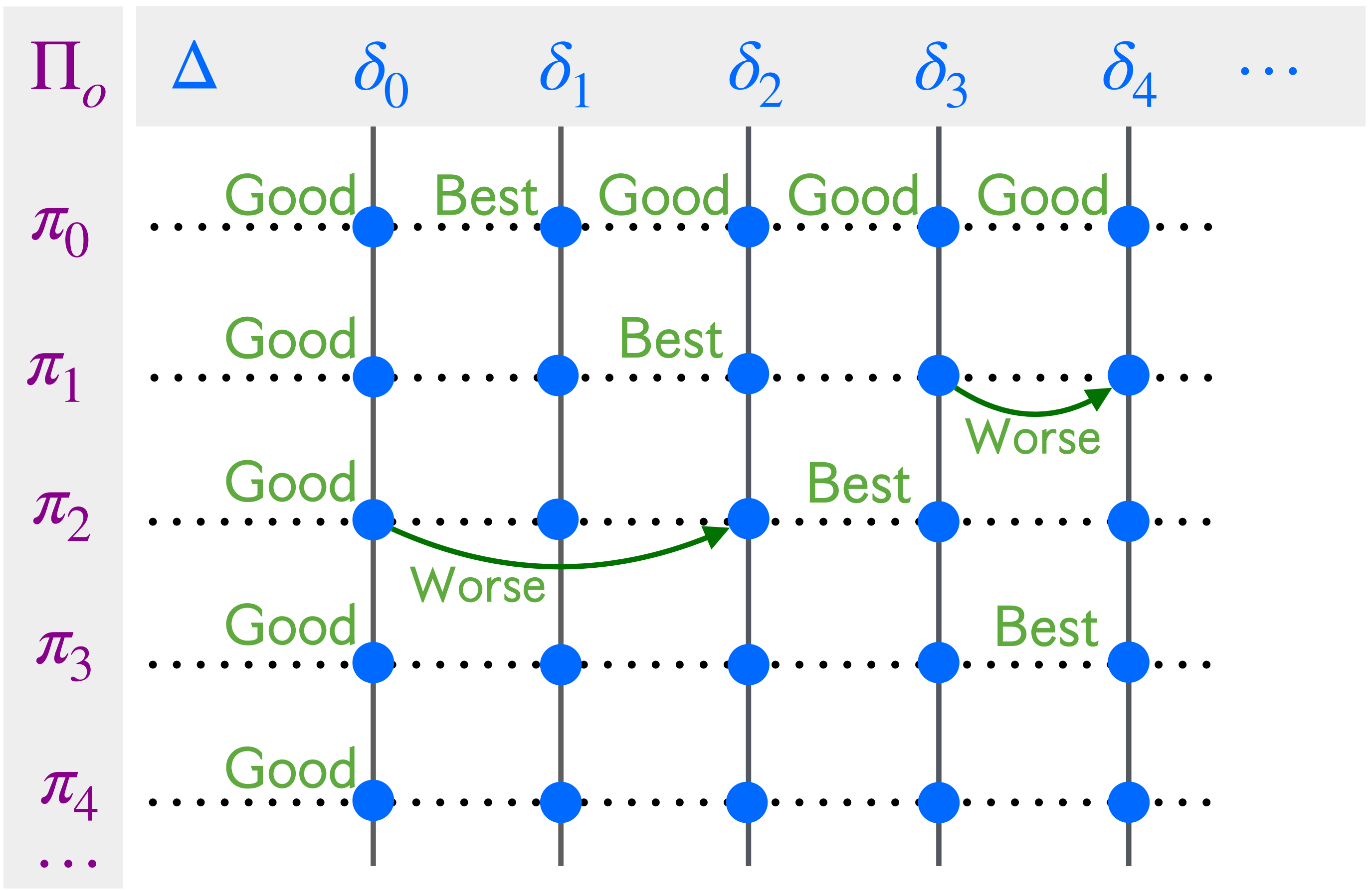

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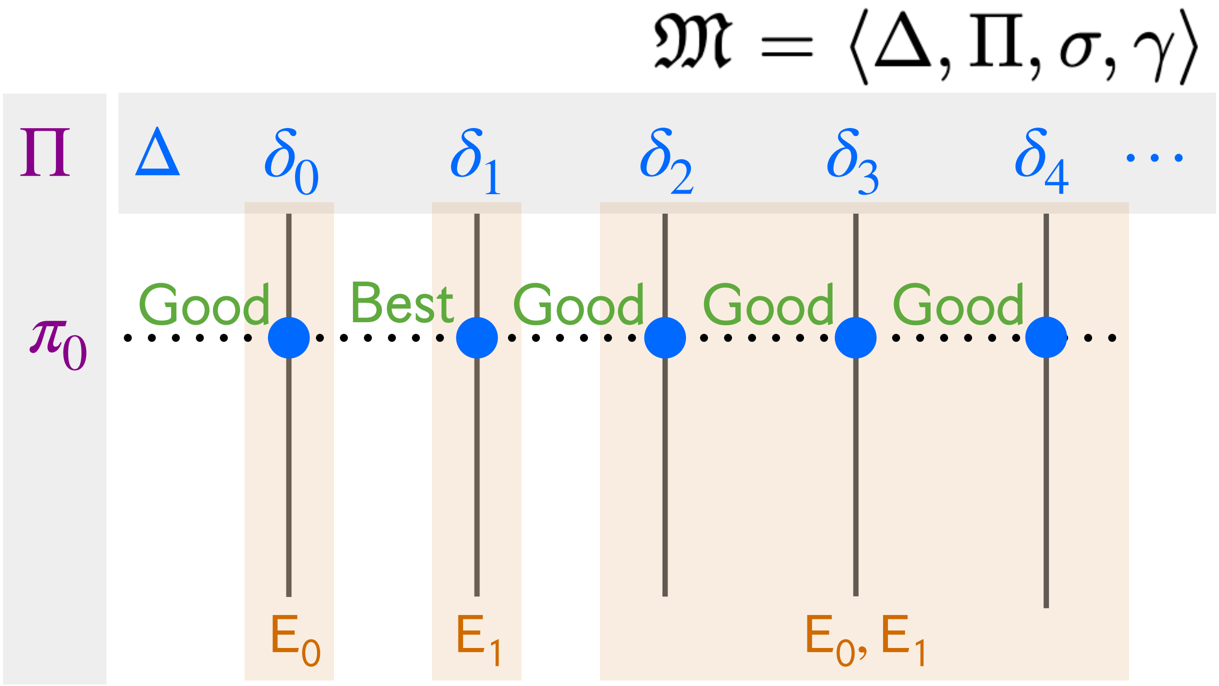

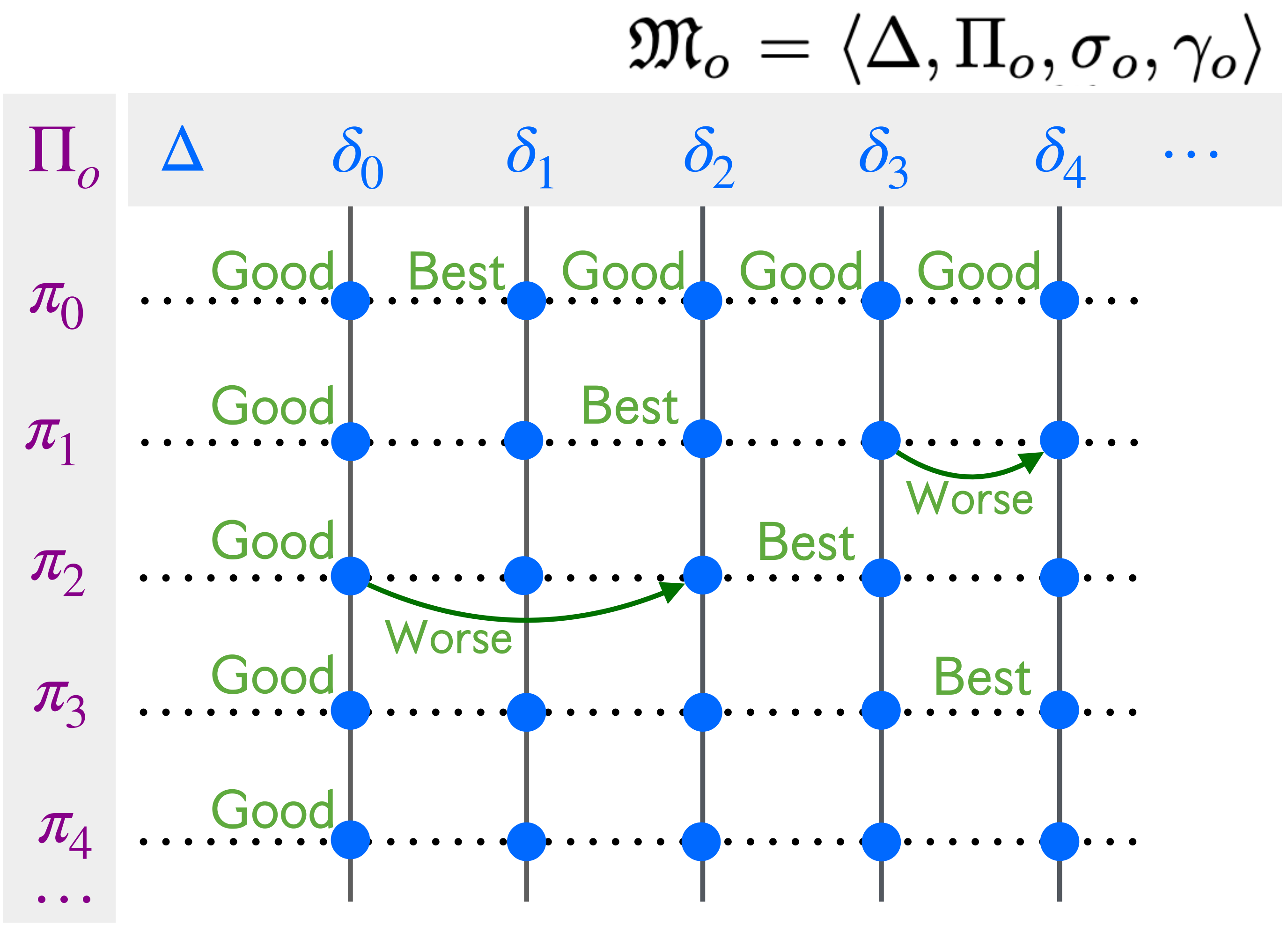

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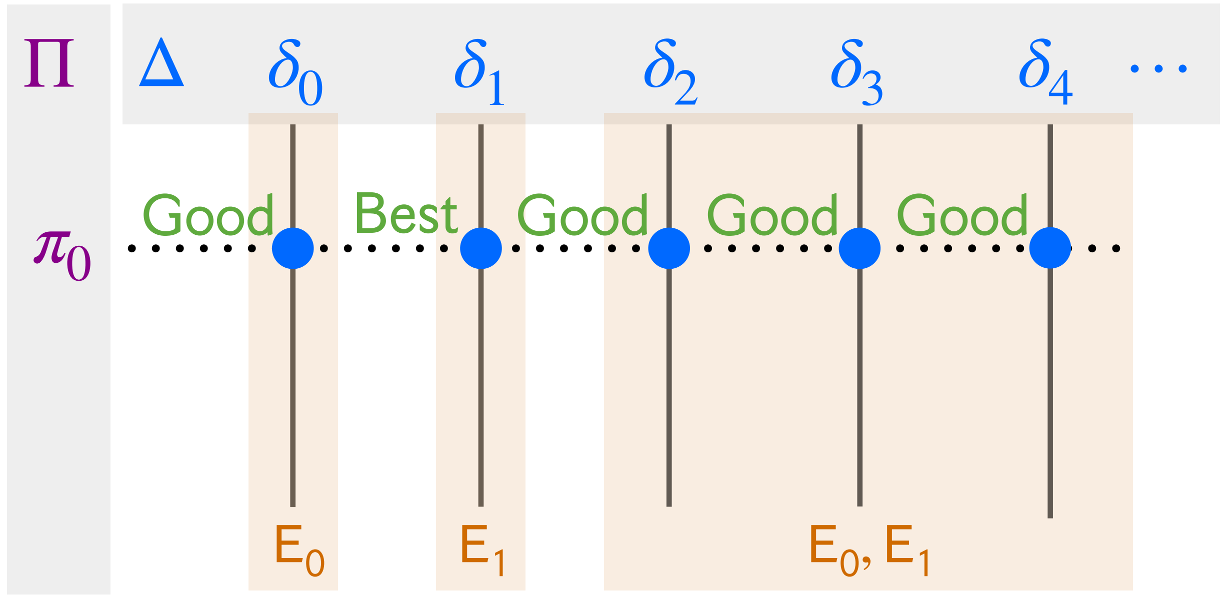

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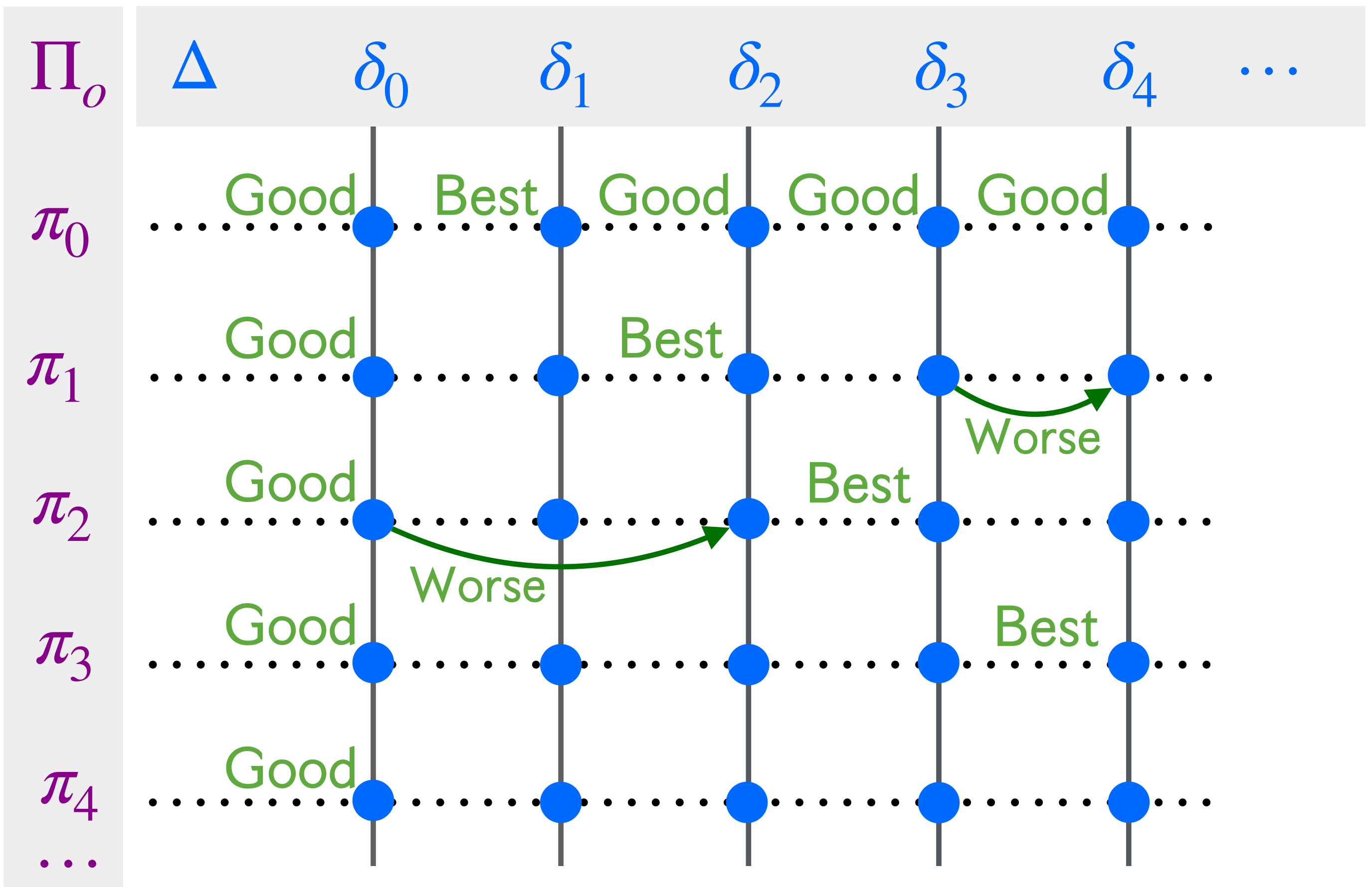

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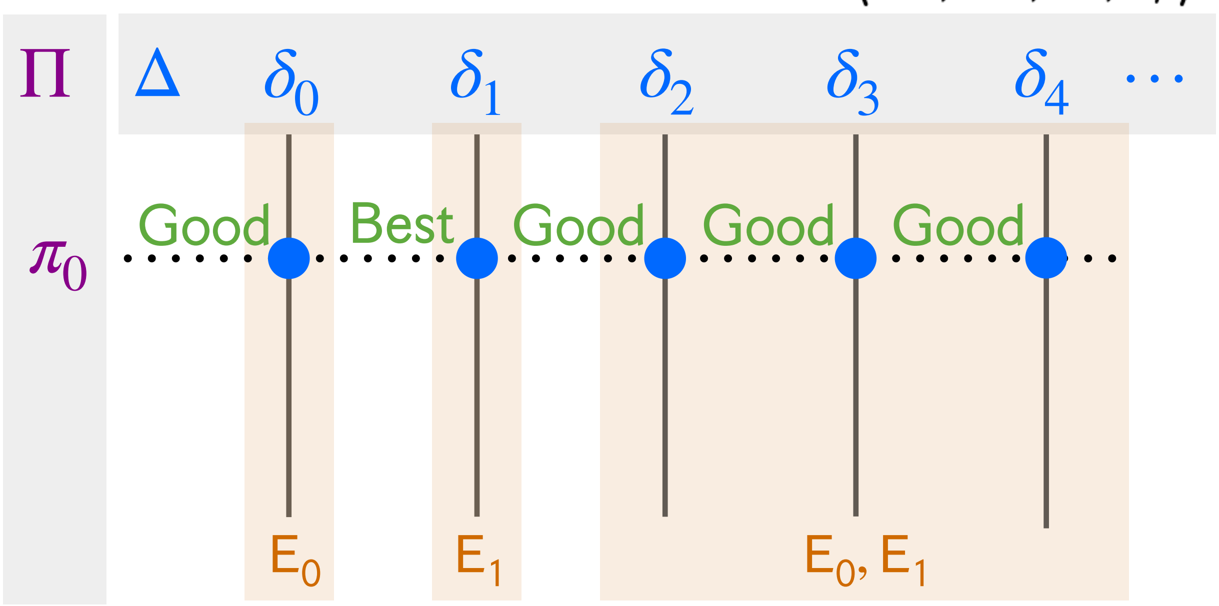

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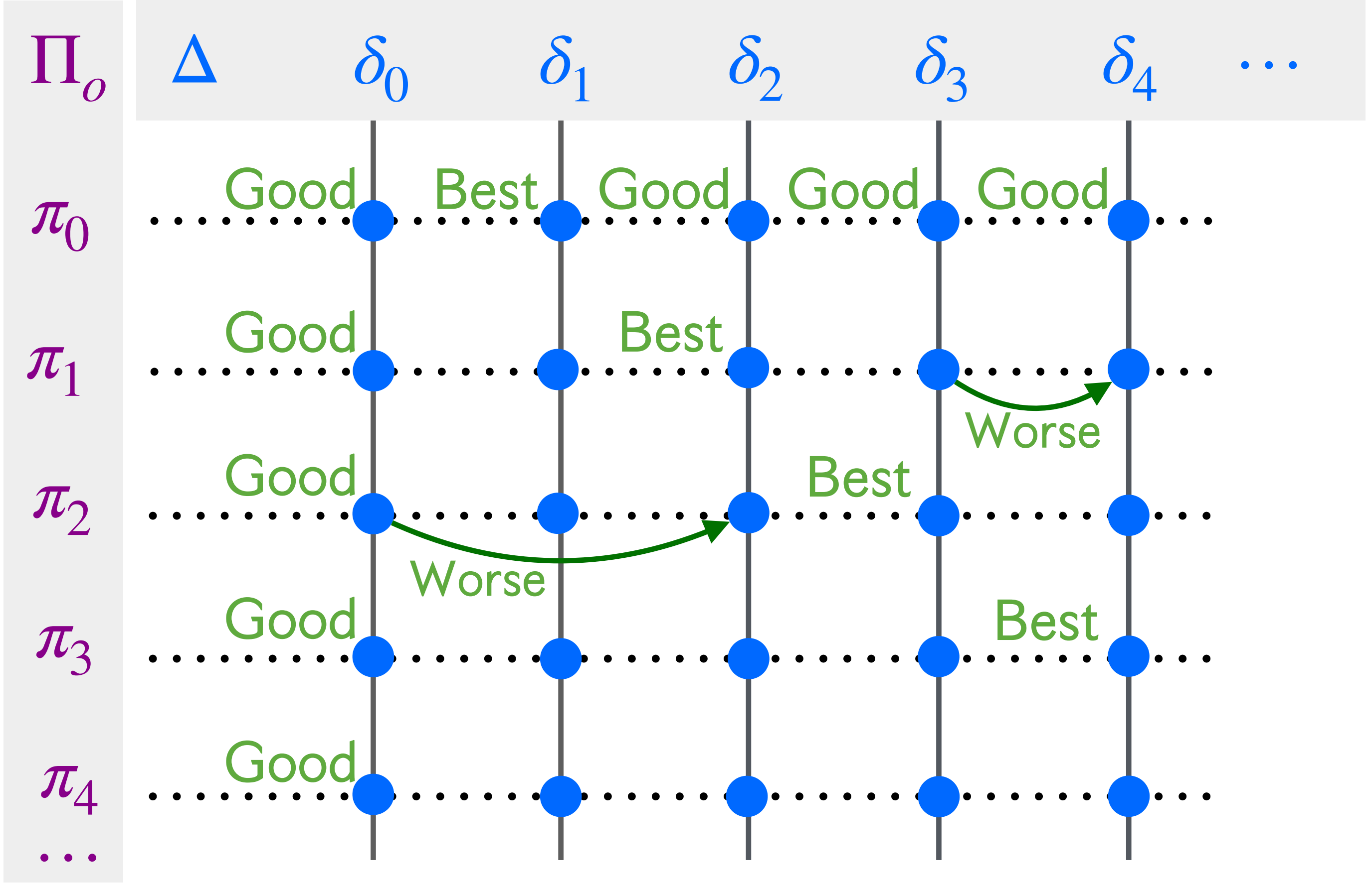

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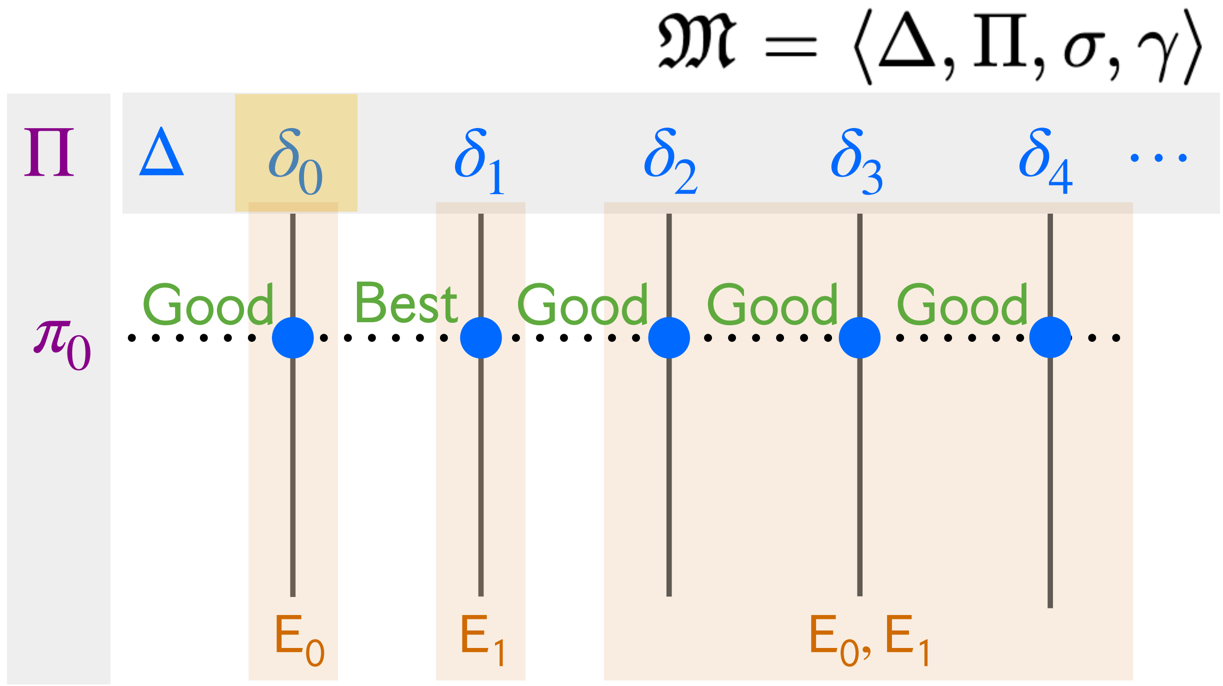

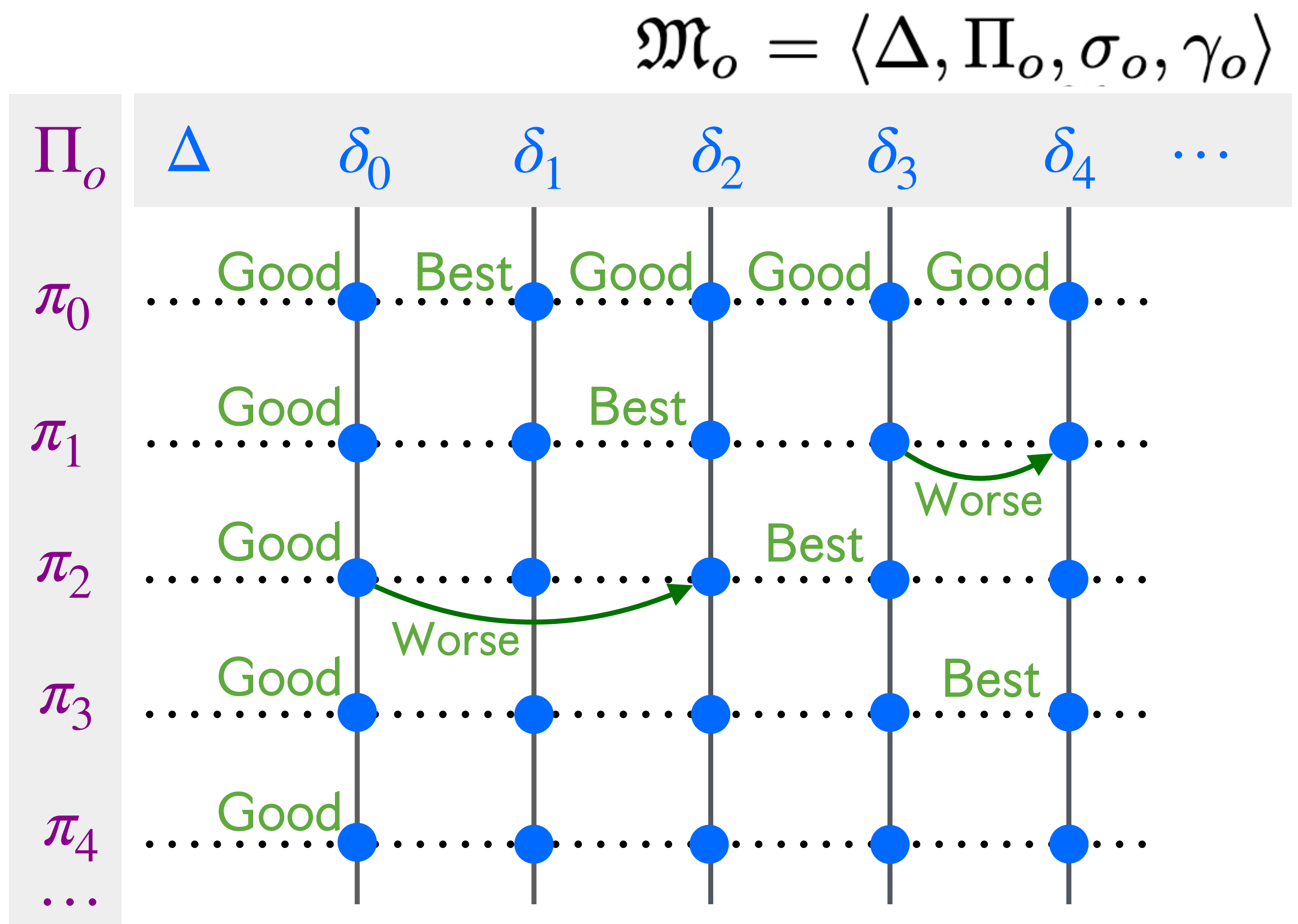

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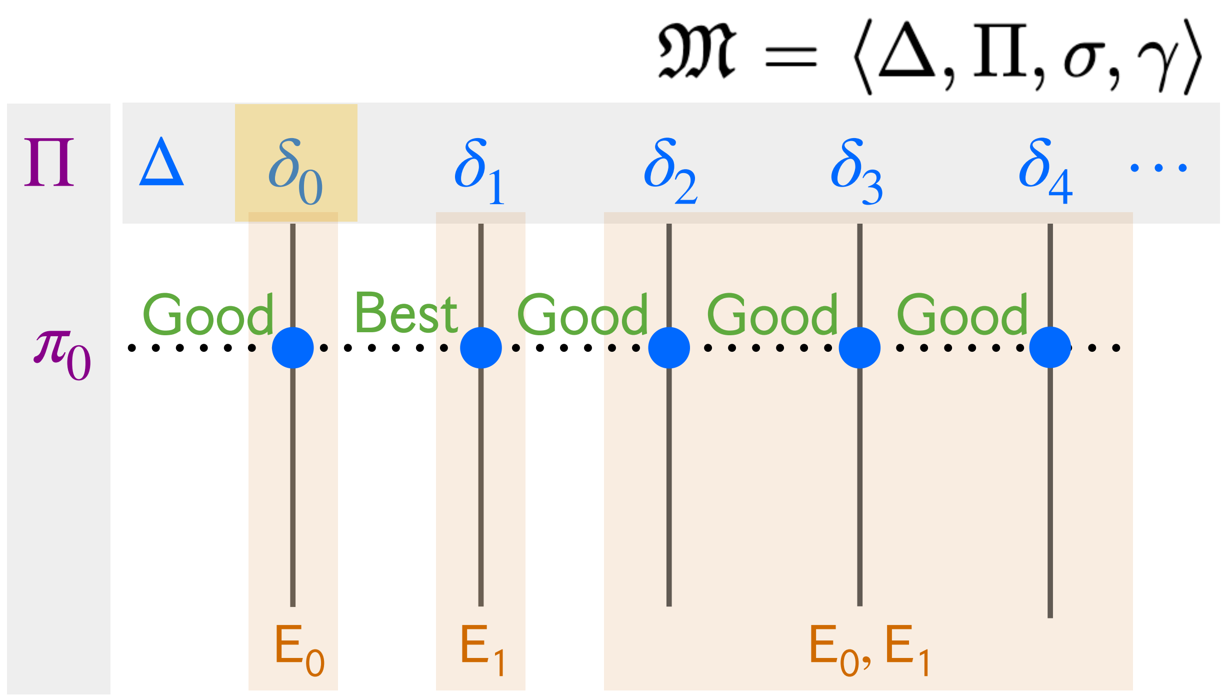

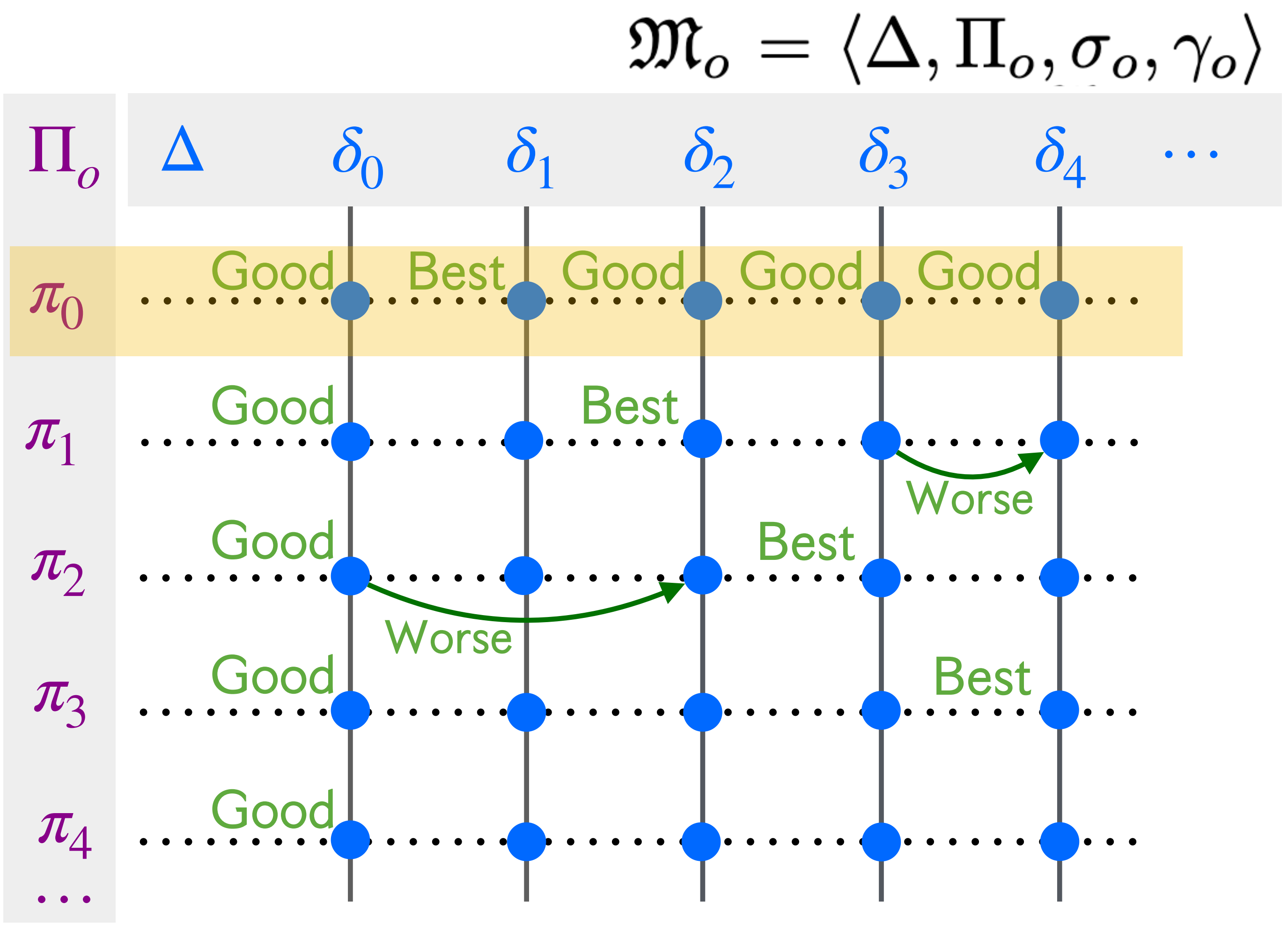

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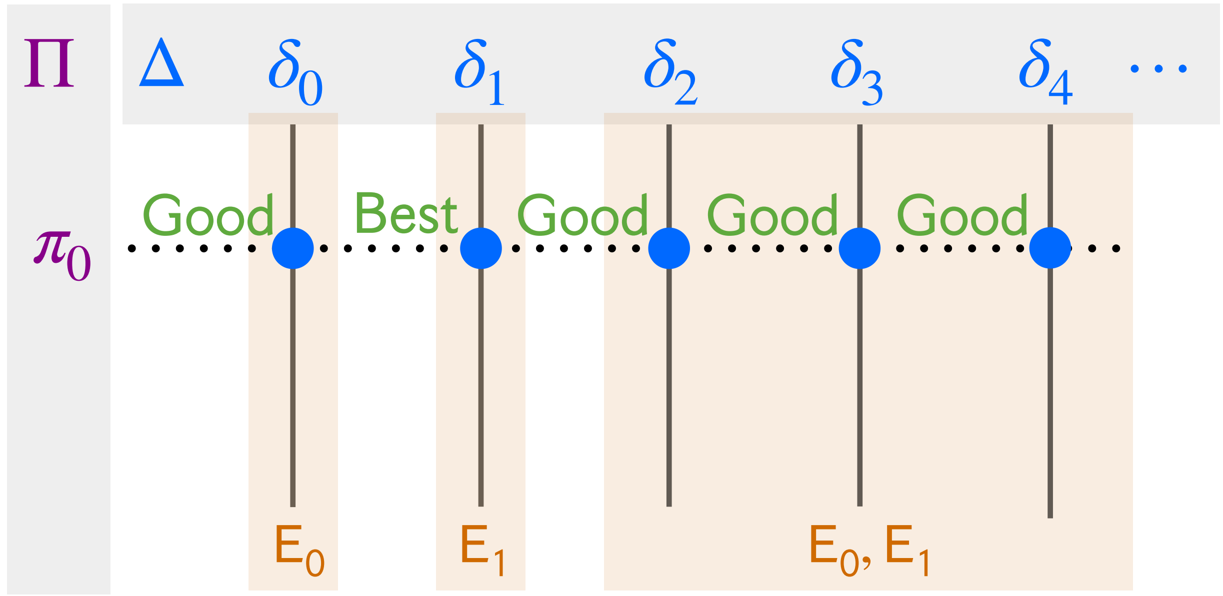

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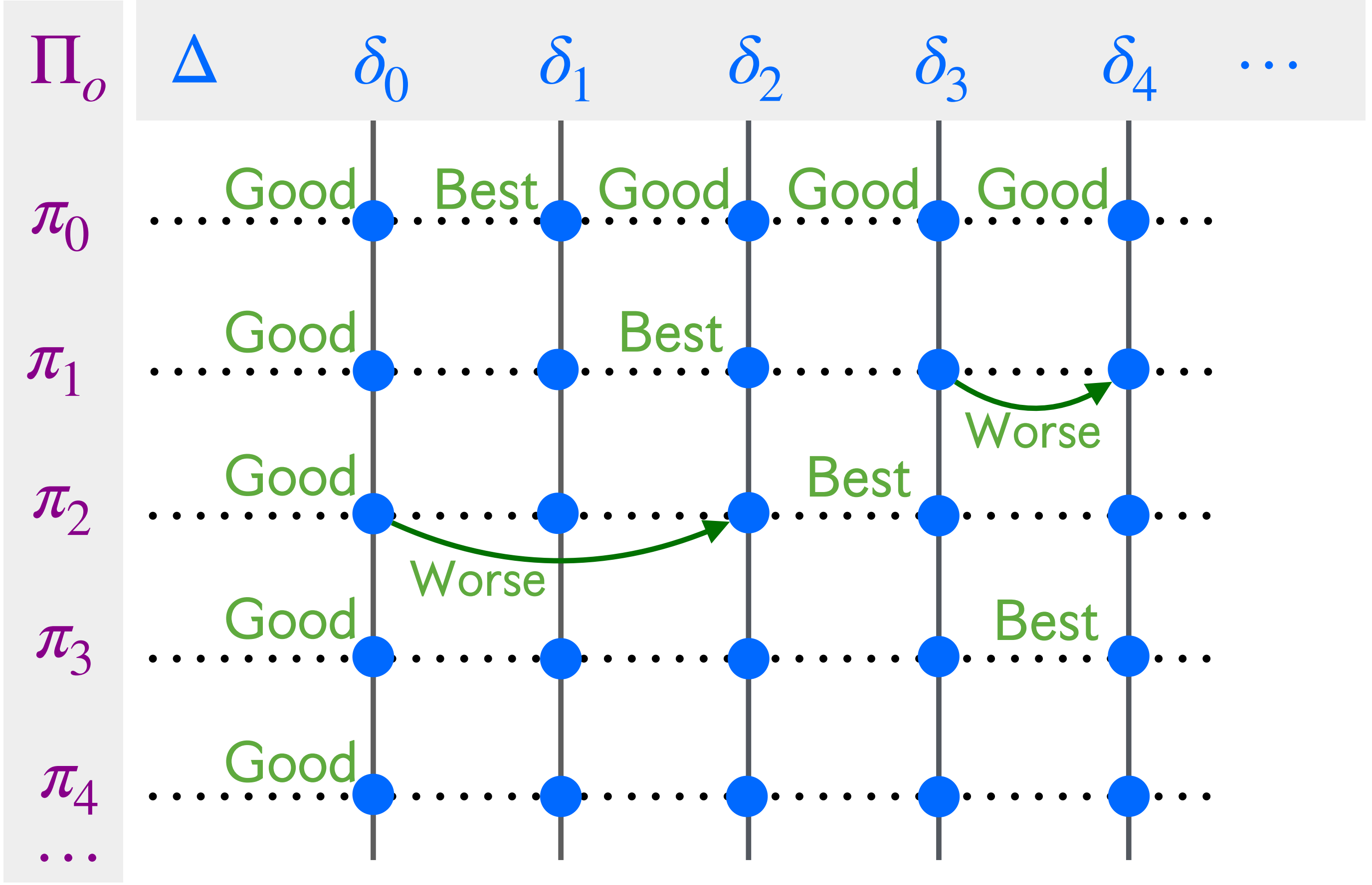

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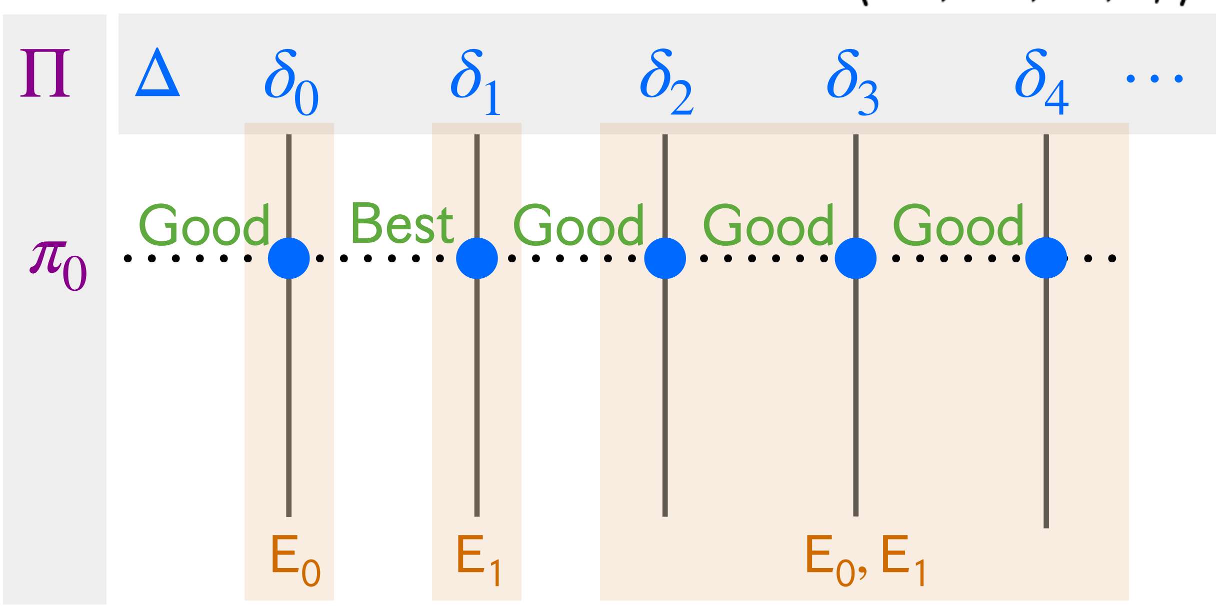

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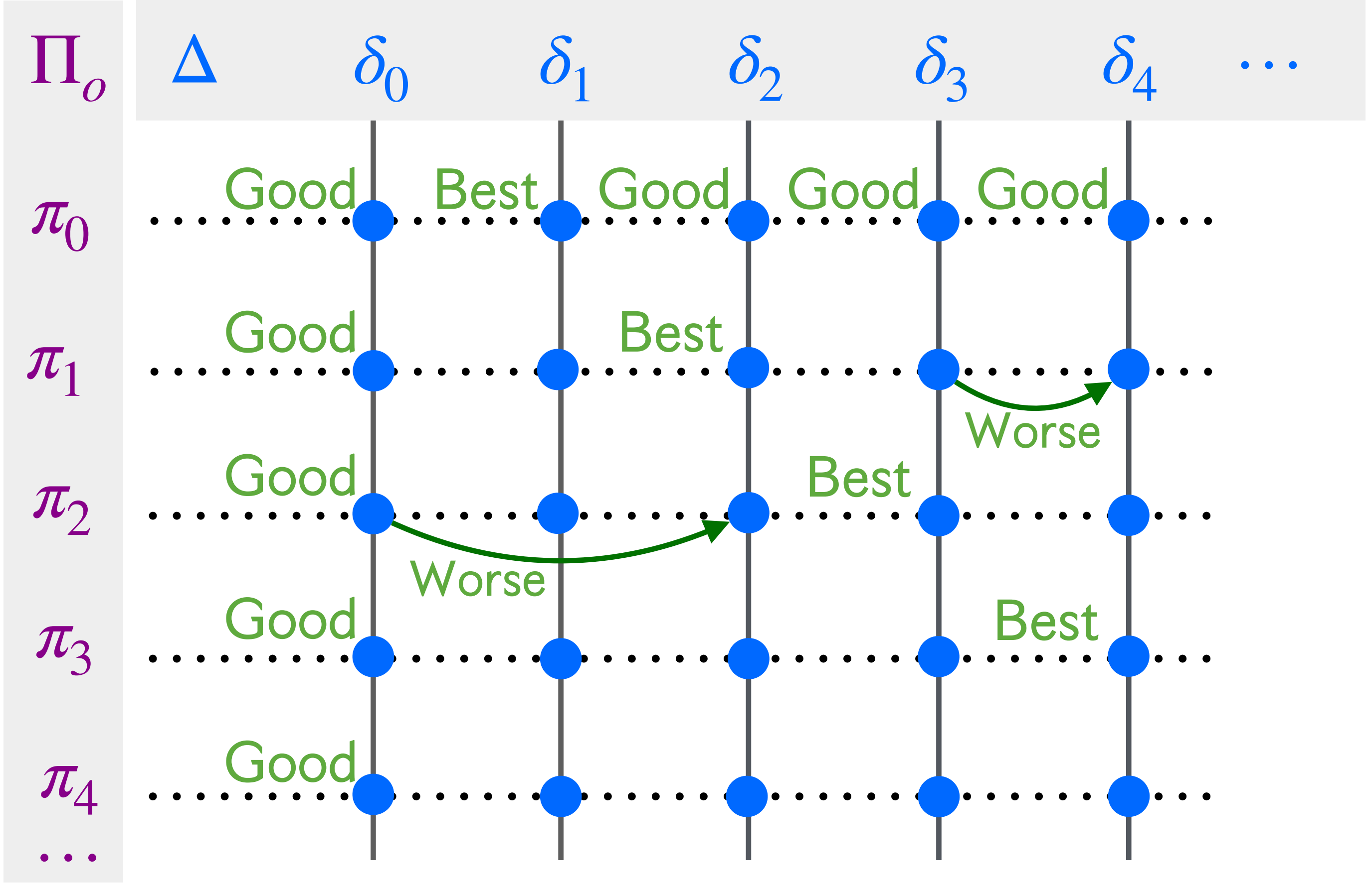

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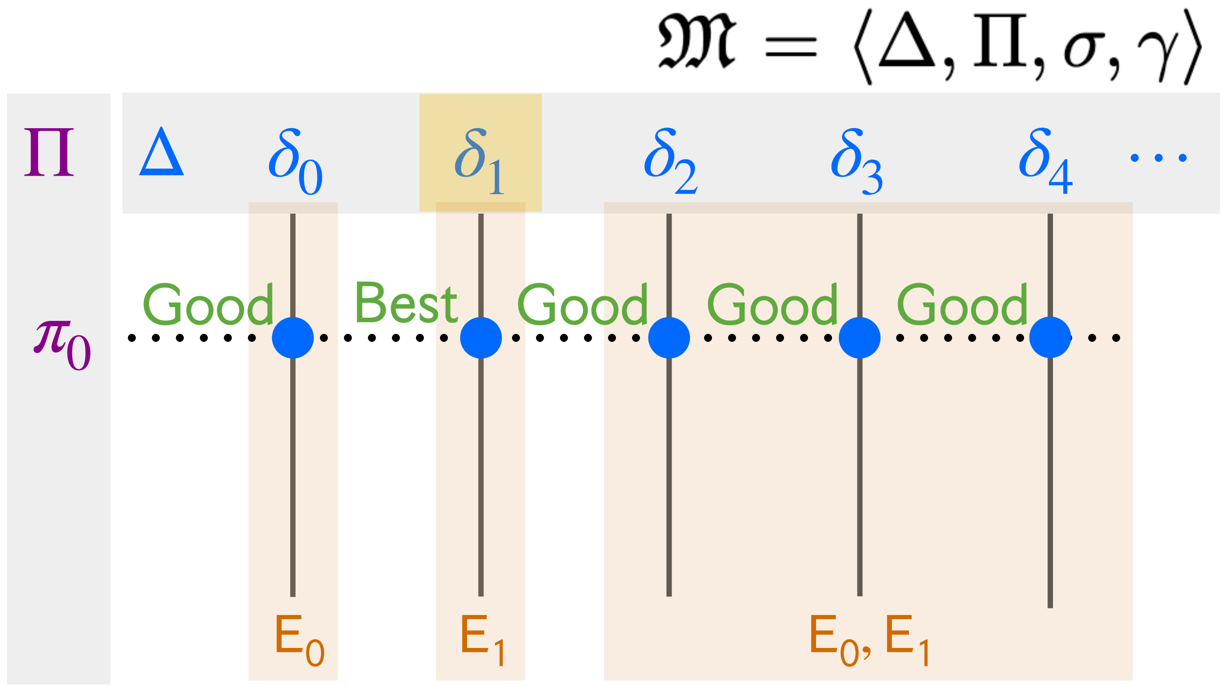

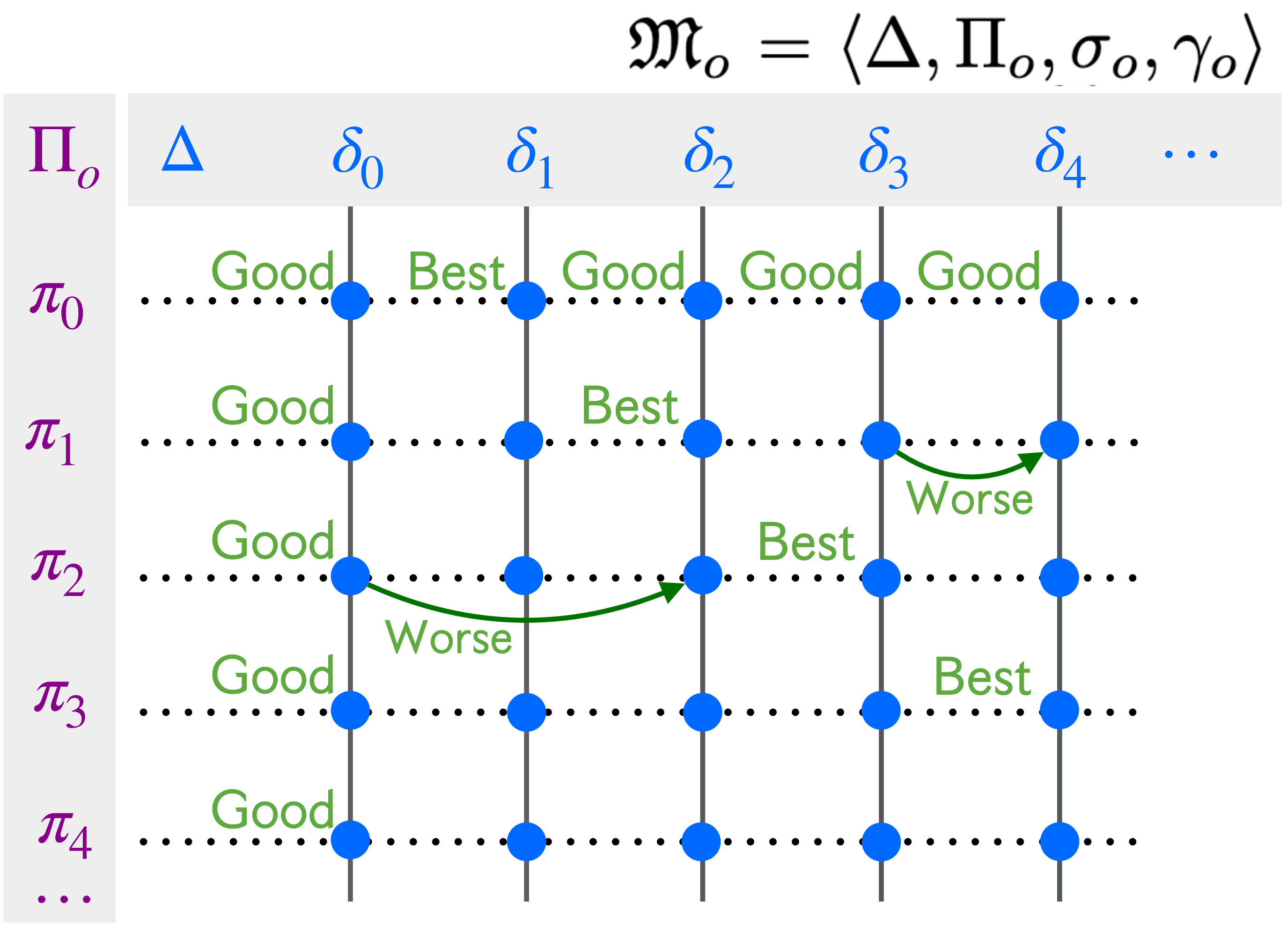

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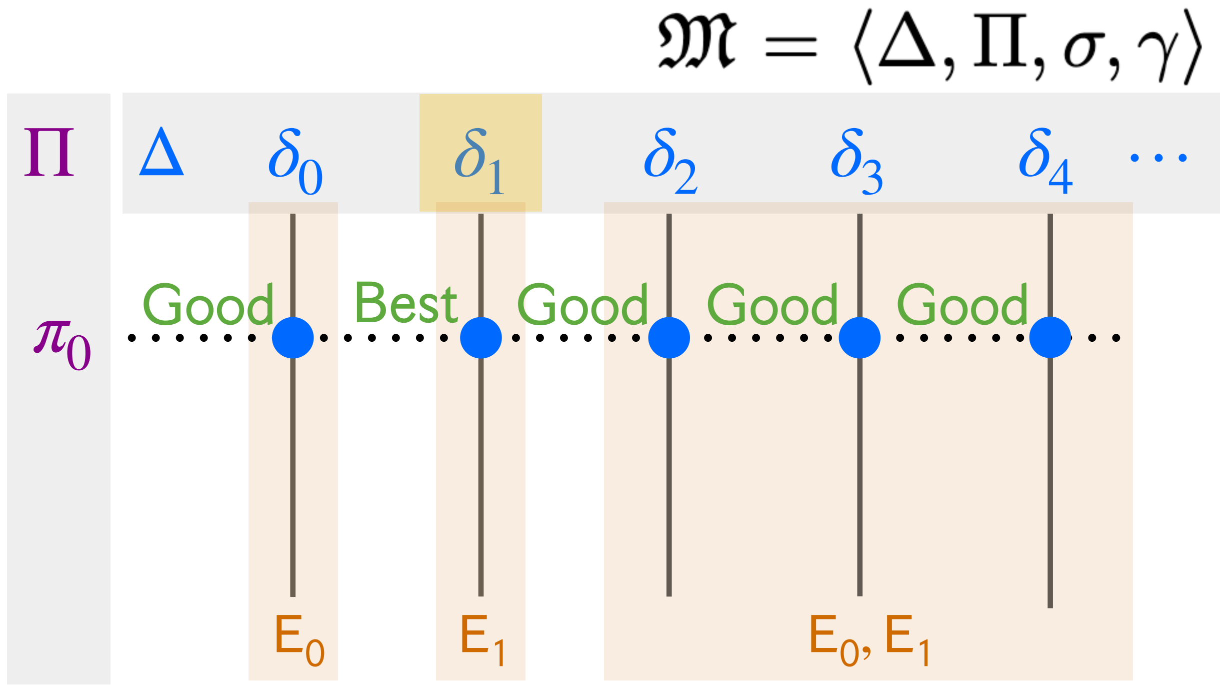

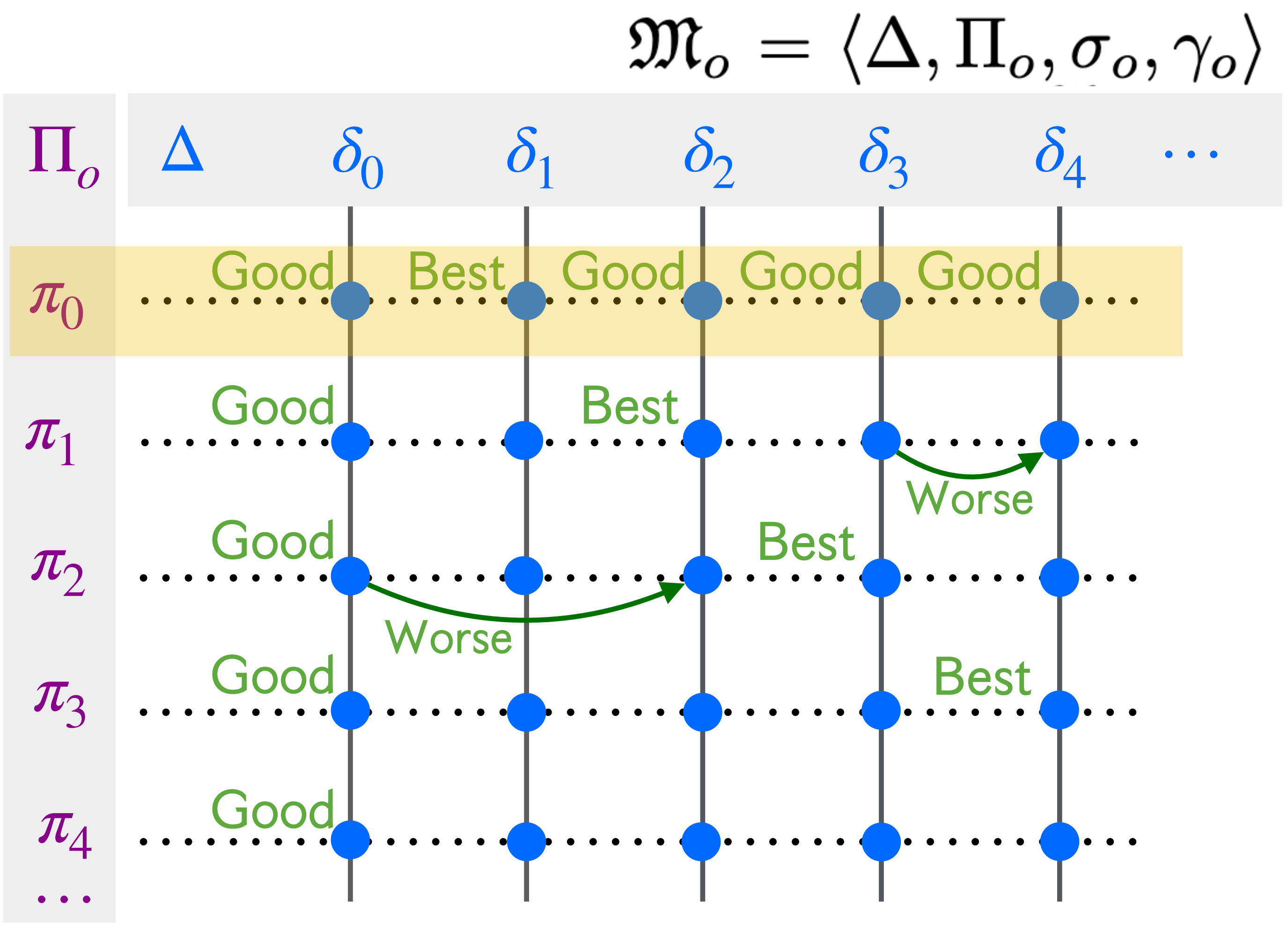

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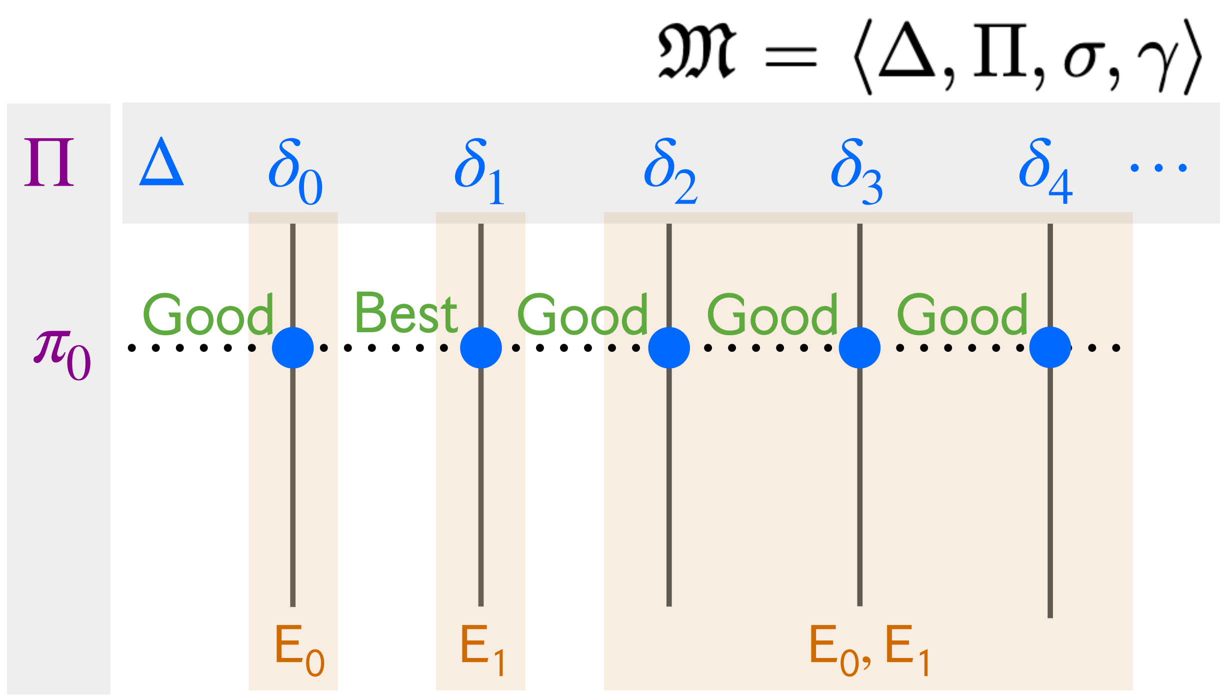

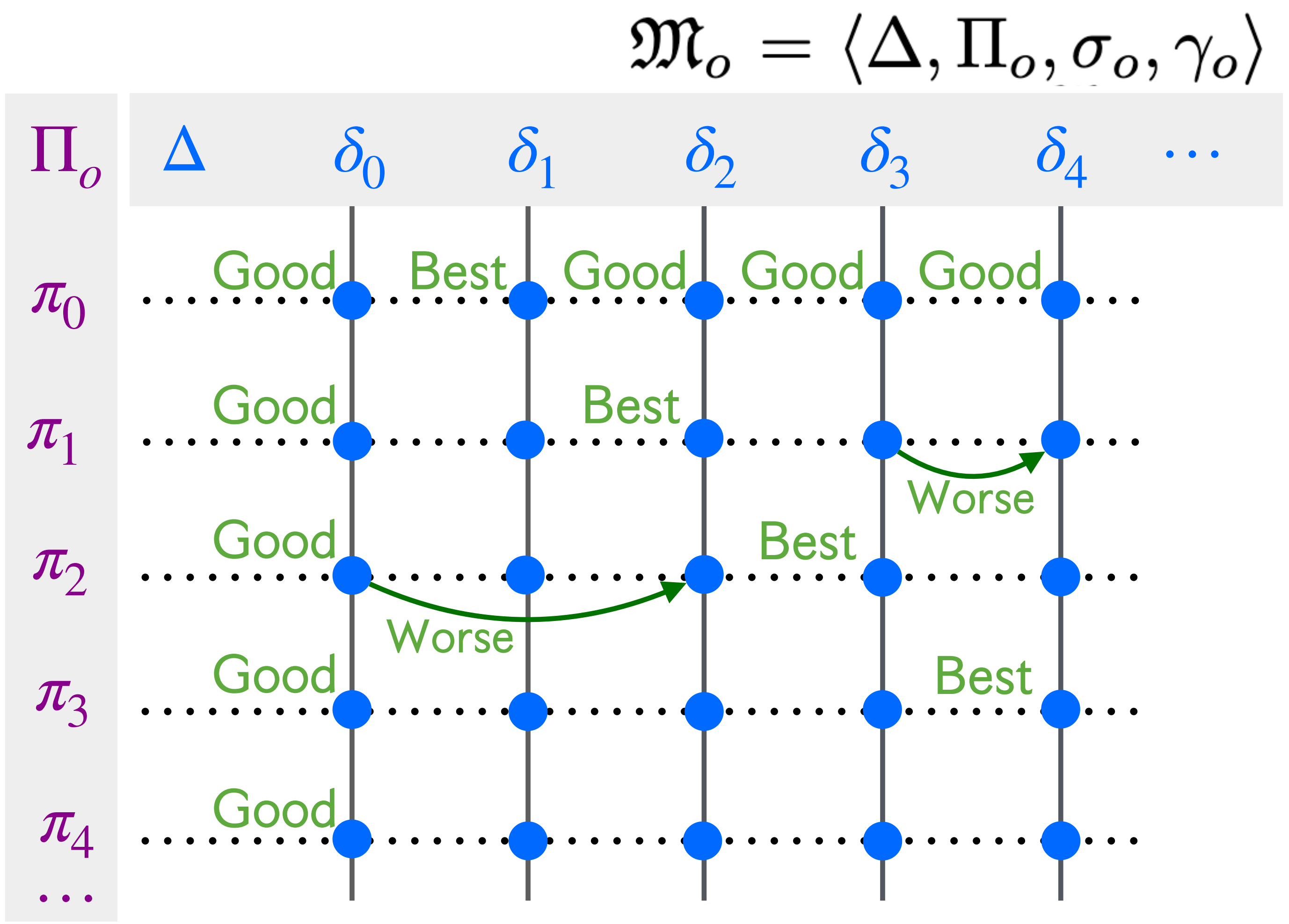

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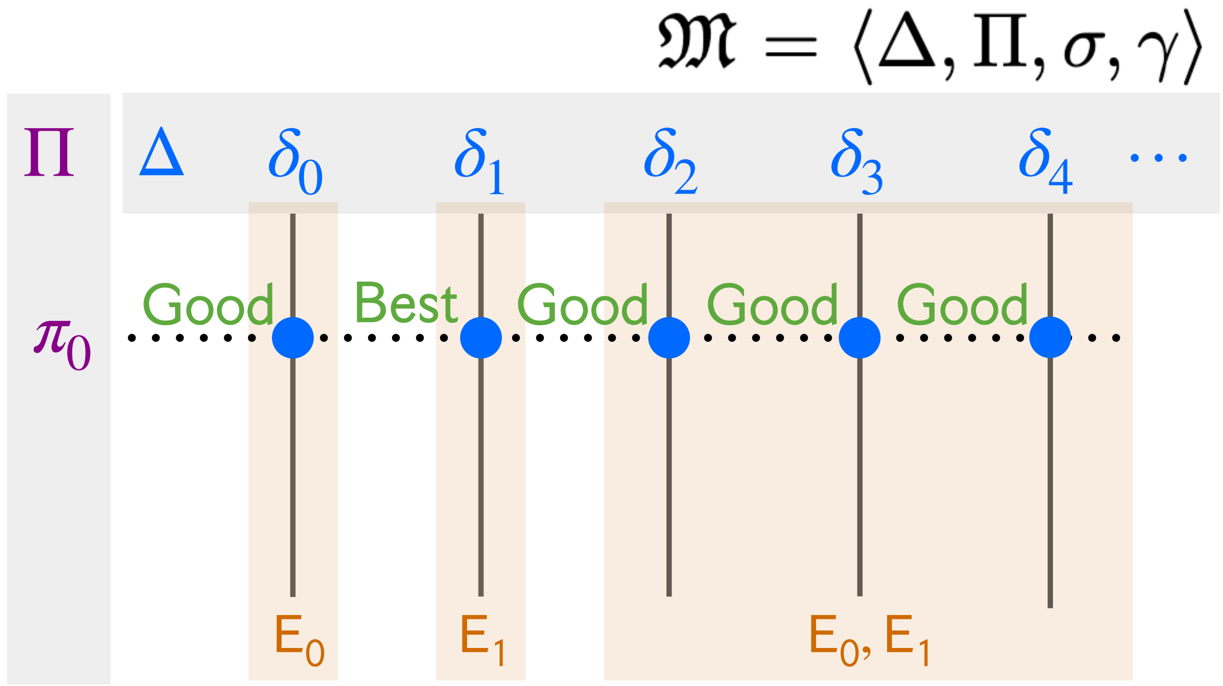

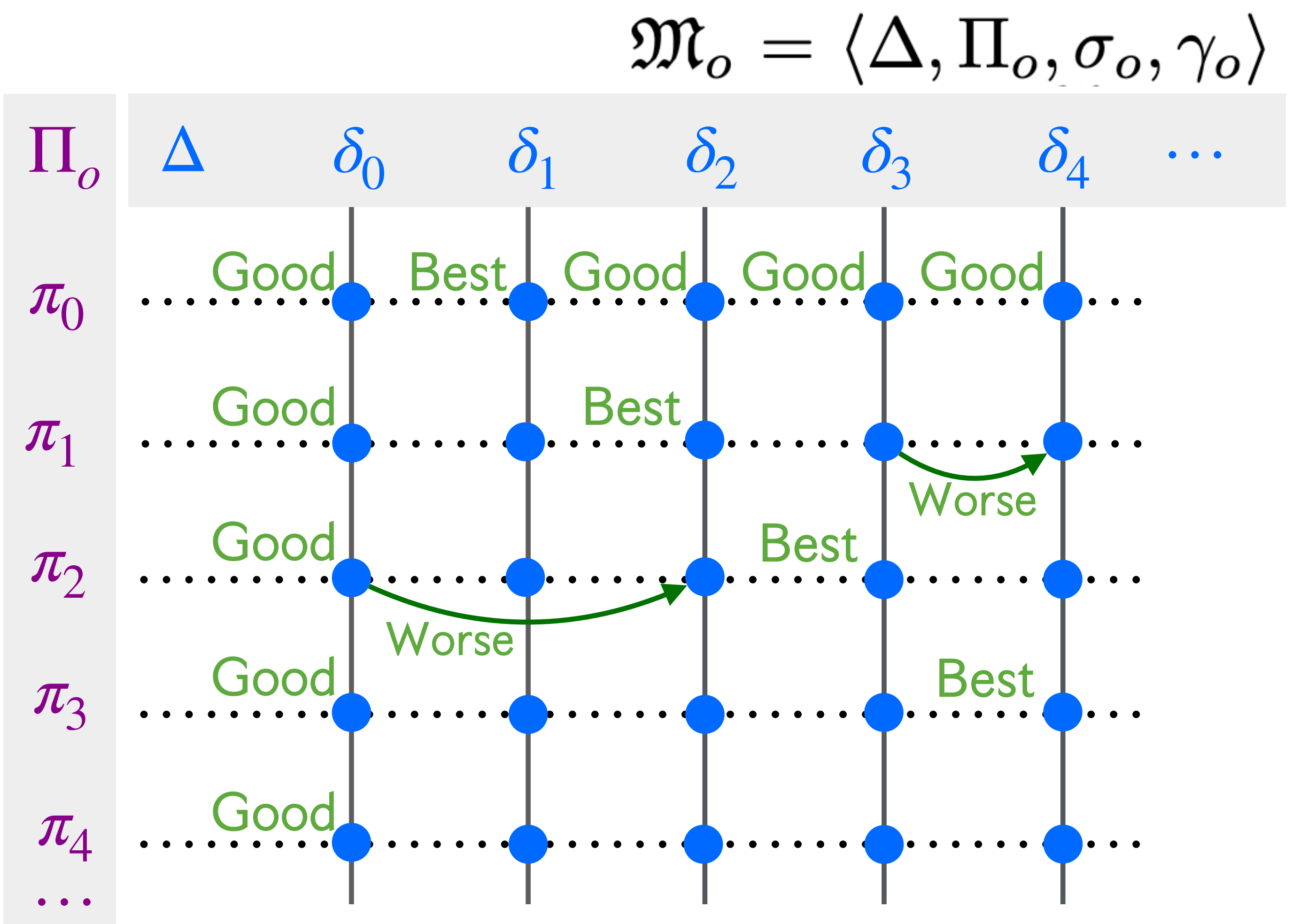

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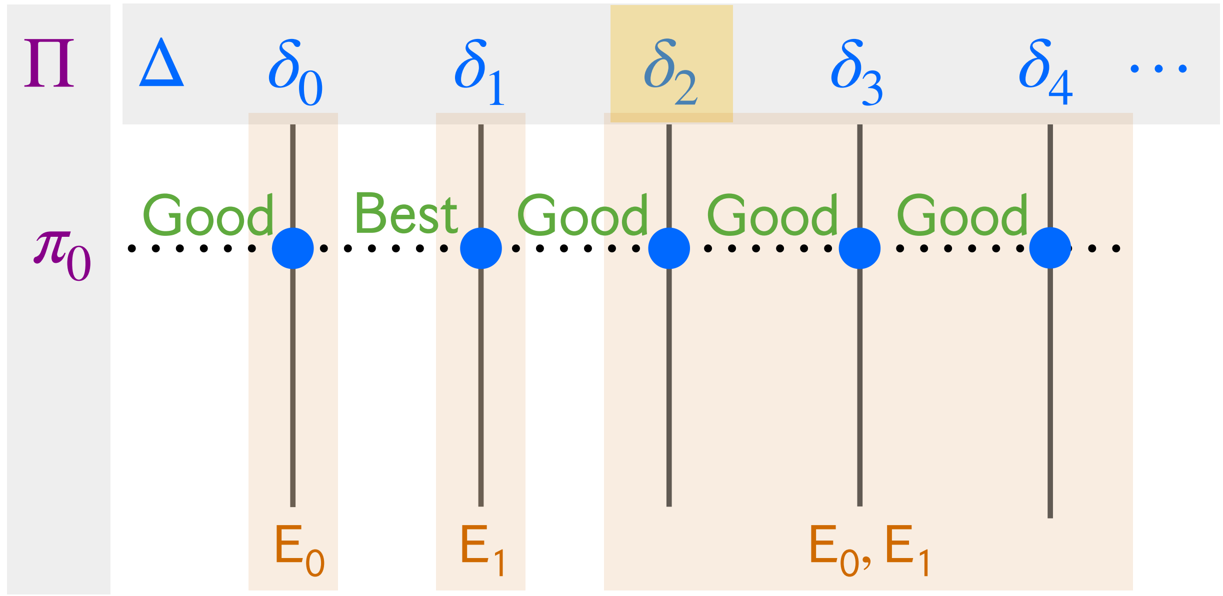

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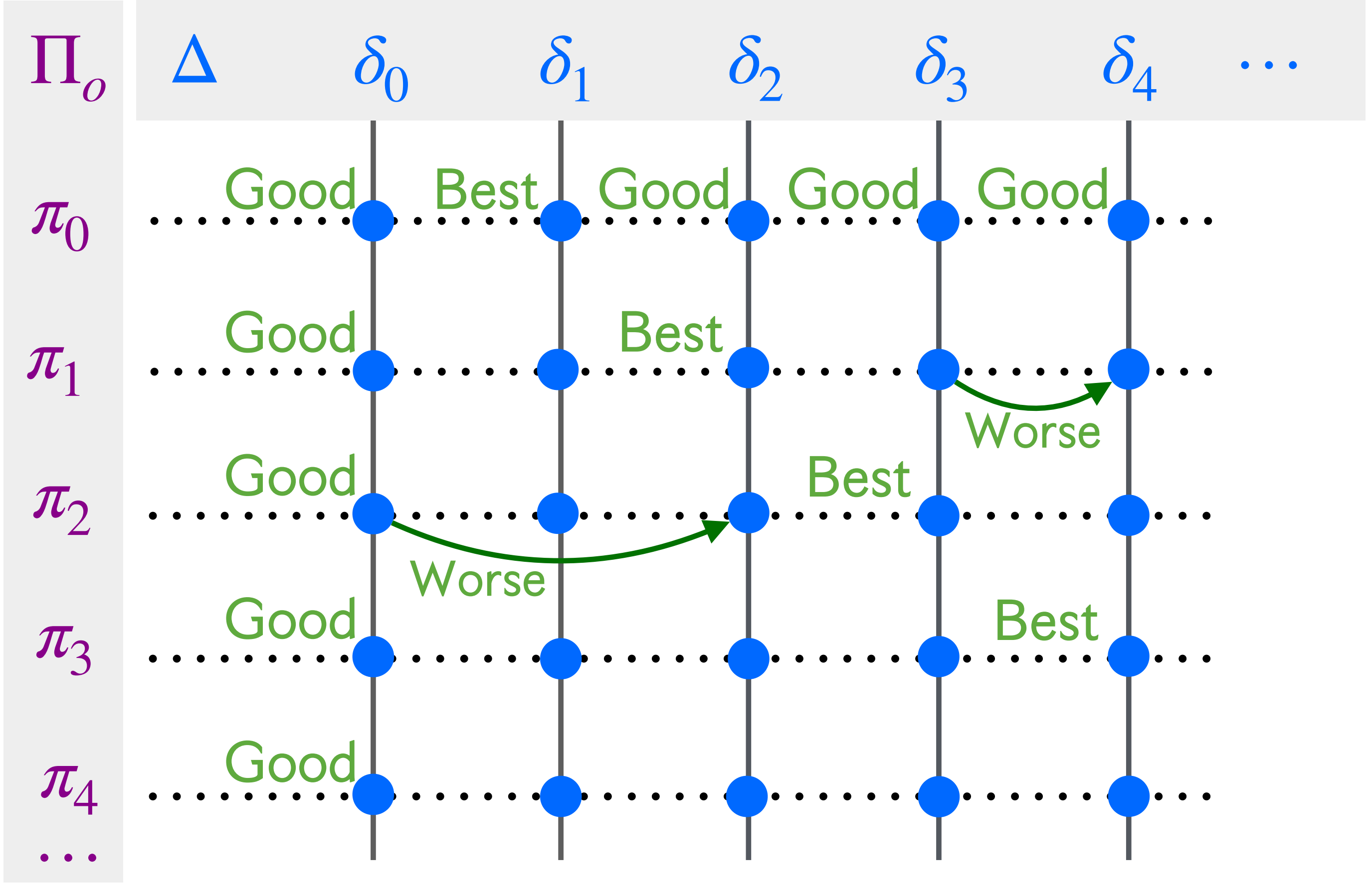

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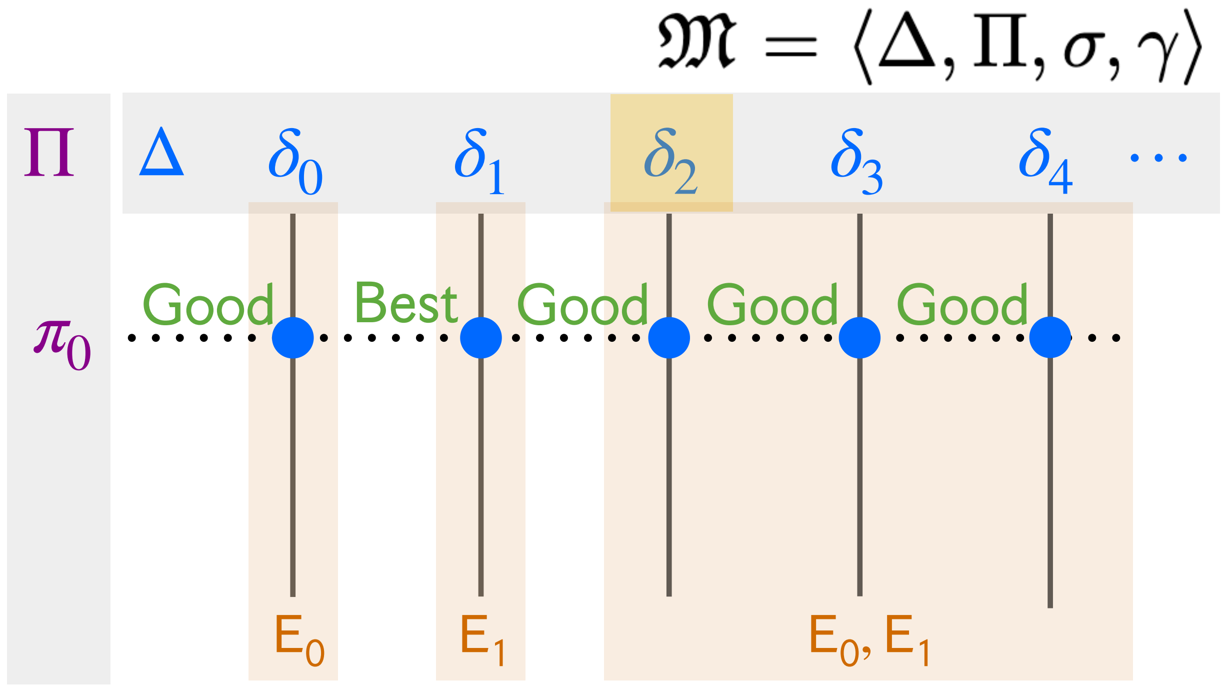

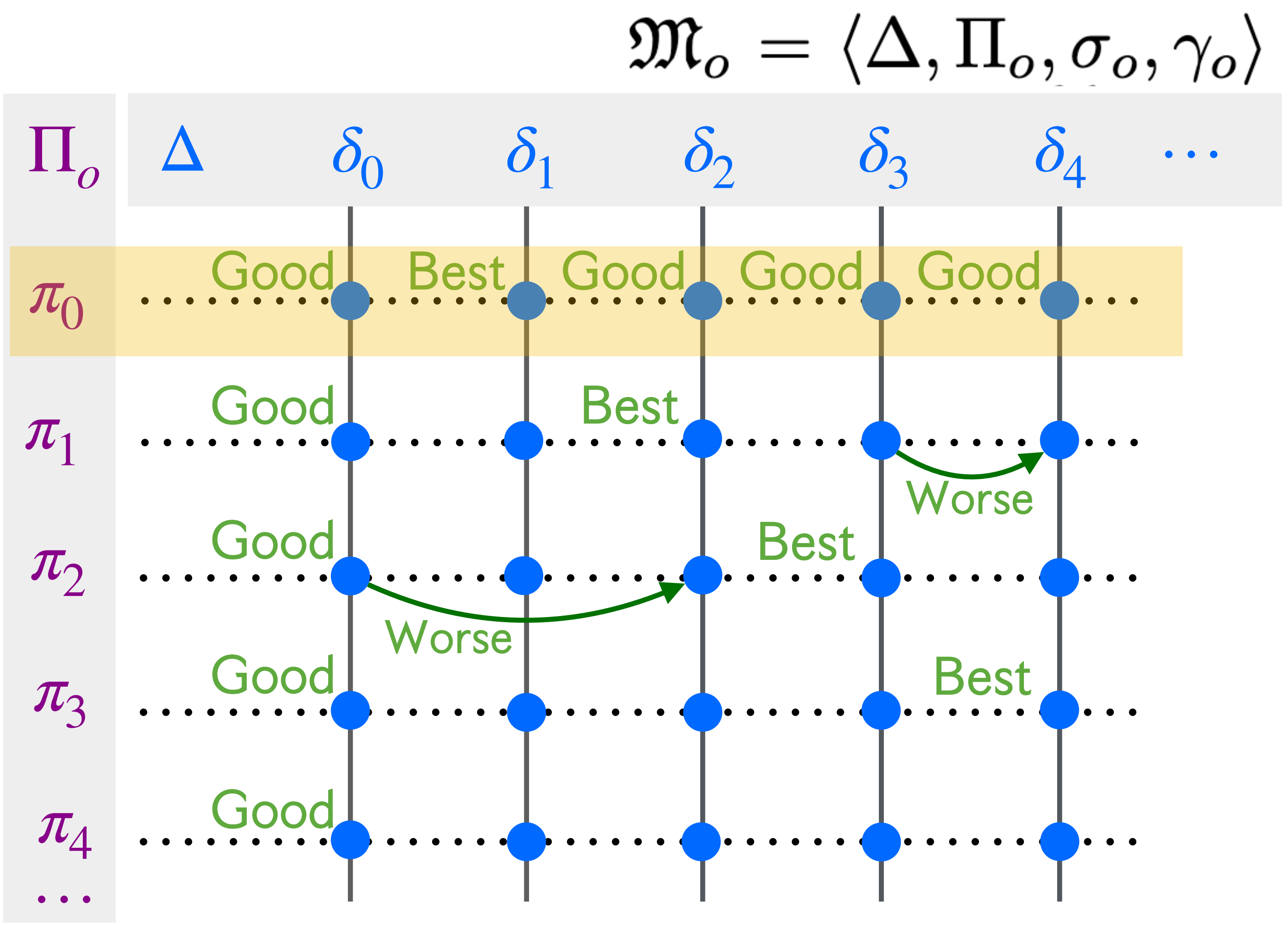

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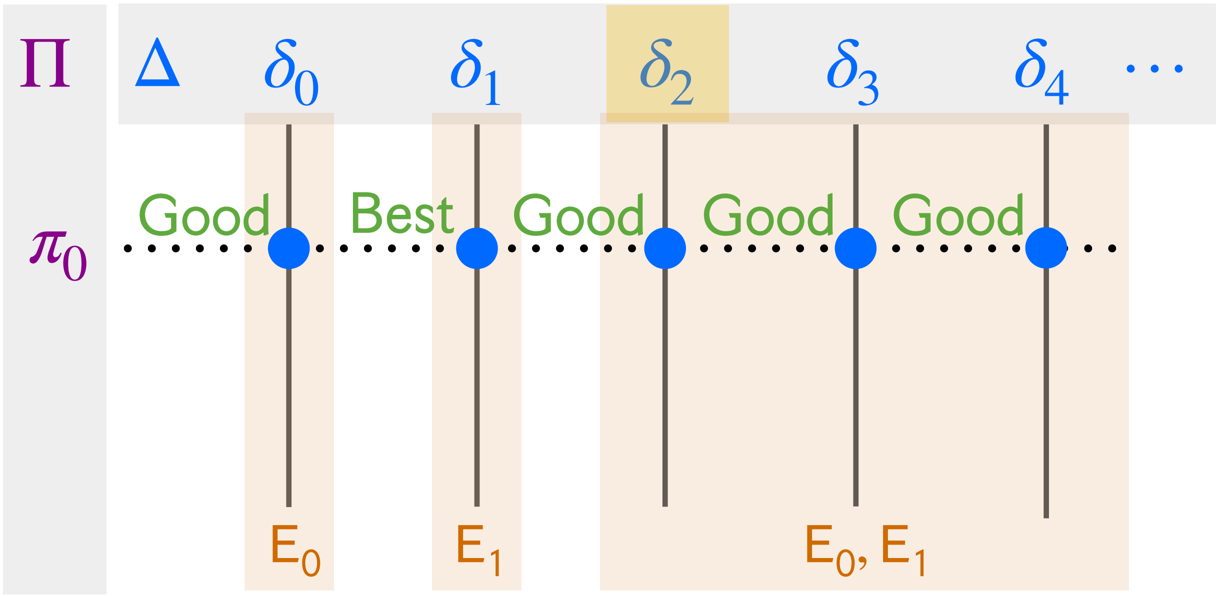

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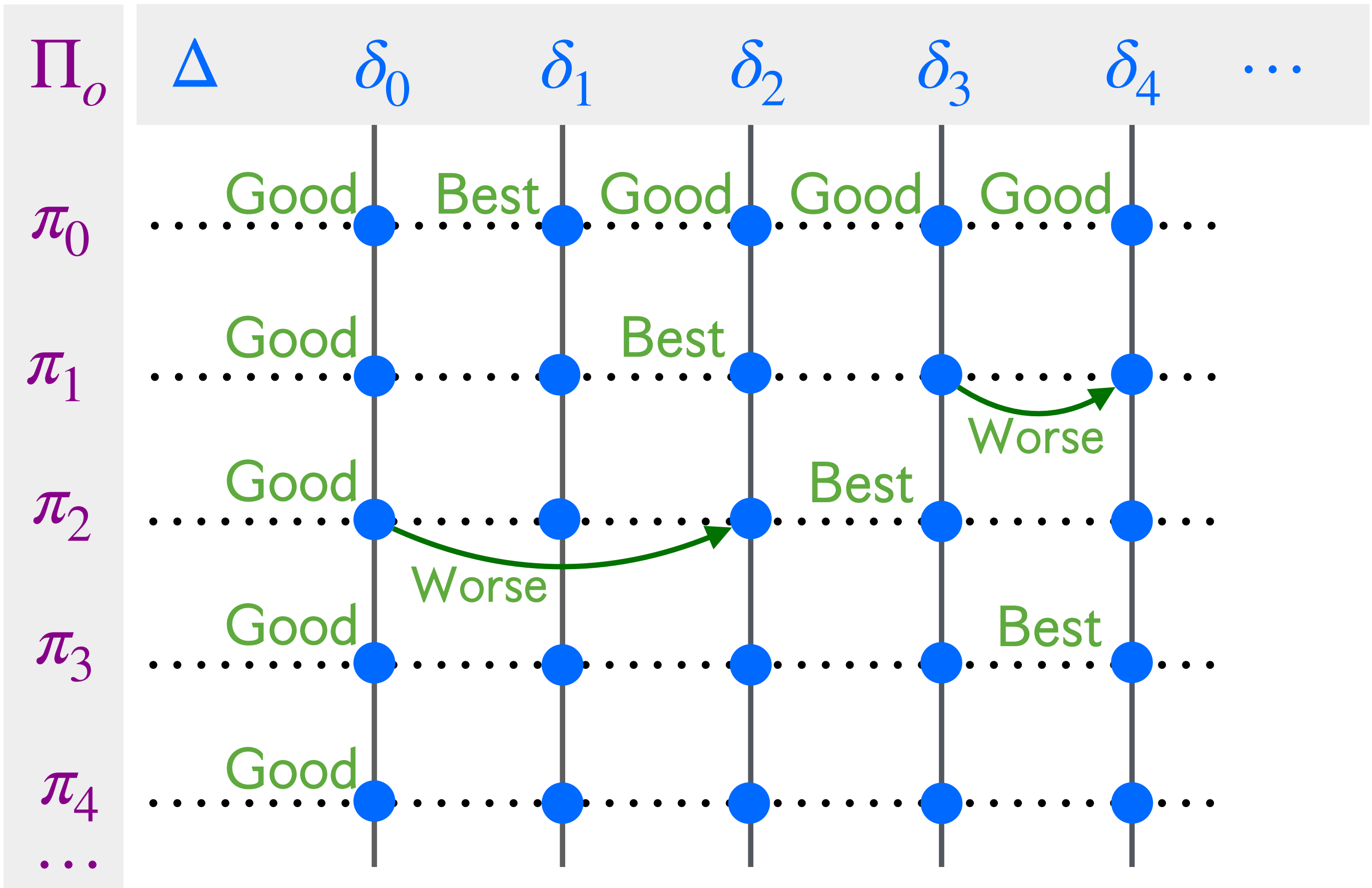

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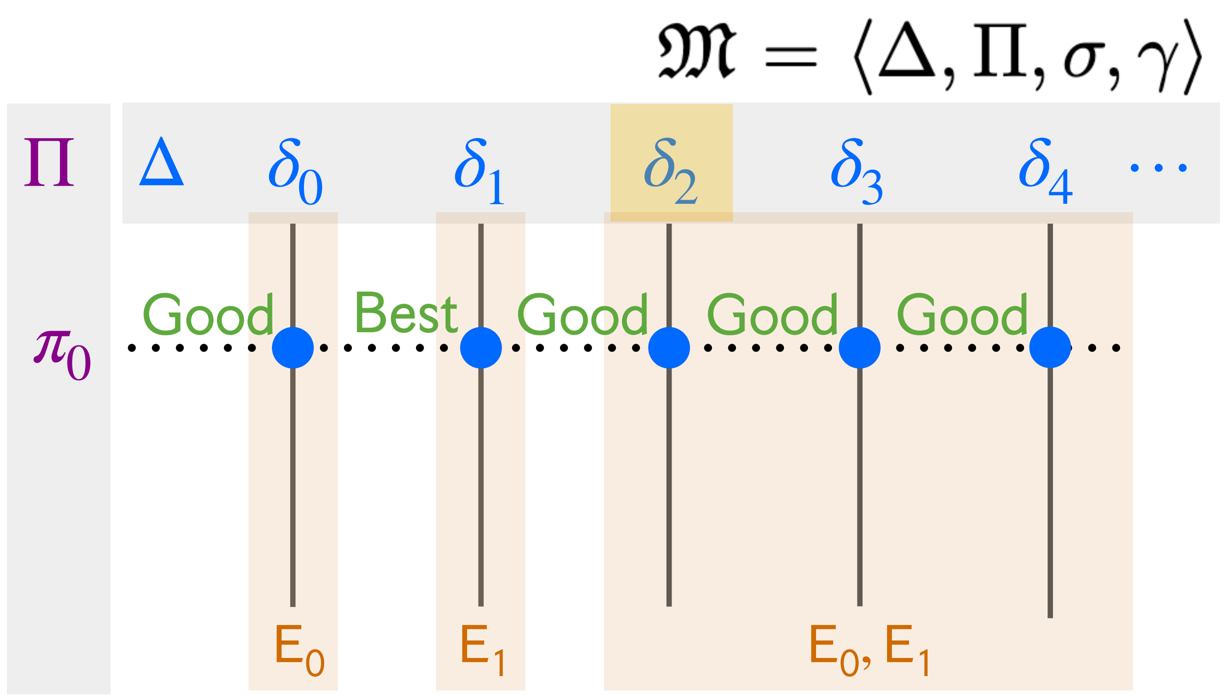

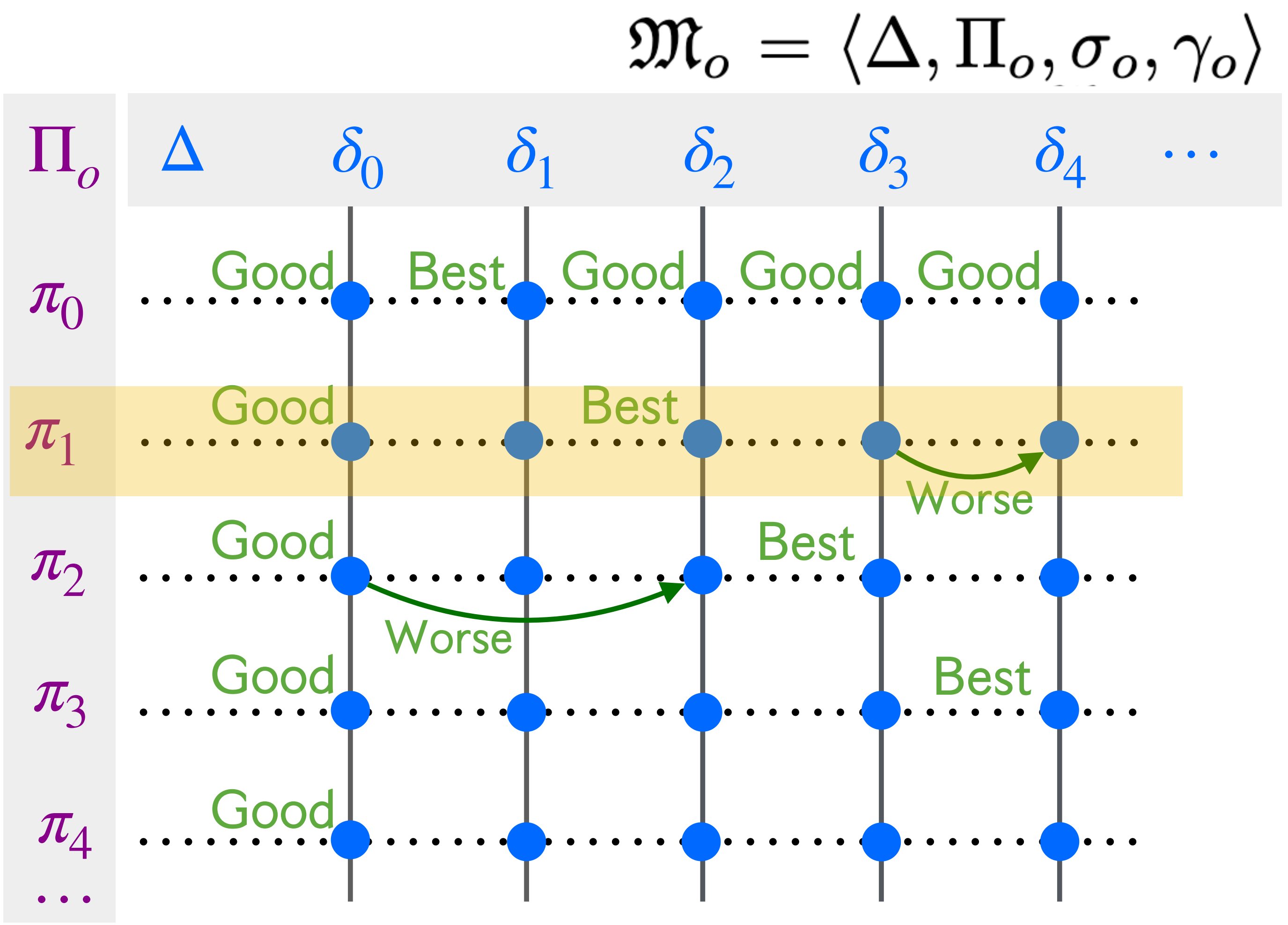

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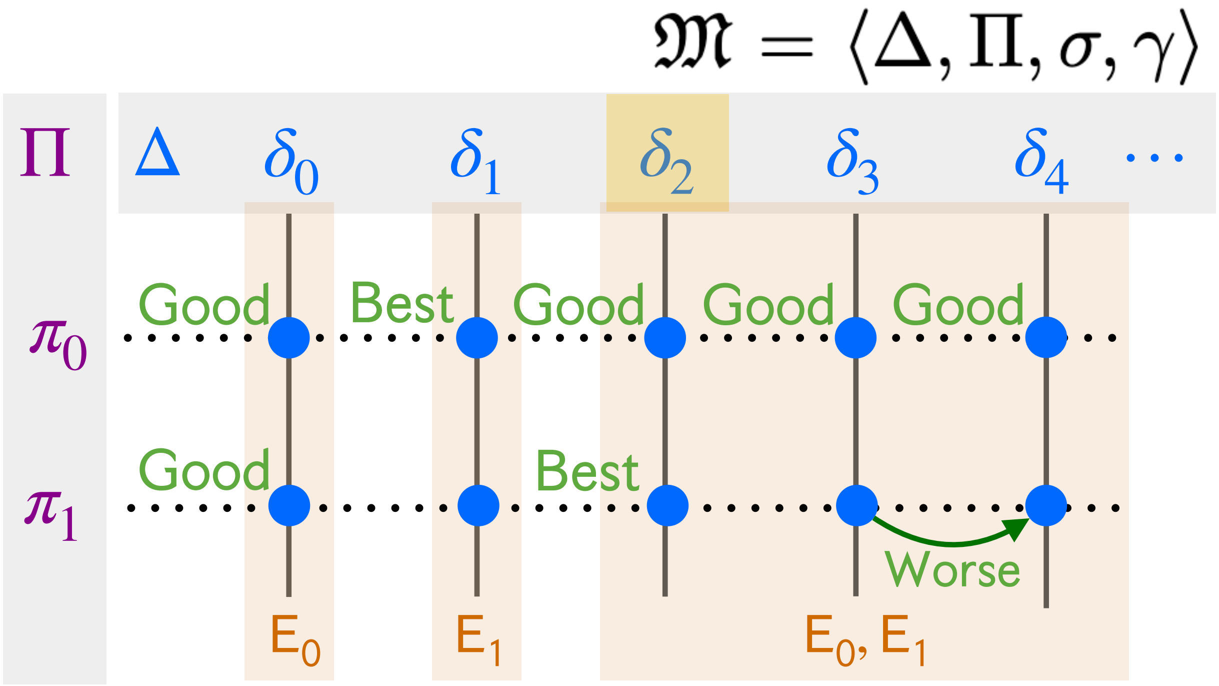

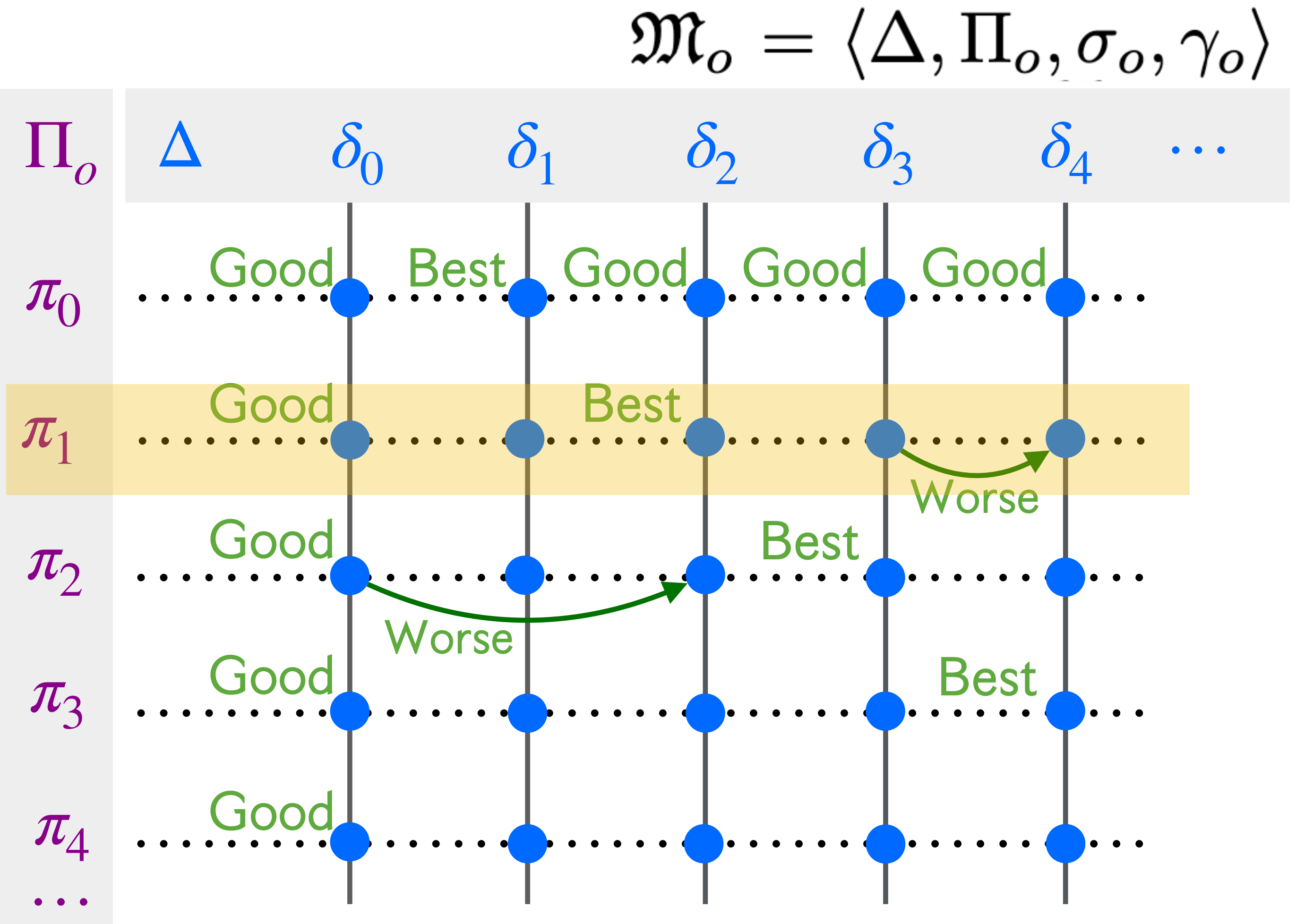

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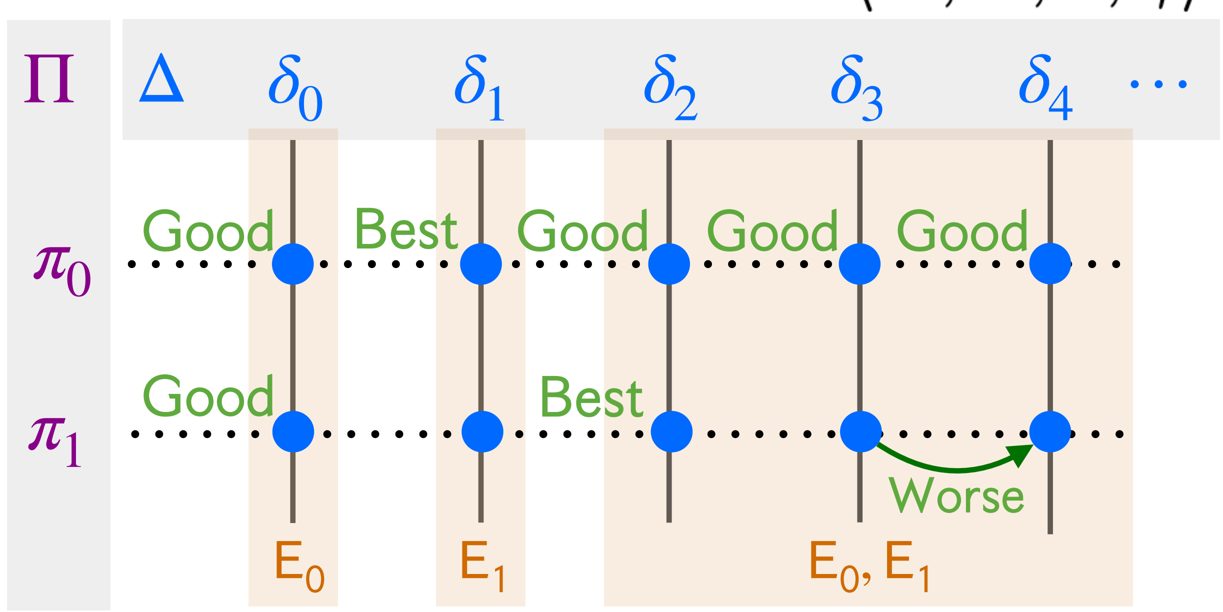

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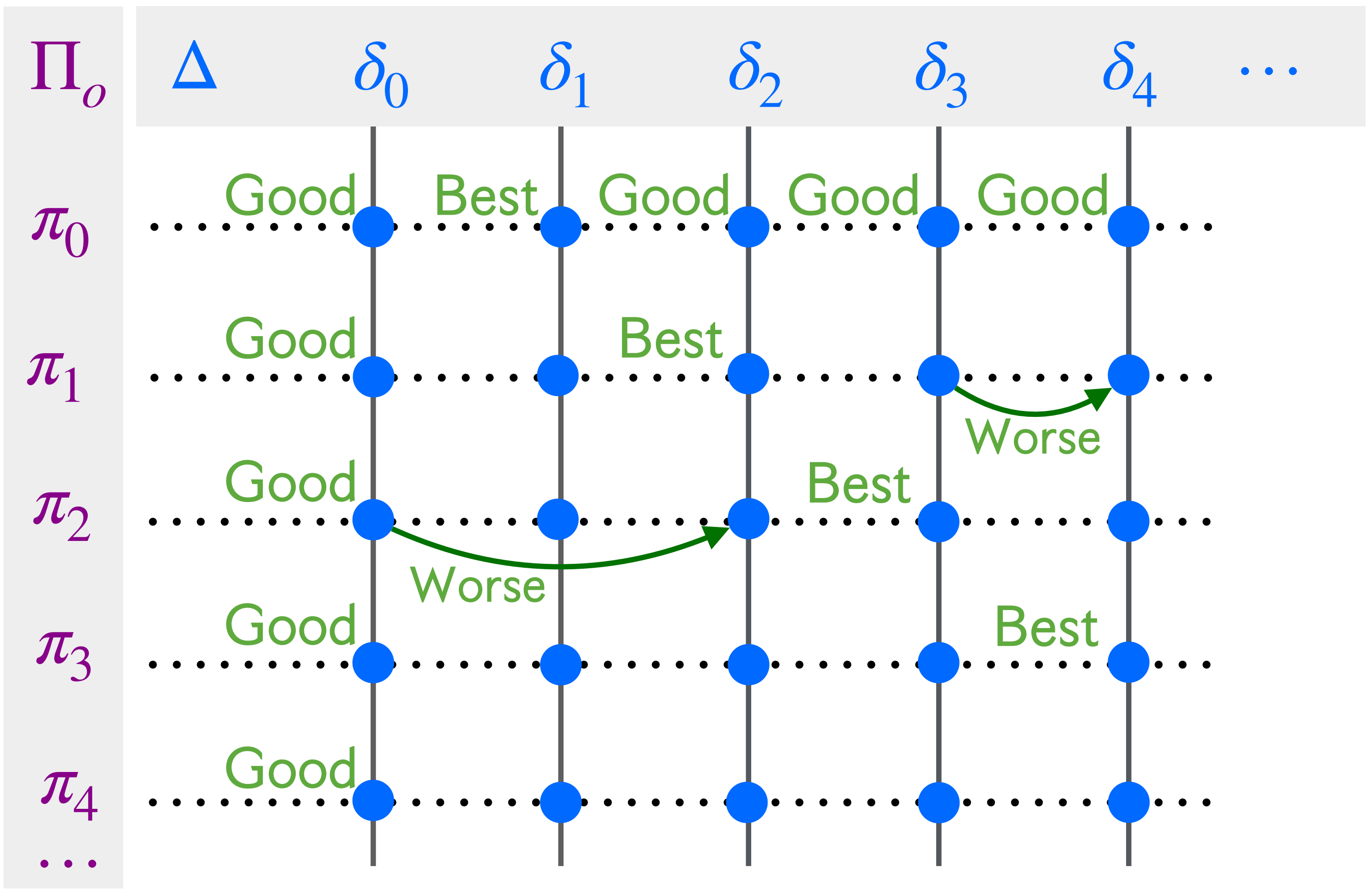

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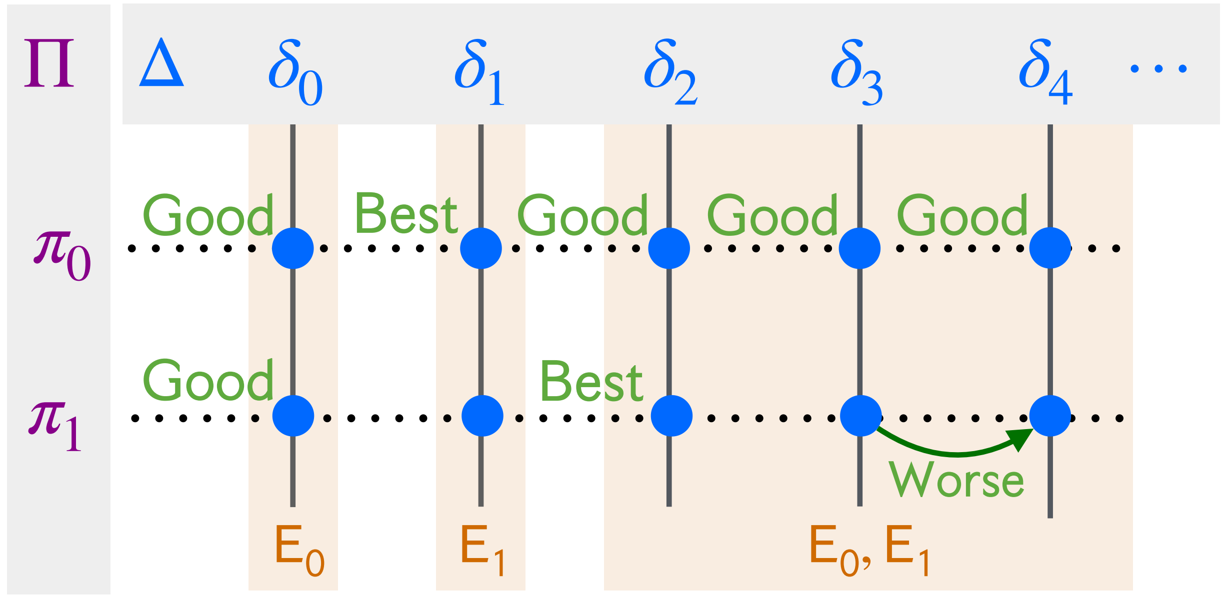

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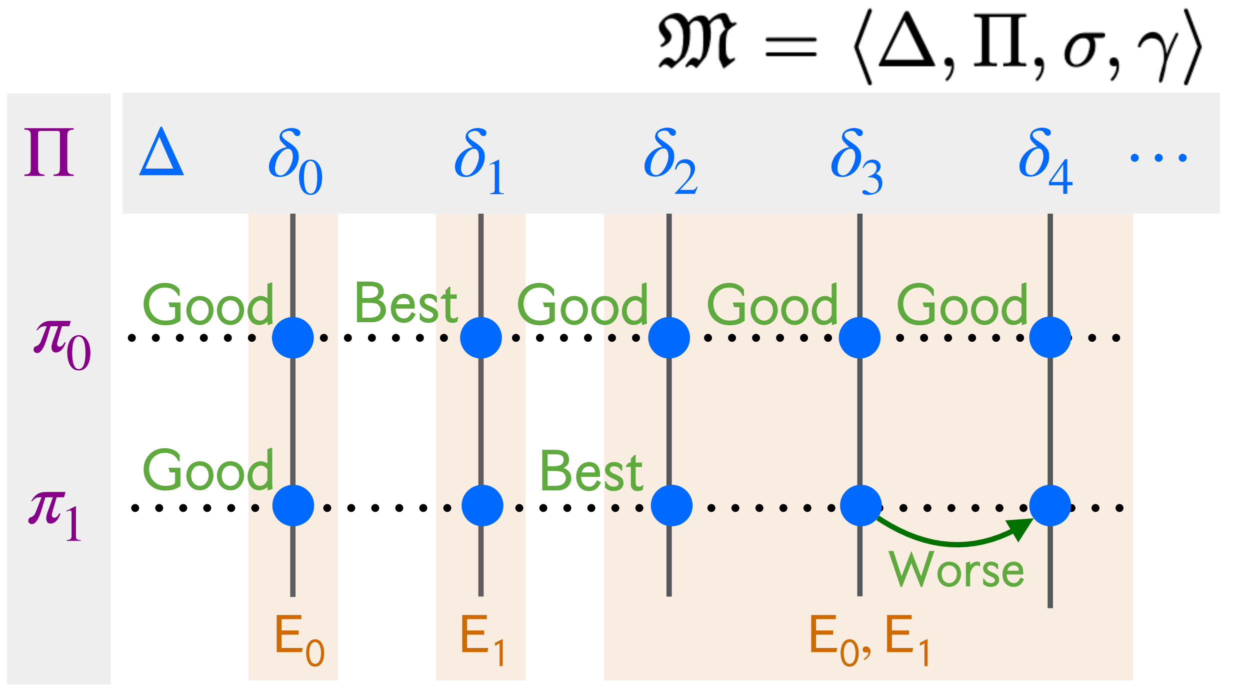

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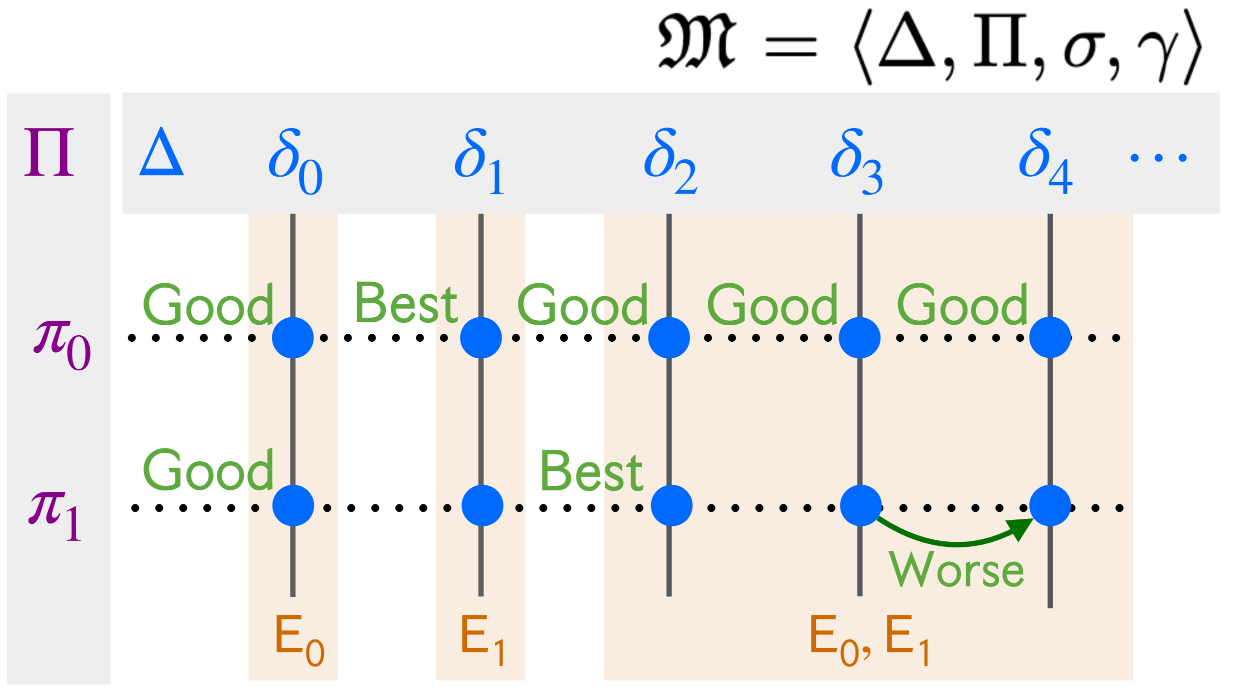
The $\mathbf{P}_{\mathrm{E}}$-stable permutational closure of $\mathfrak{M}$



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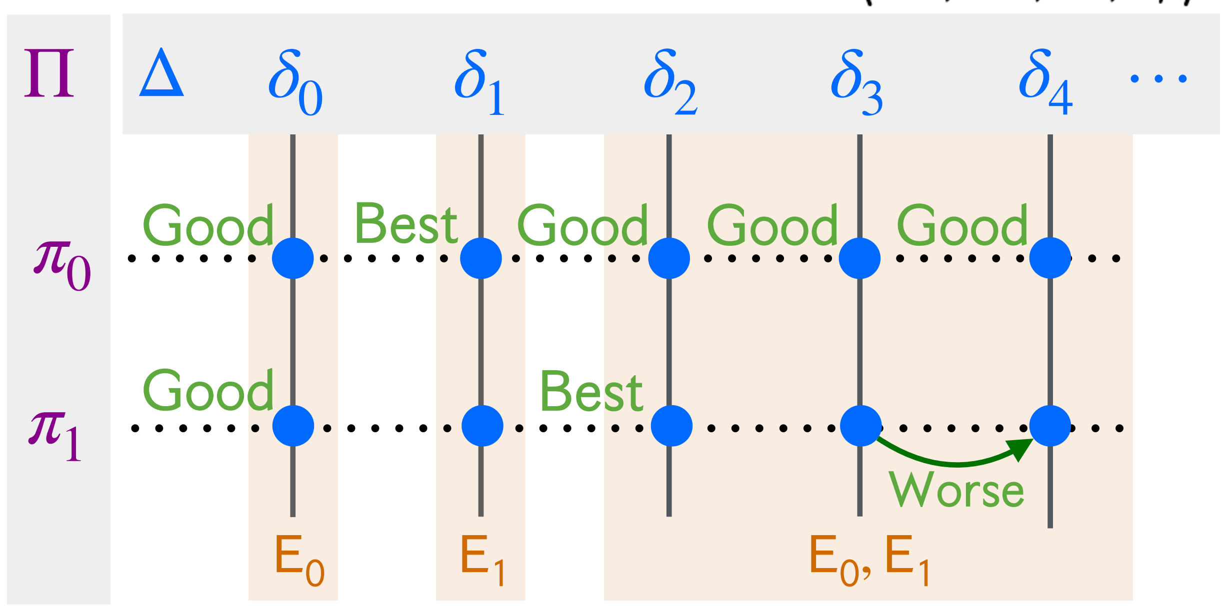

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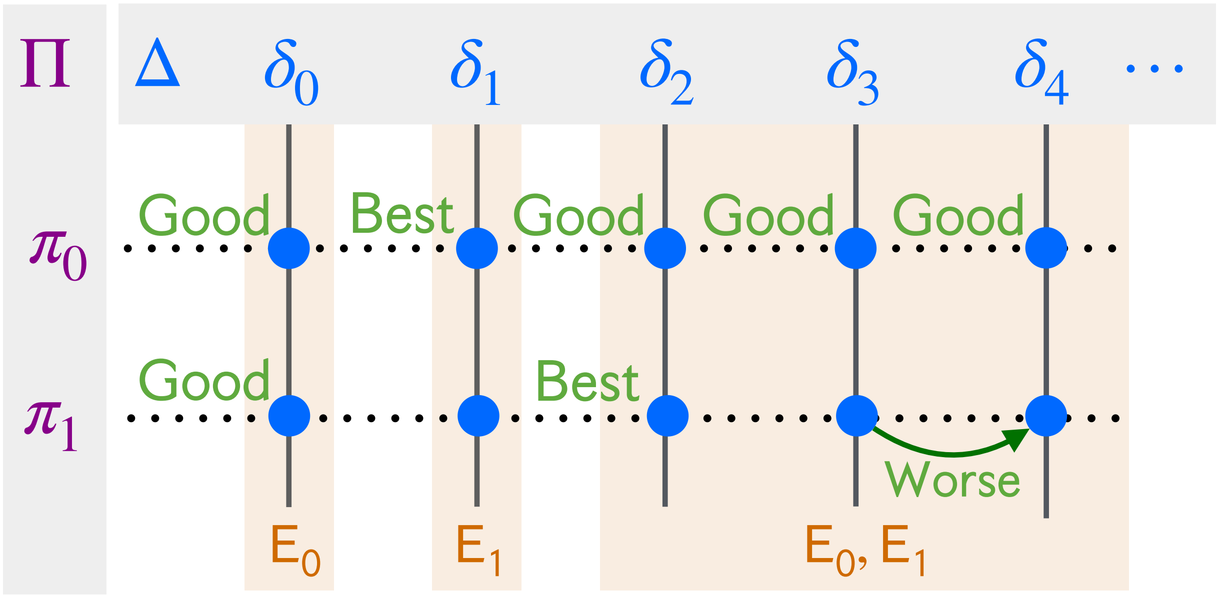

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<u>The $\mathbf{P}_{\mathsf{E}}$-stable permutational closure of $\mathfrak{M}$</u>

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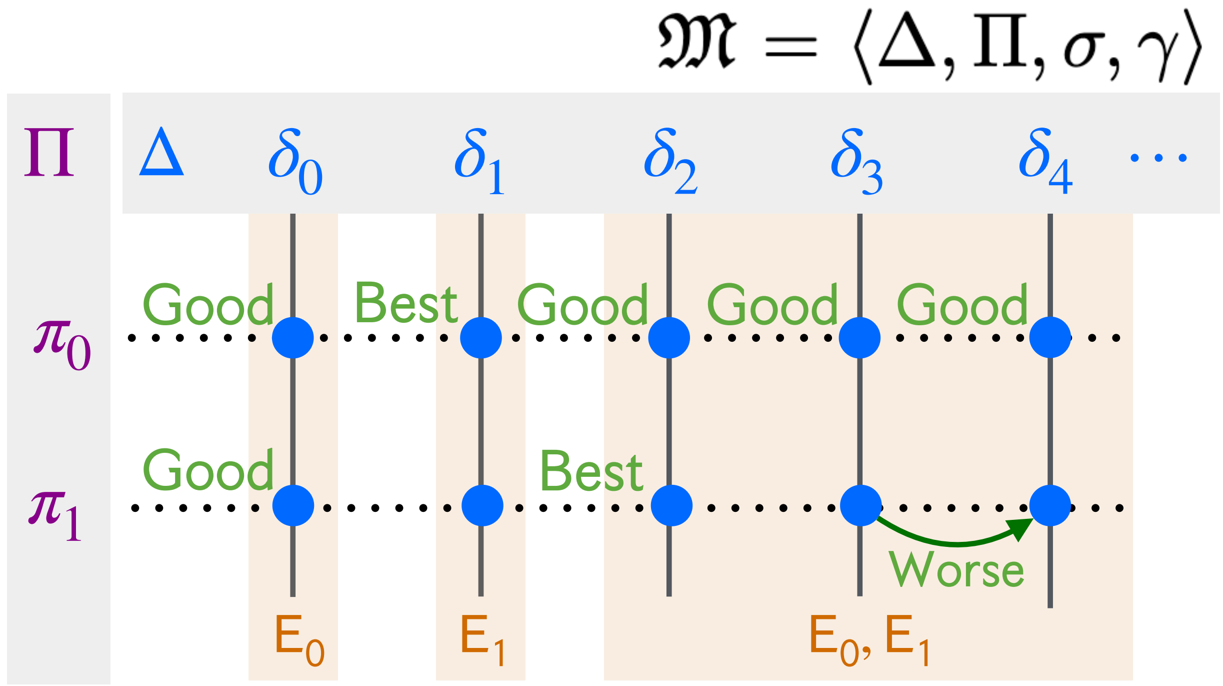

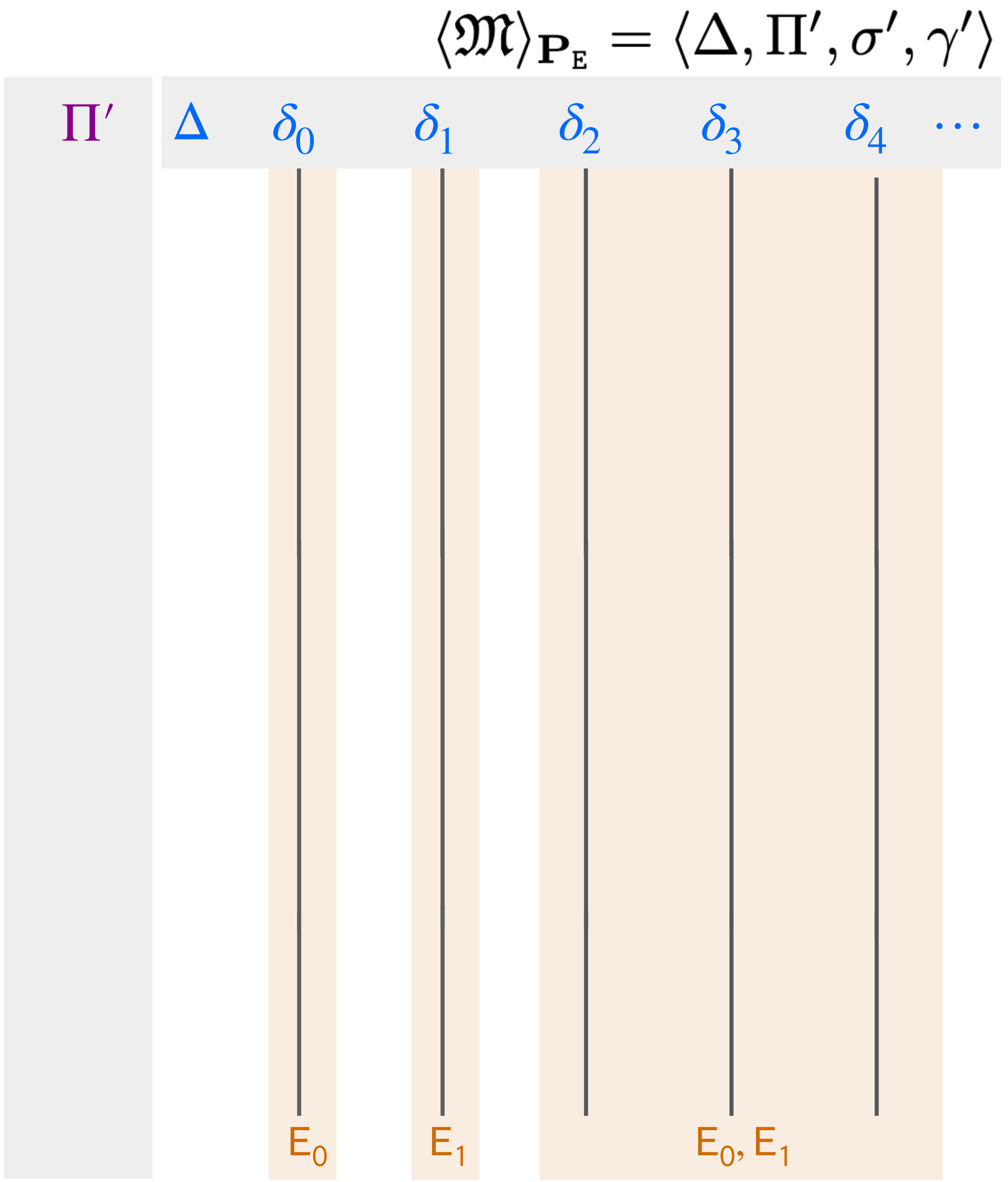

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Elements swapped around,
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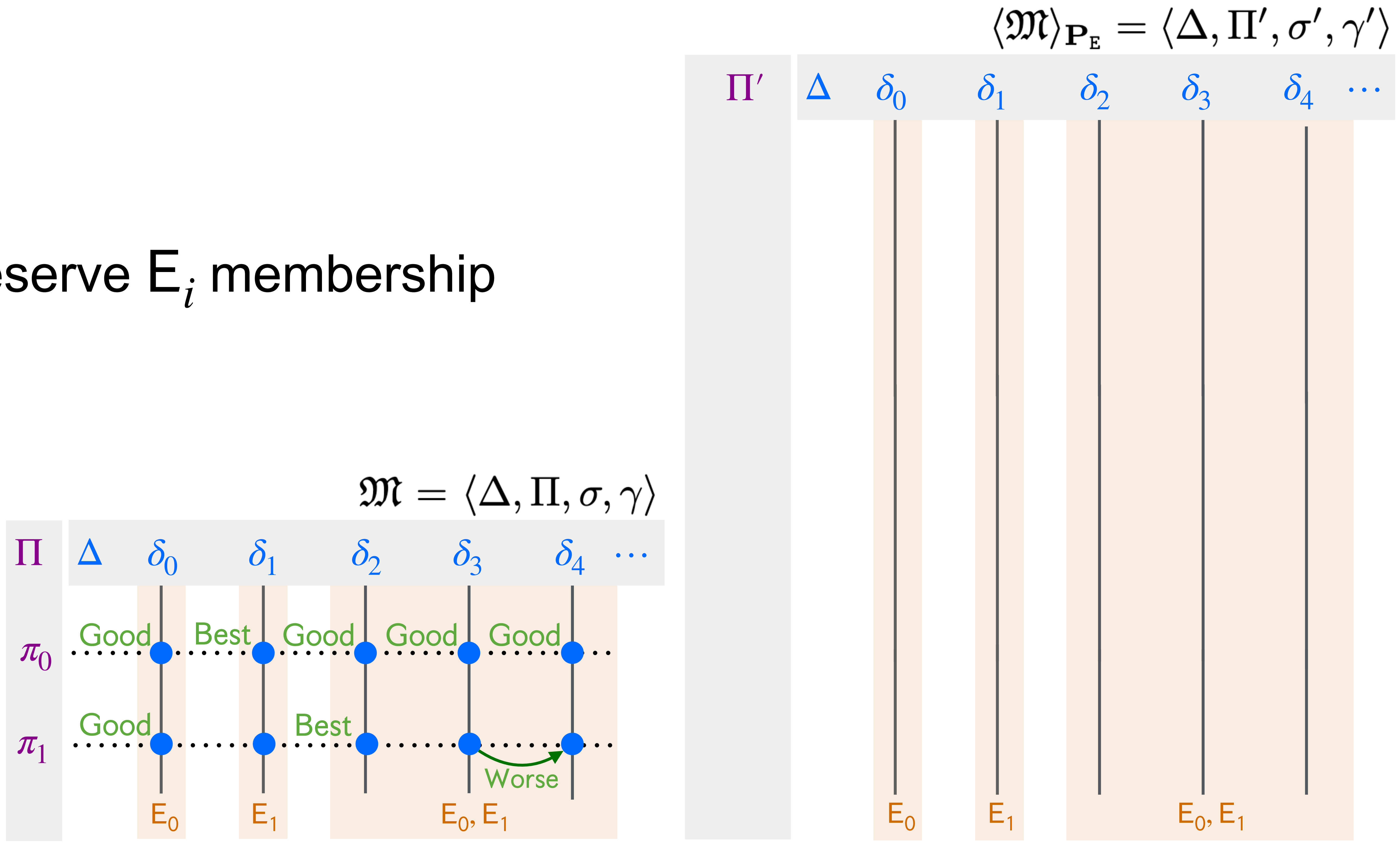

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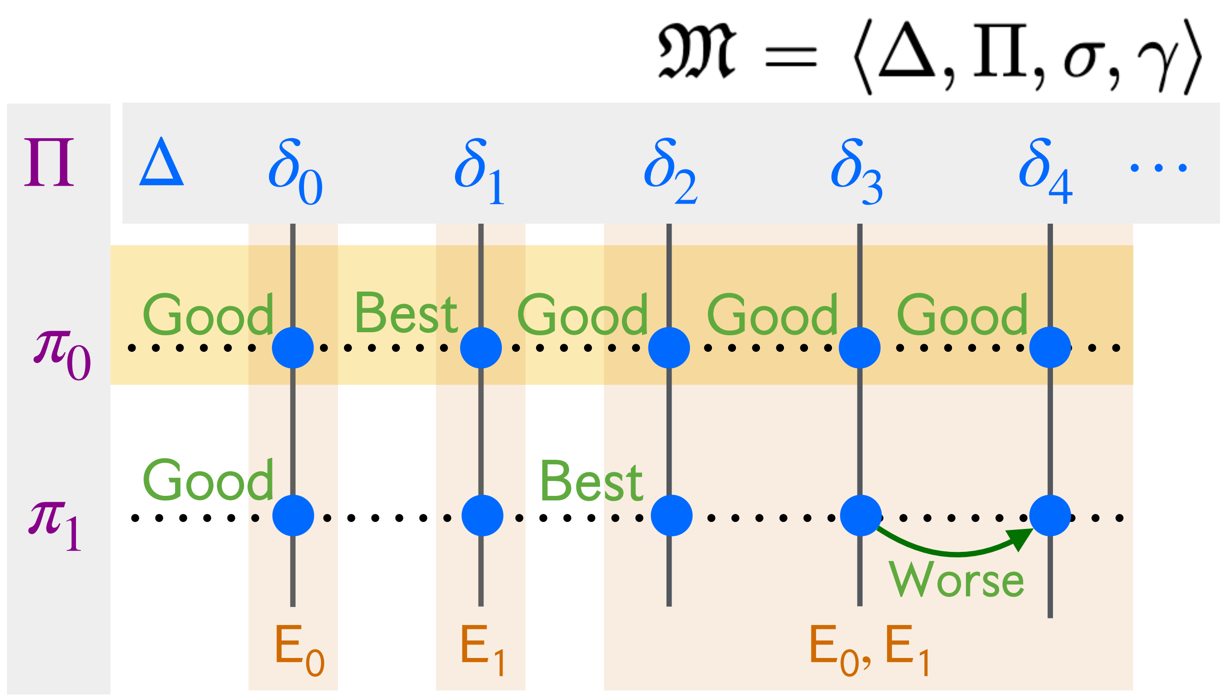

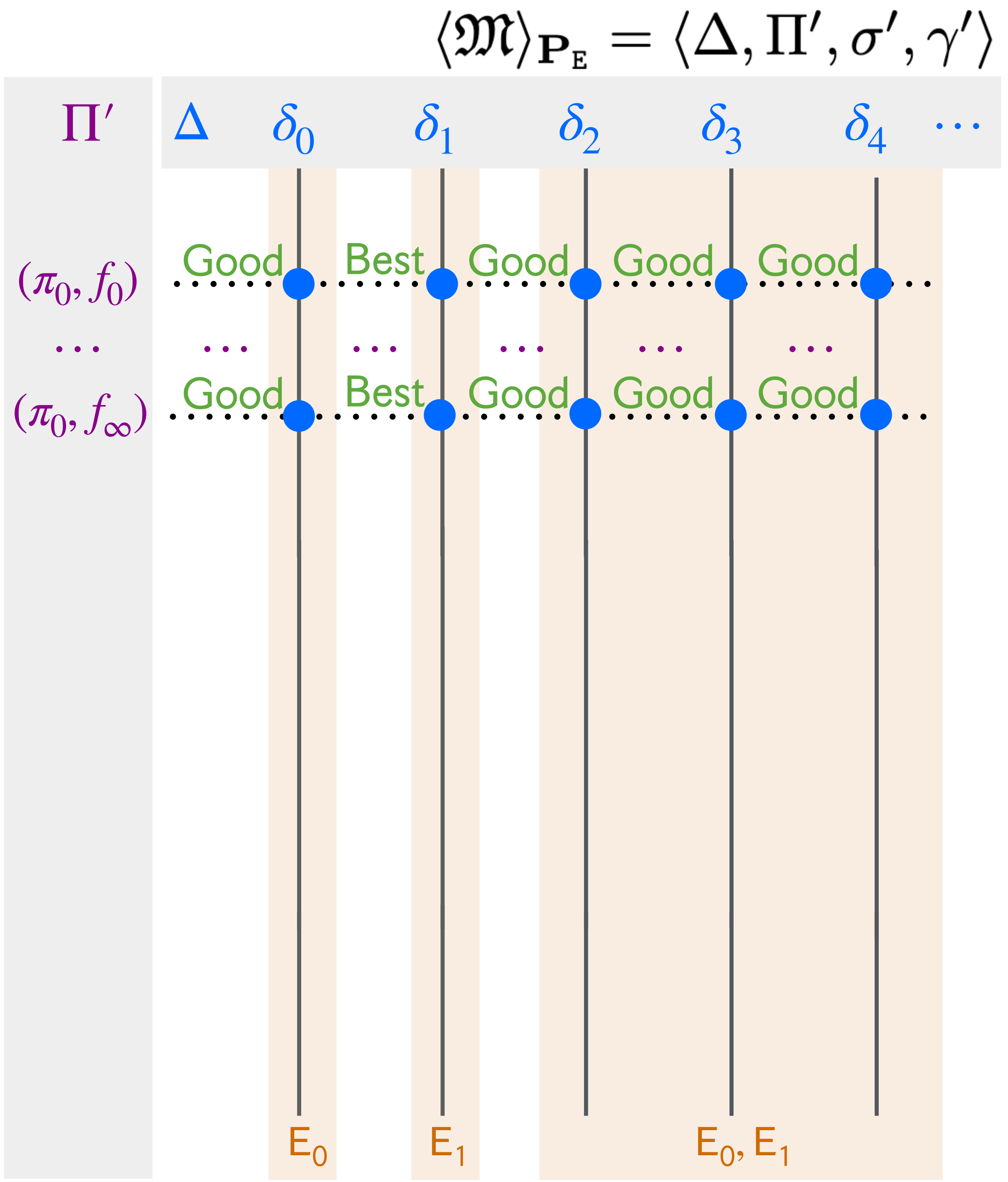

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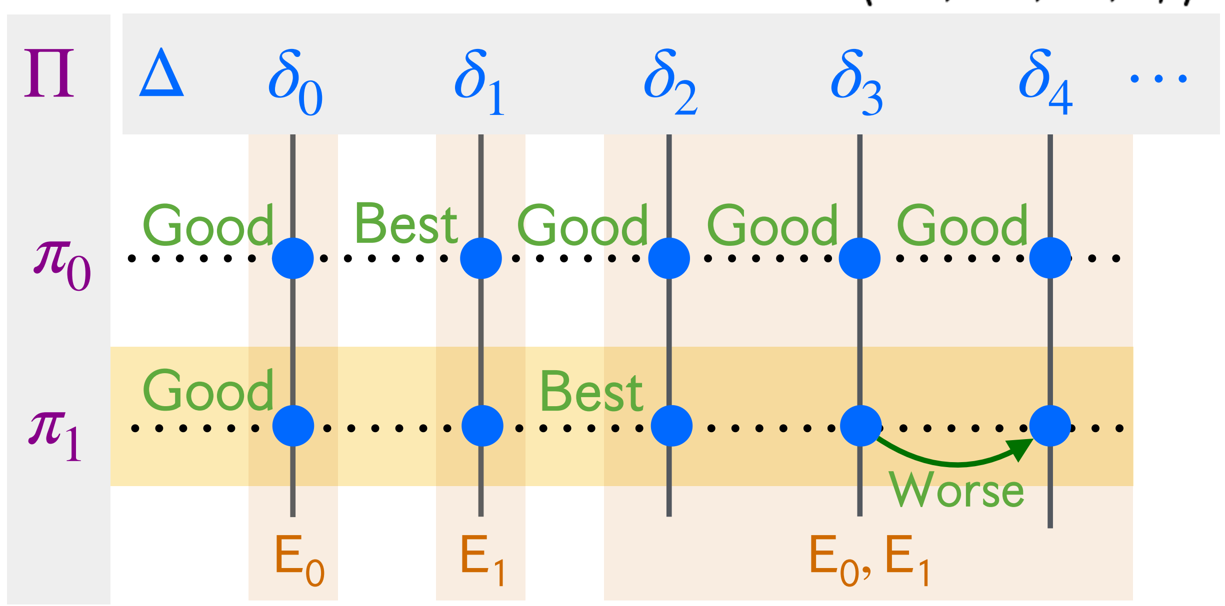

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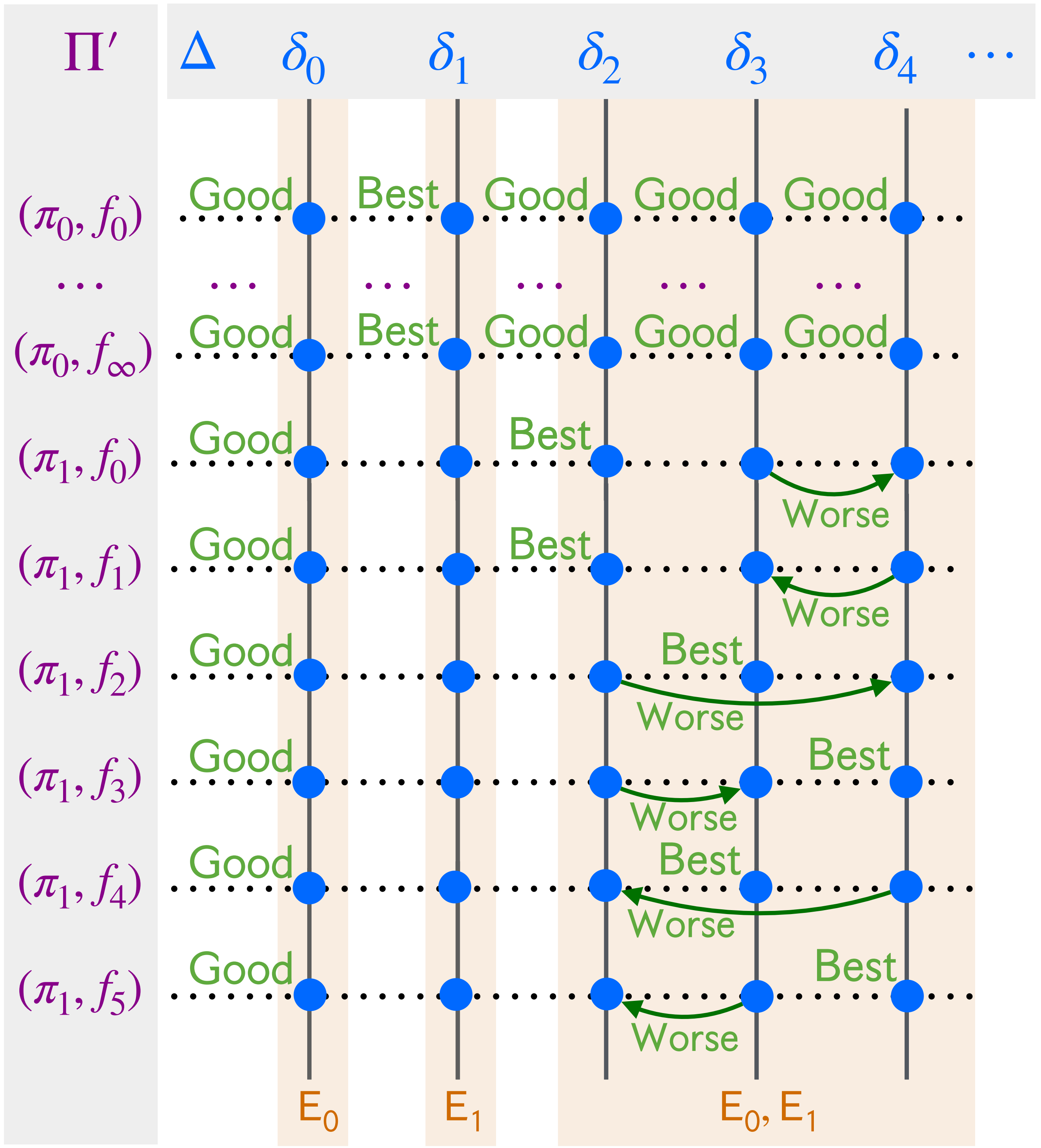

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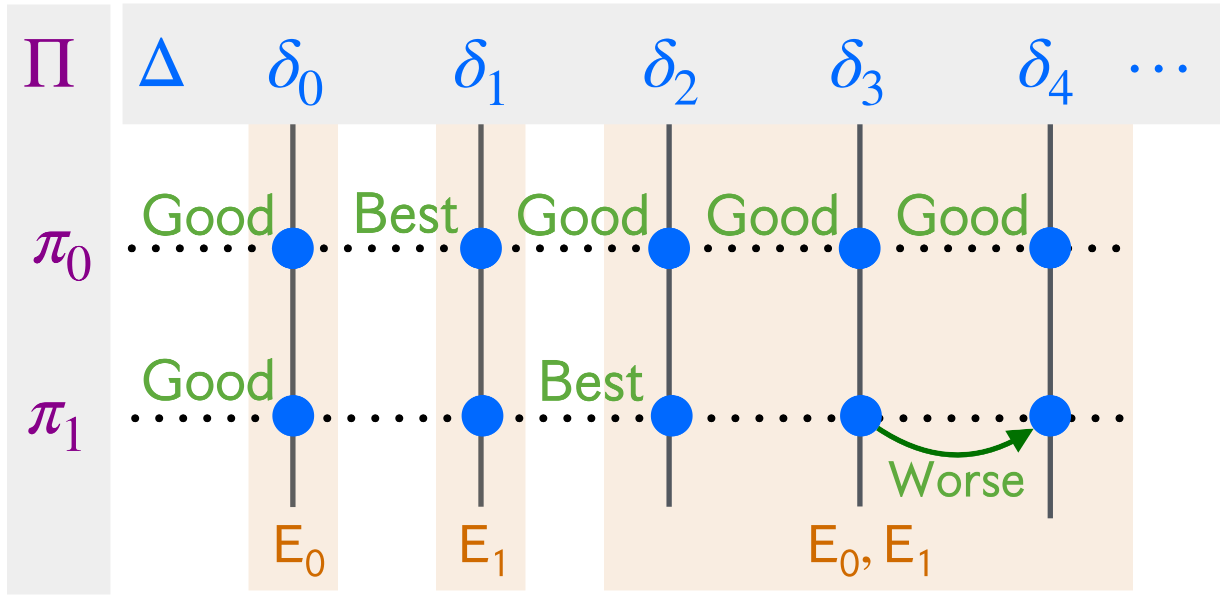

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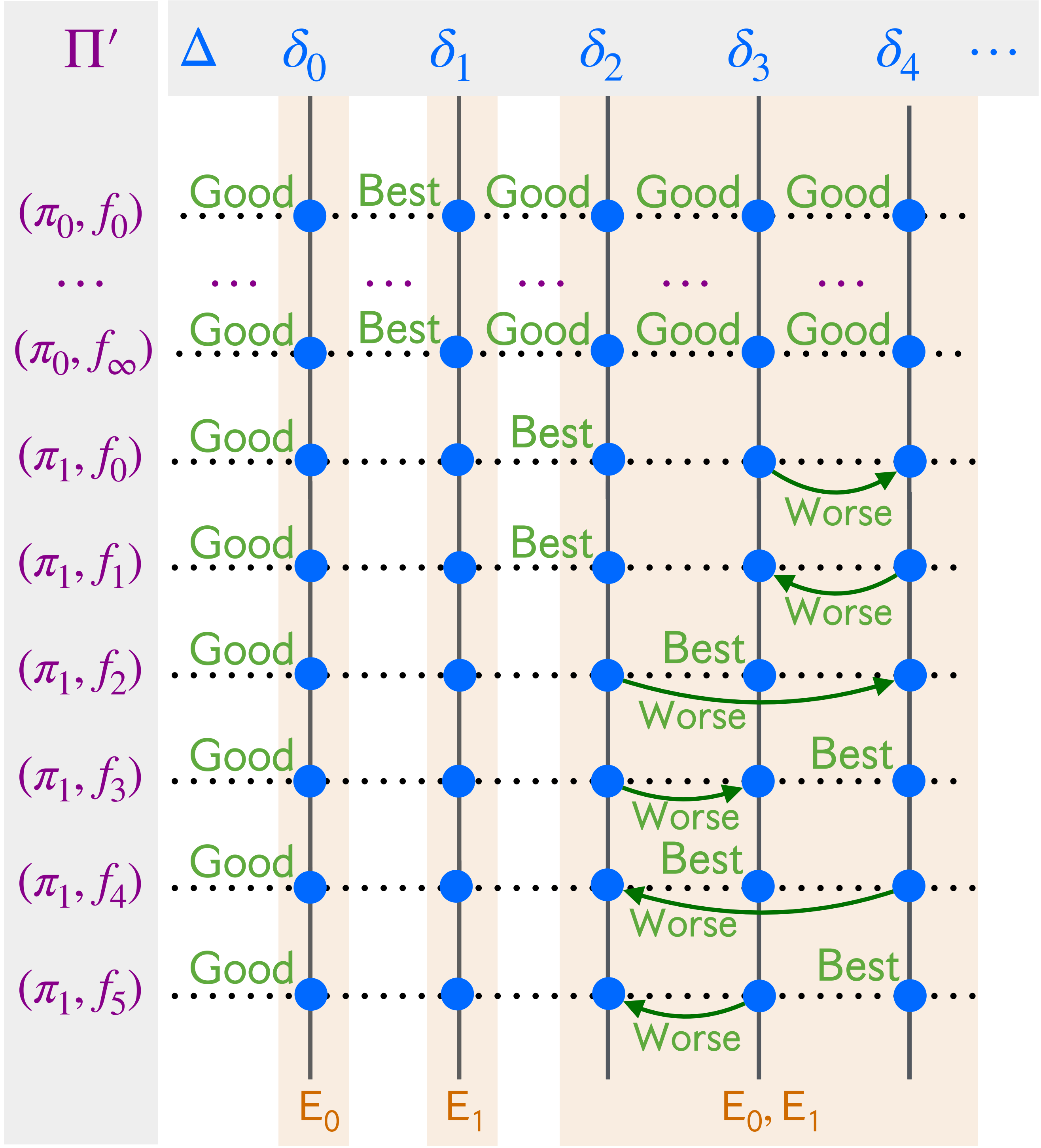

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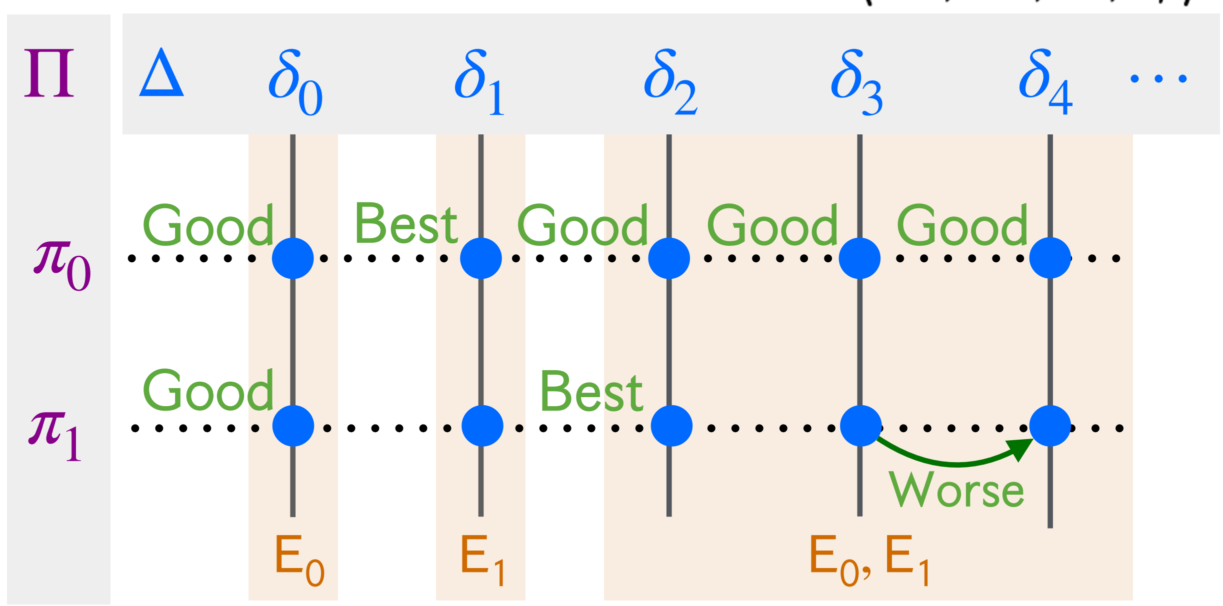

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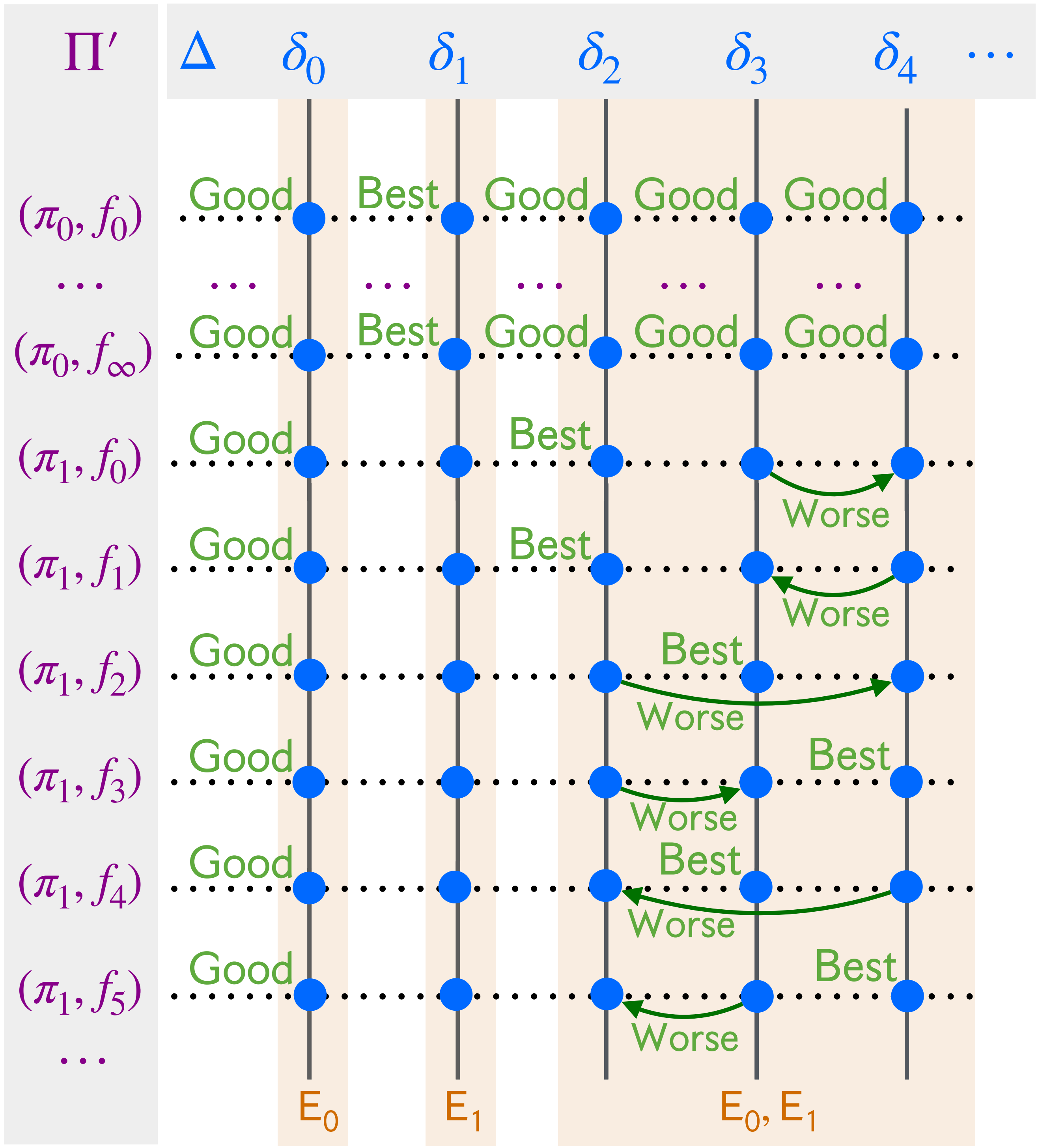

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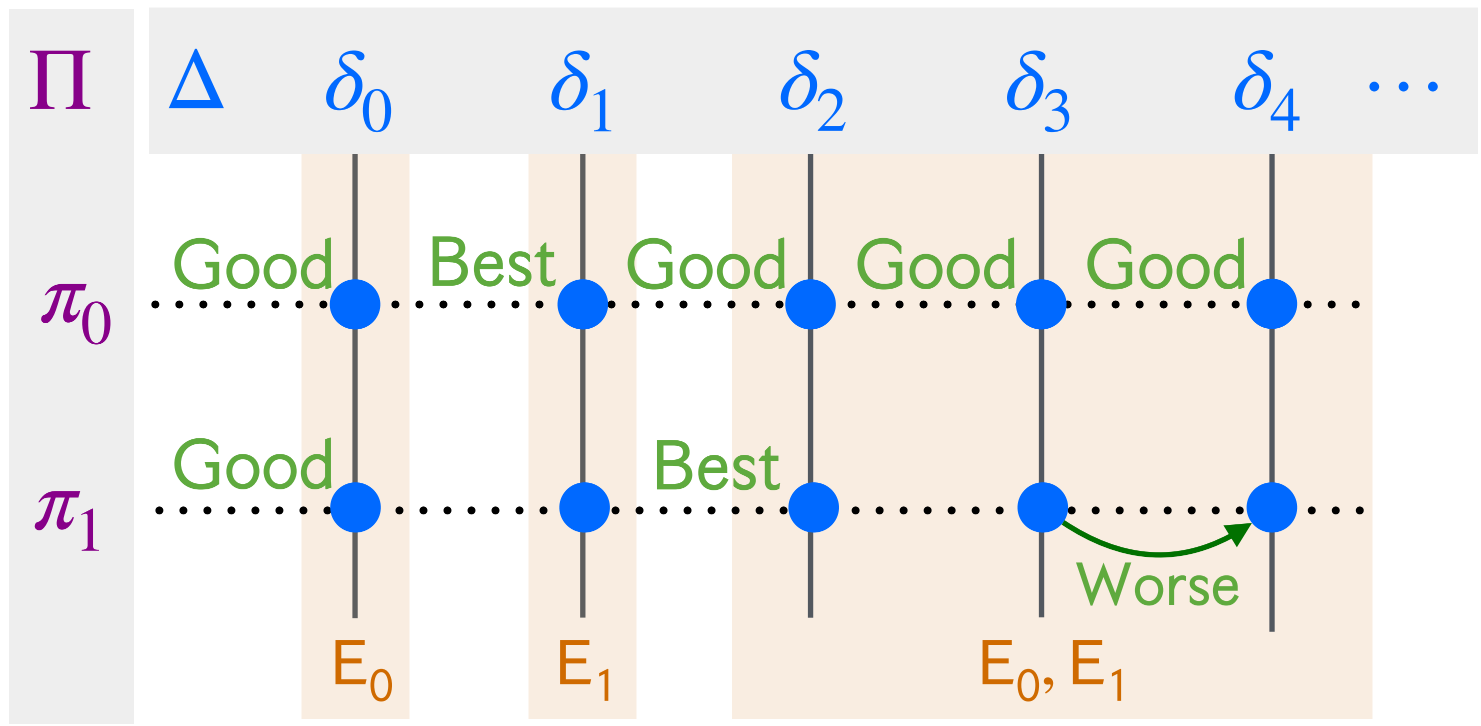

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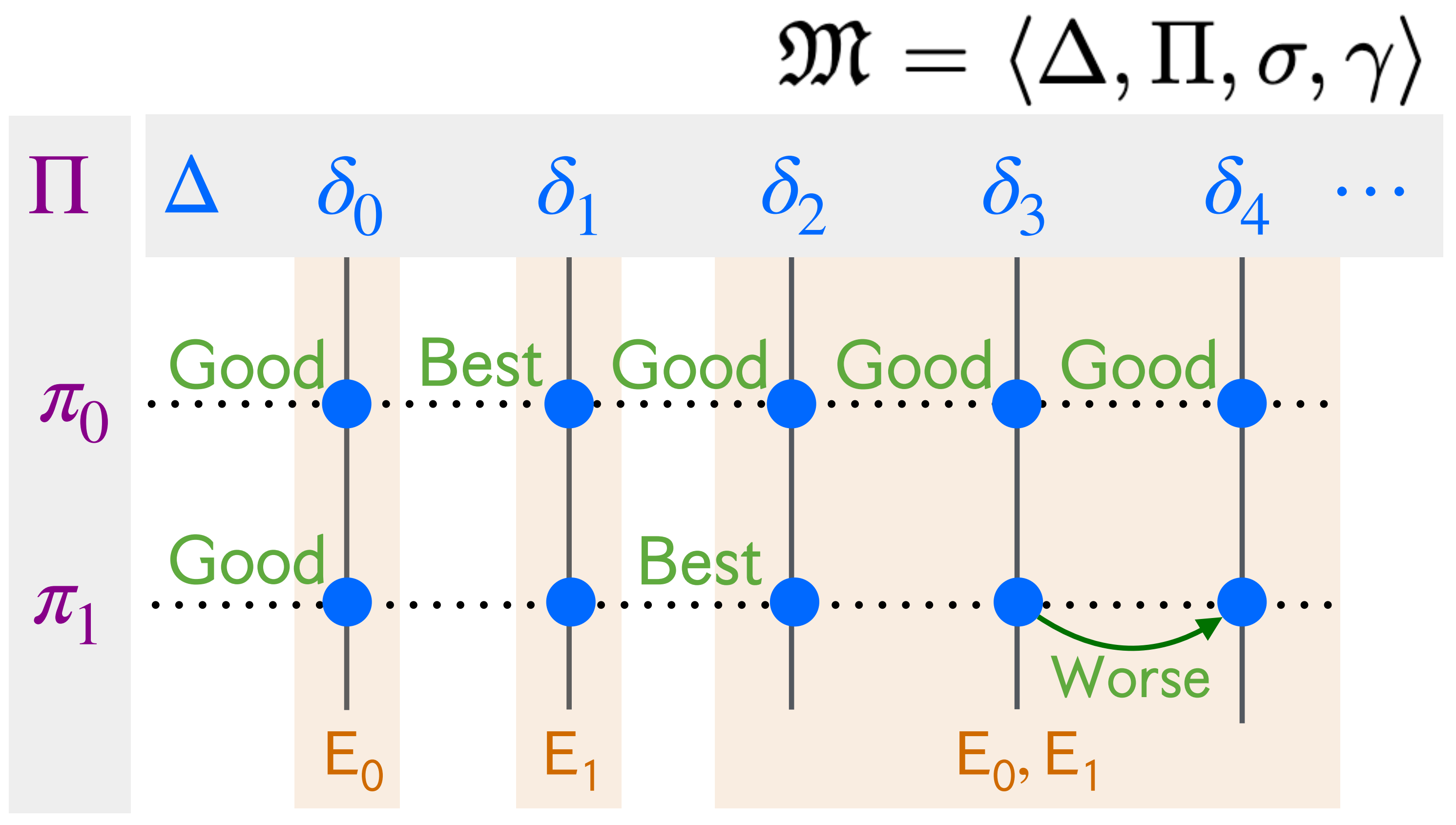

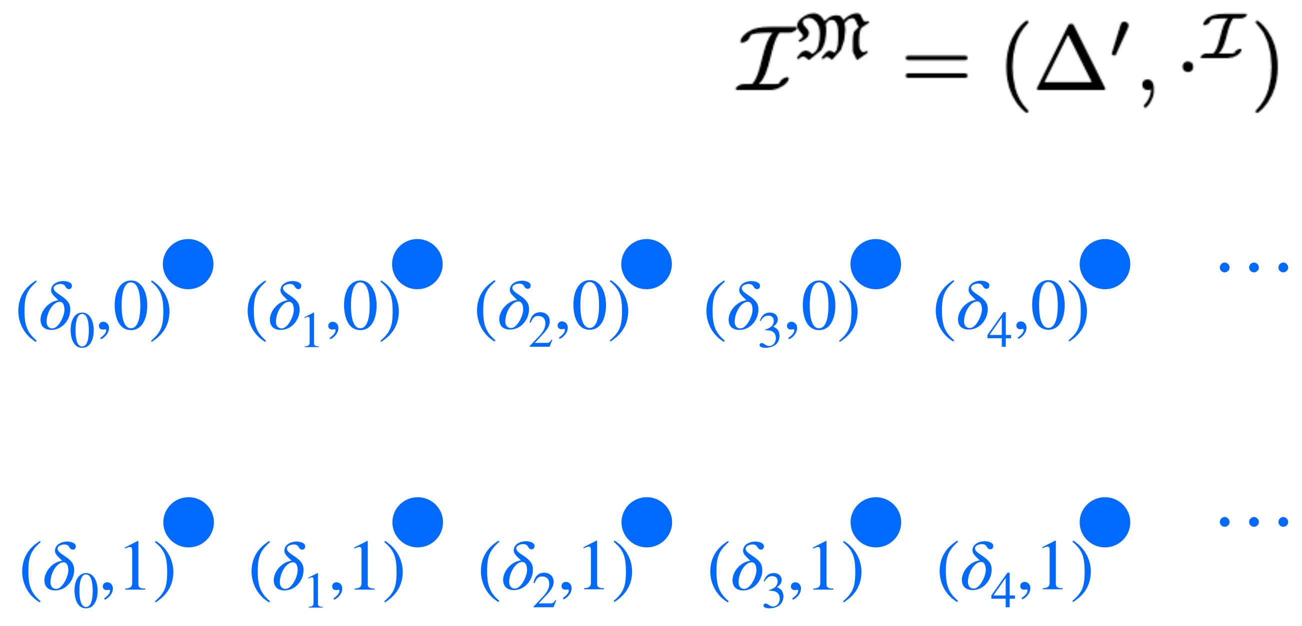

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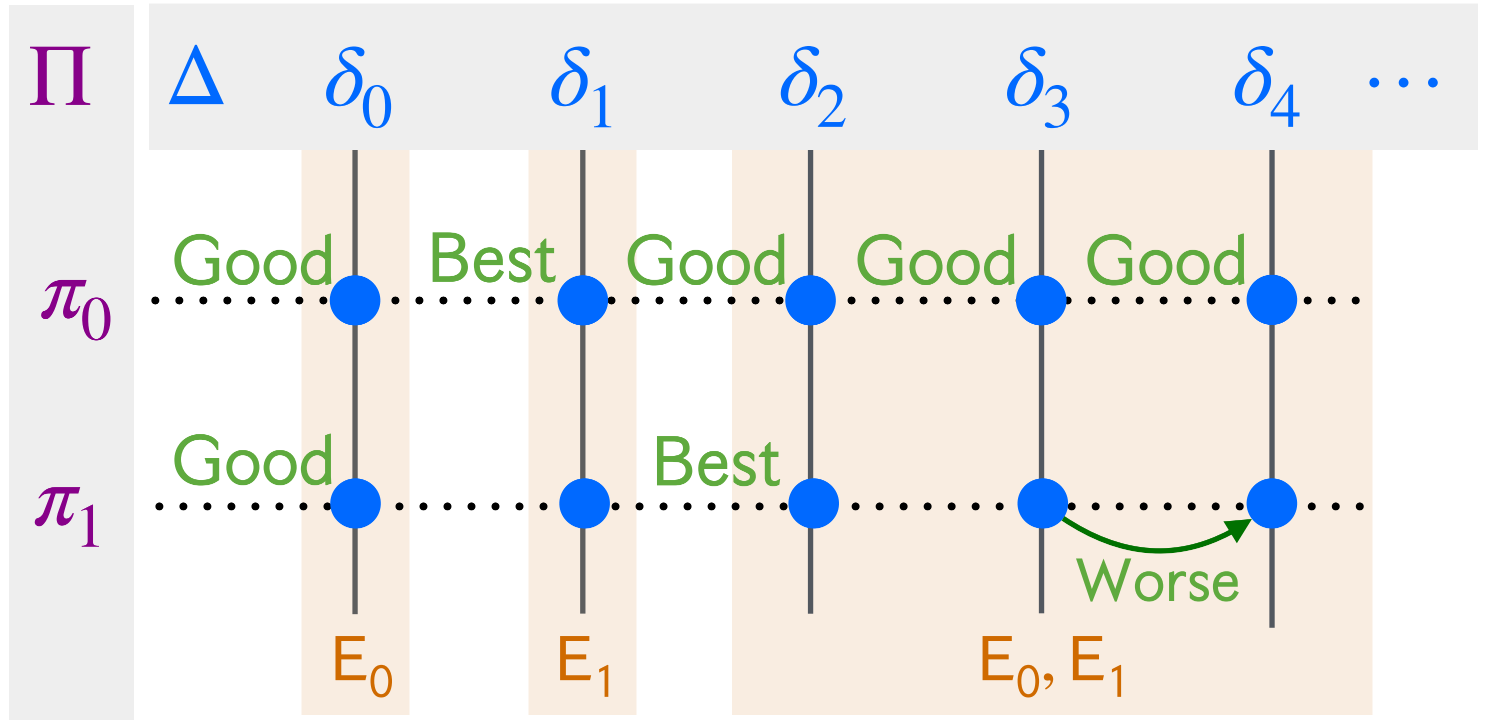

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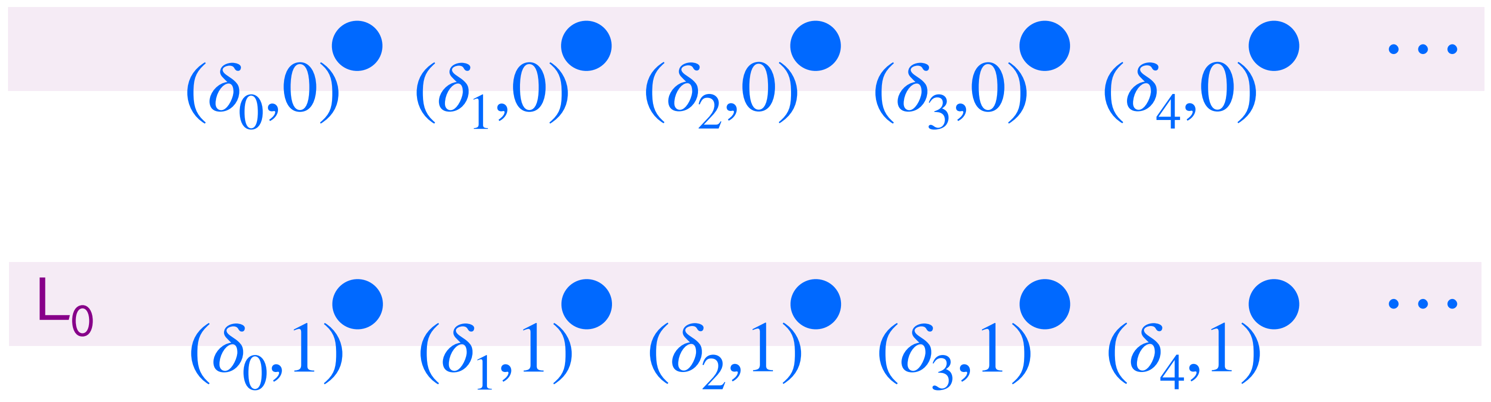

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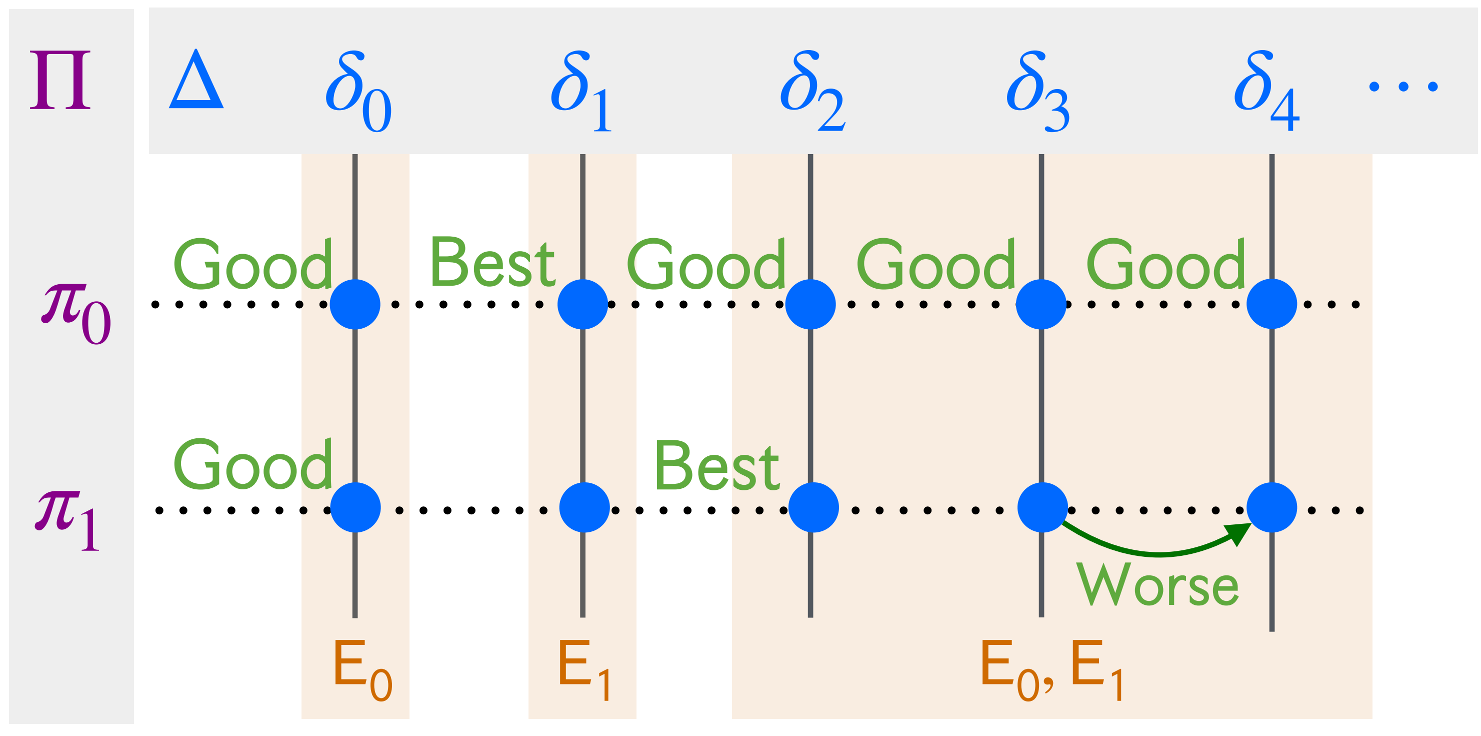

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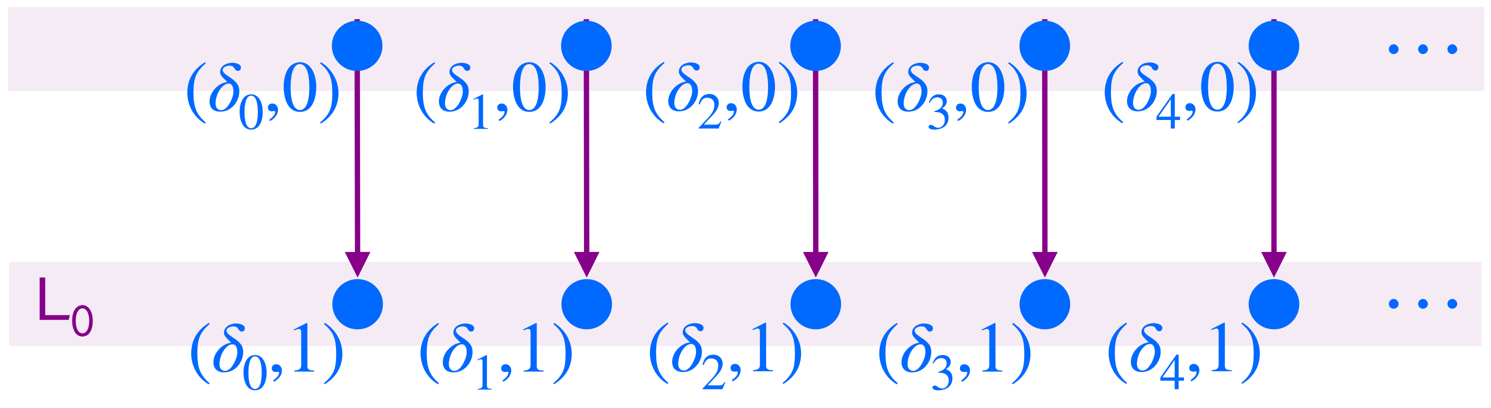

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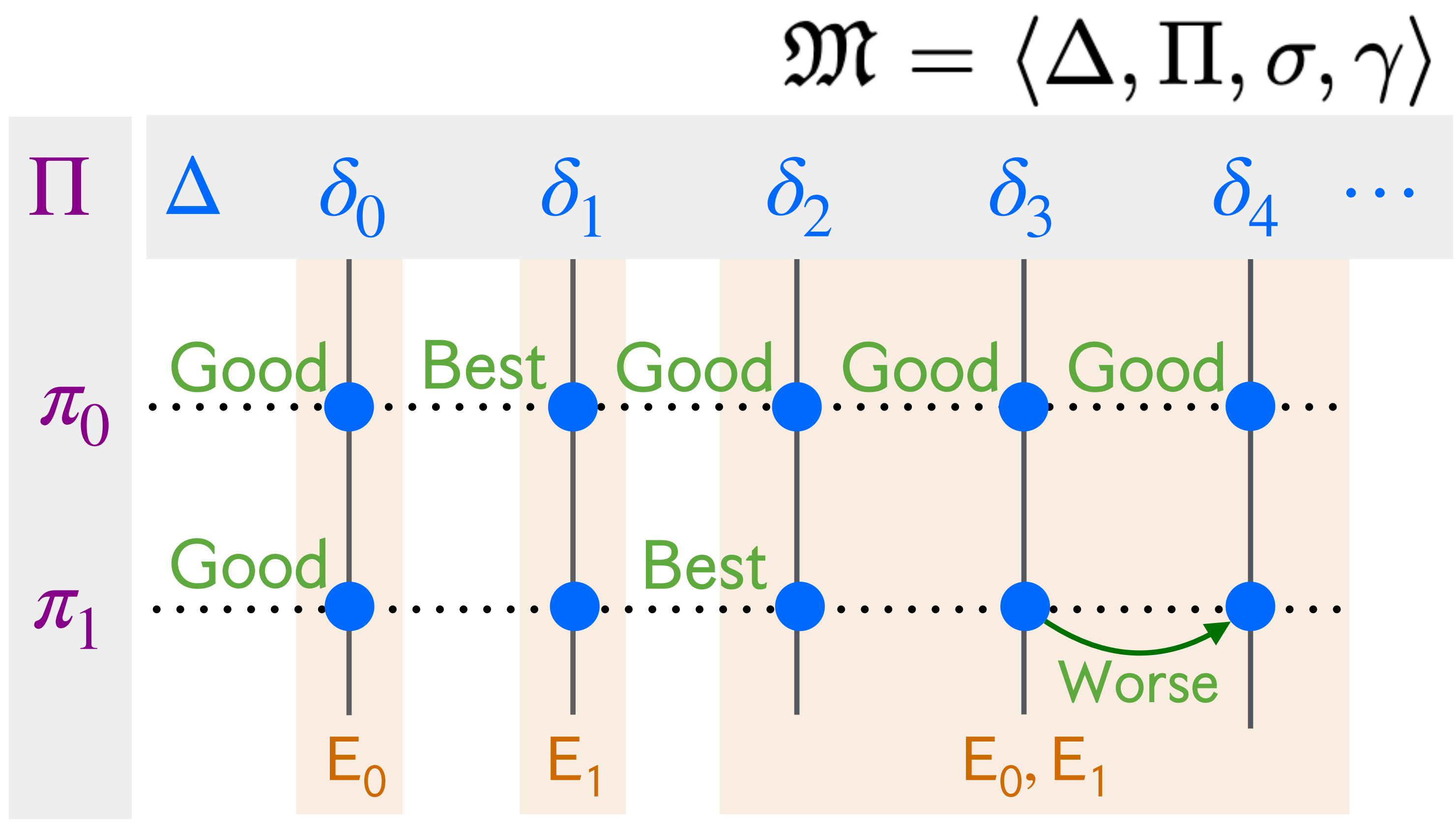

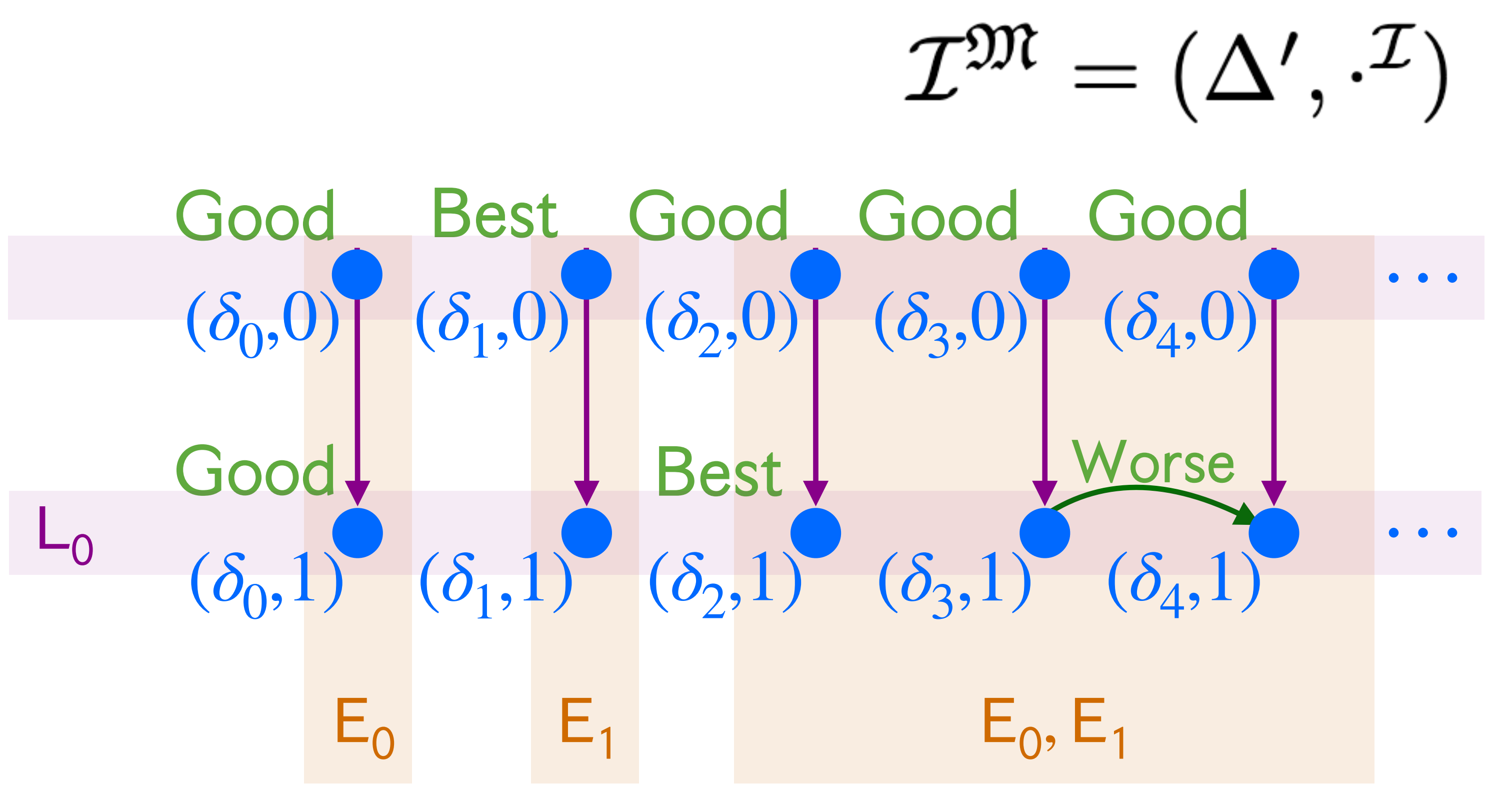

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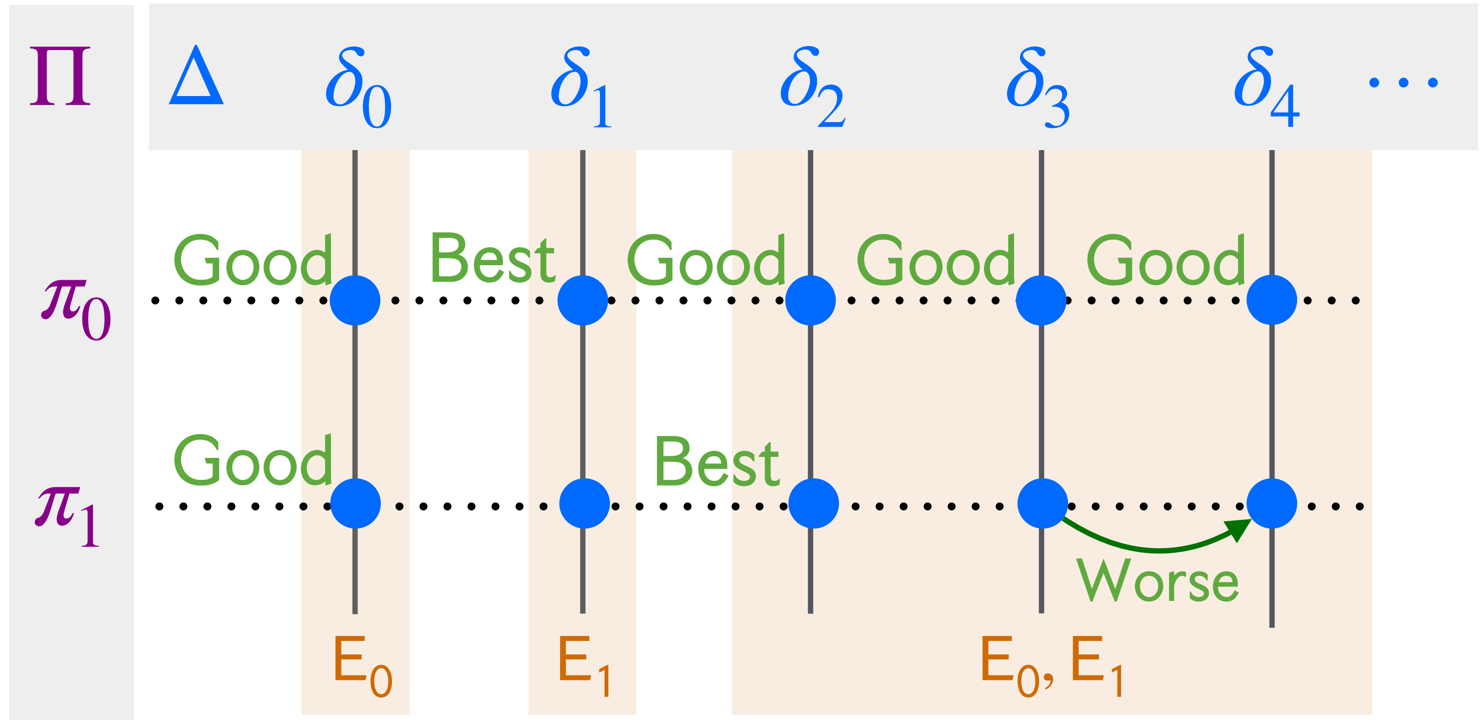

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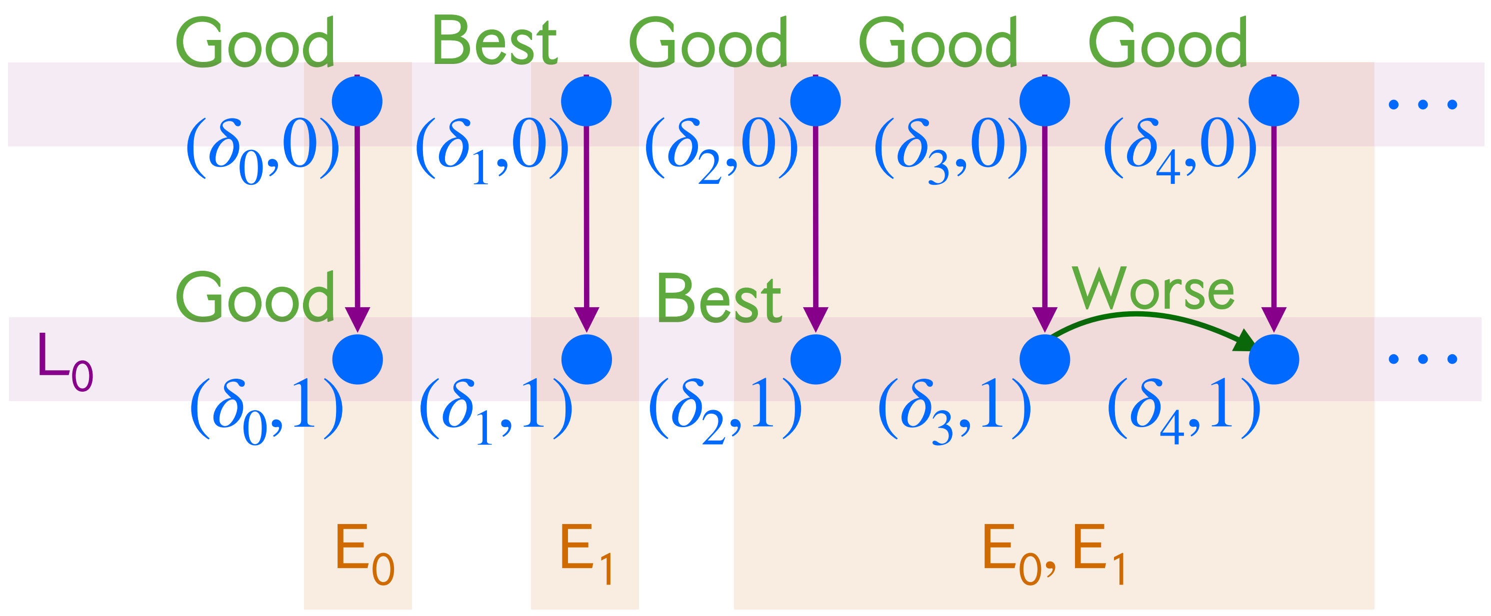

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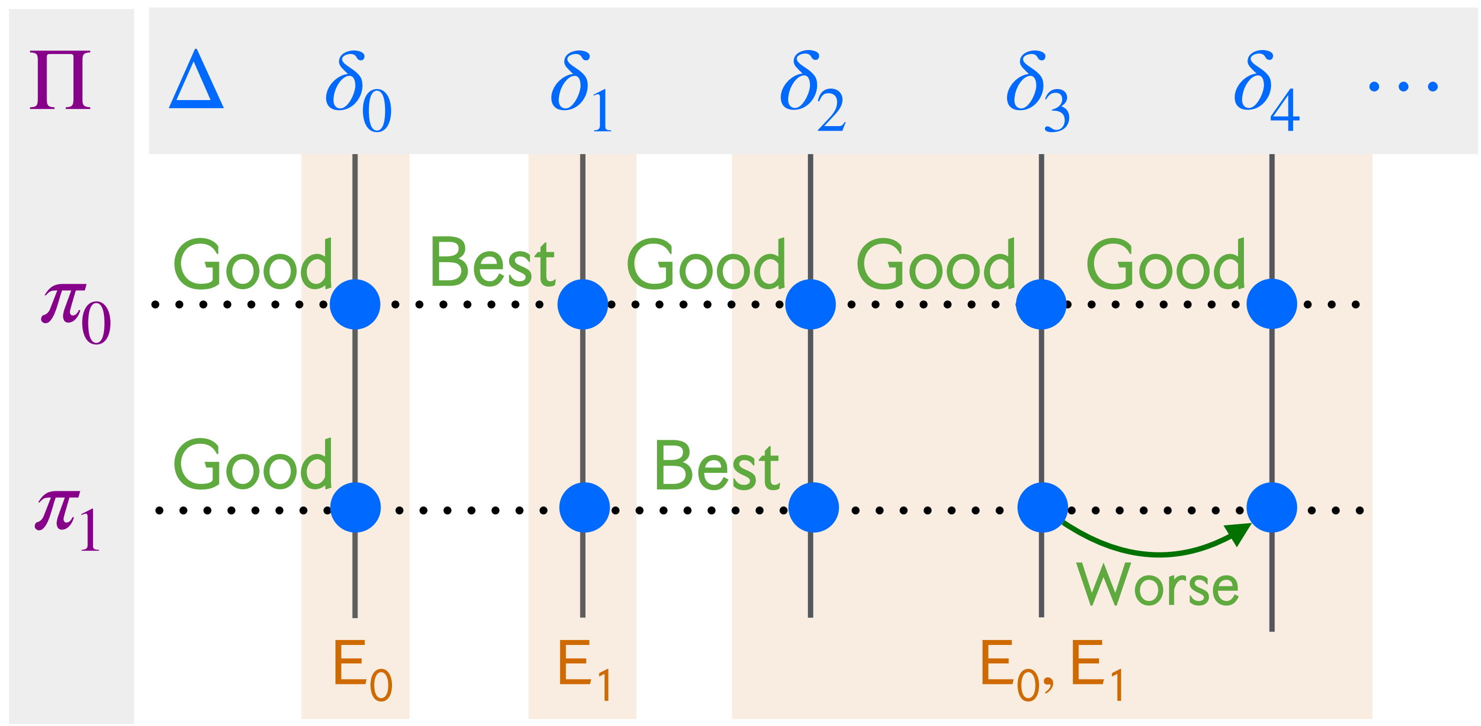

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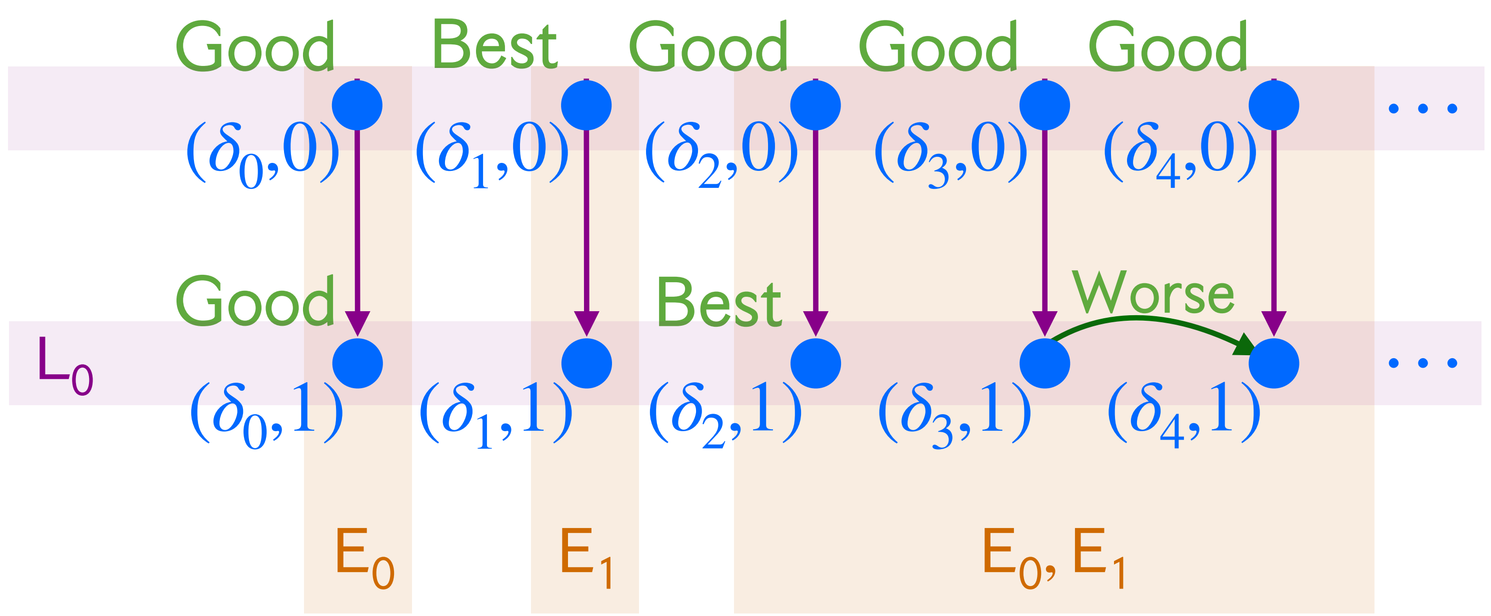

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- 1 F-successor unless they have the last index
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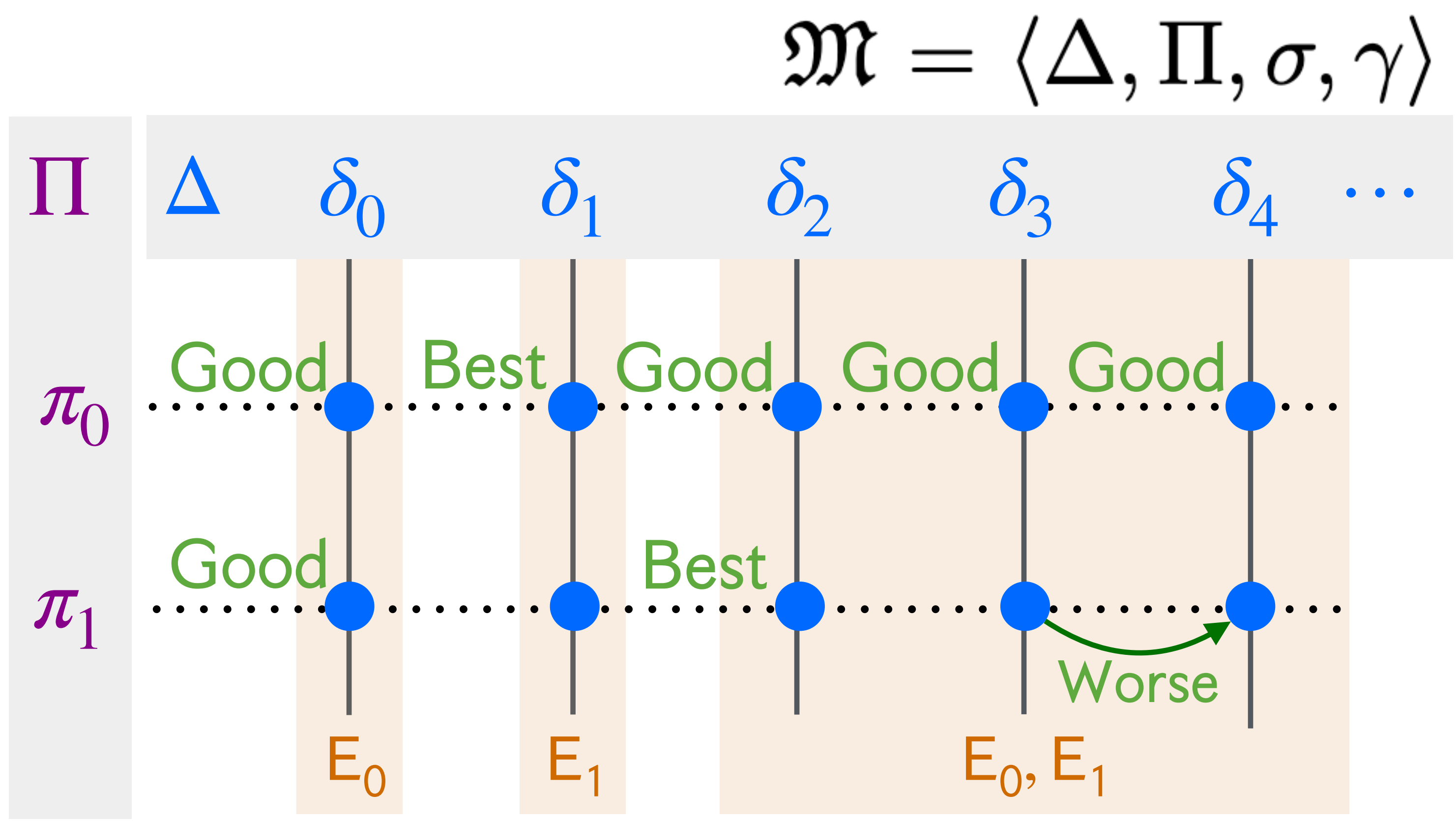

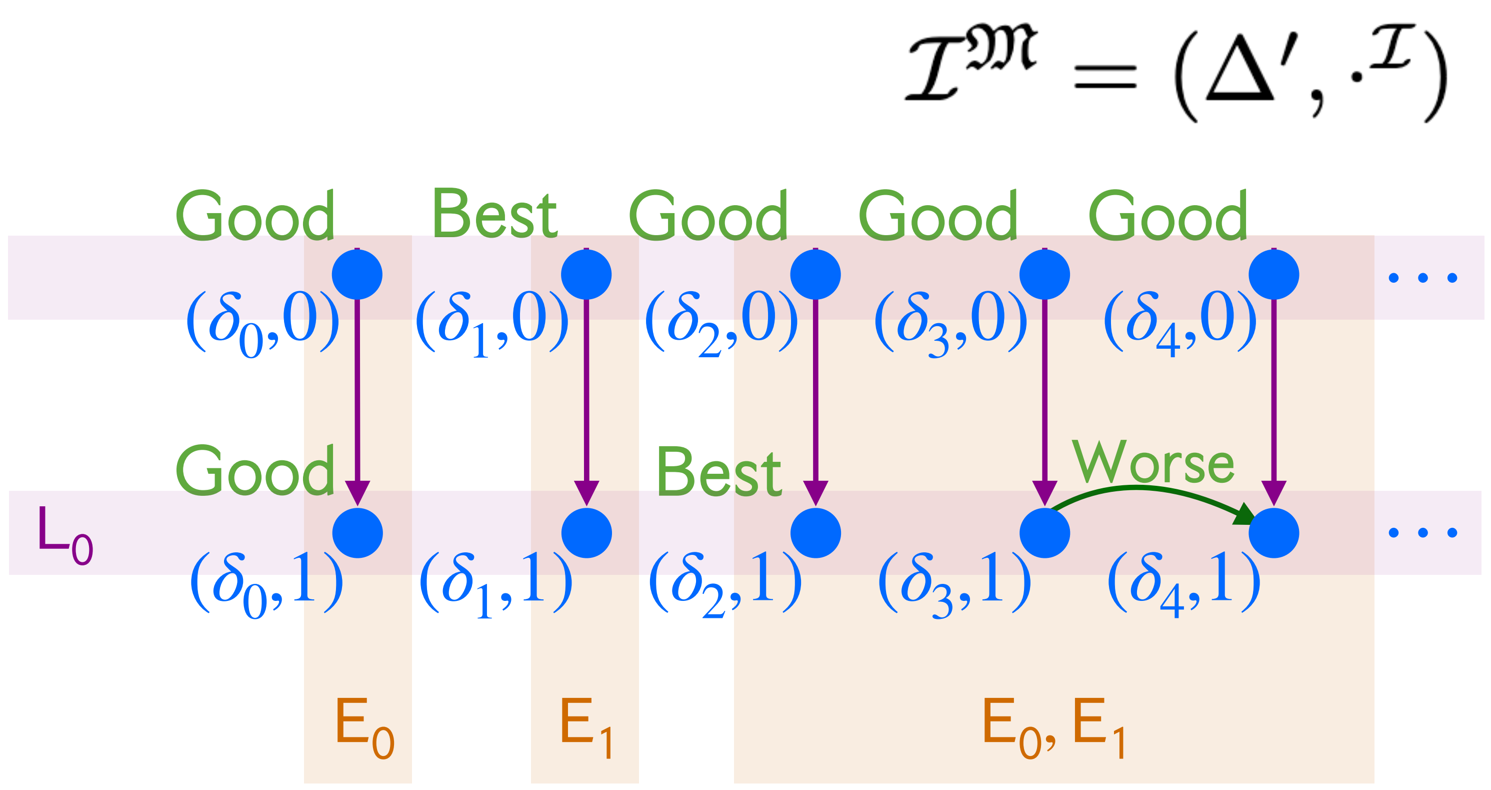

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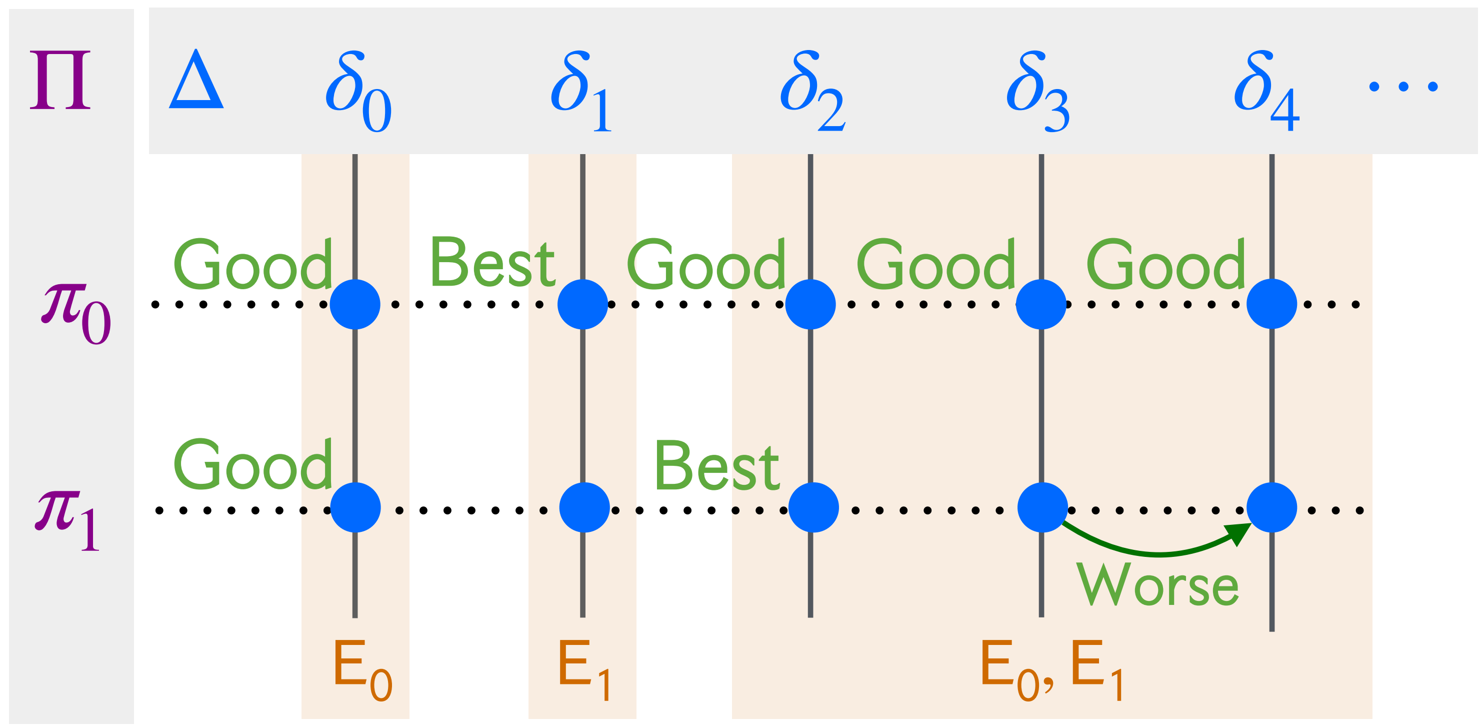

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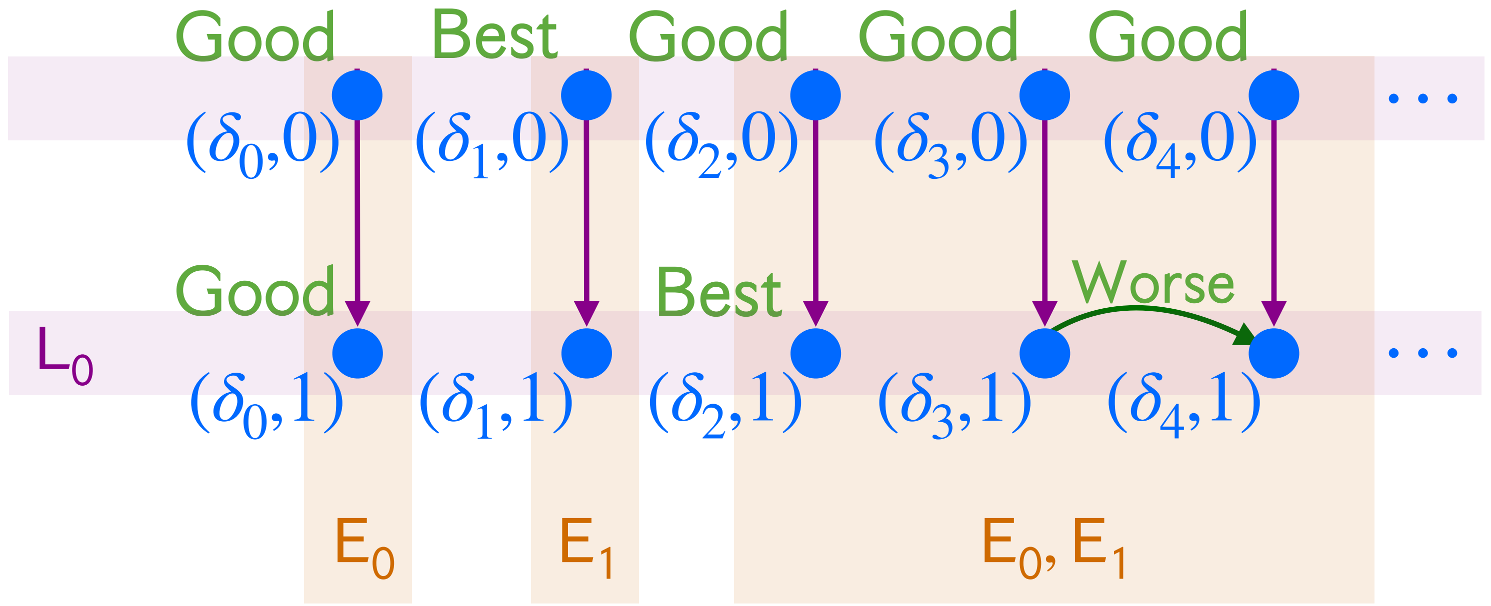

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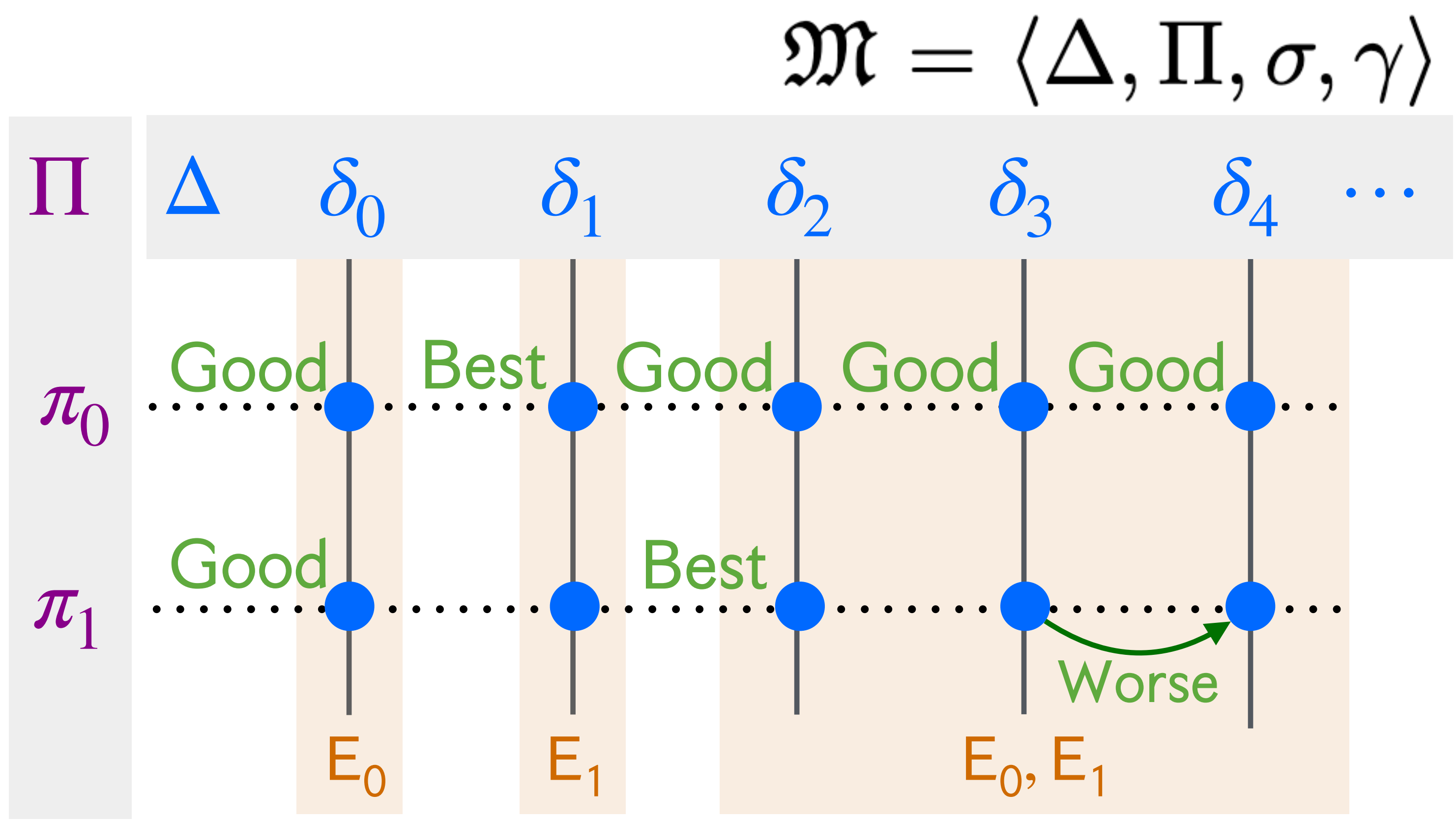

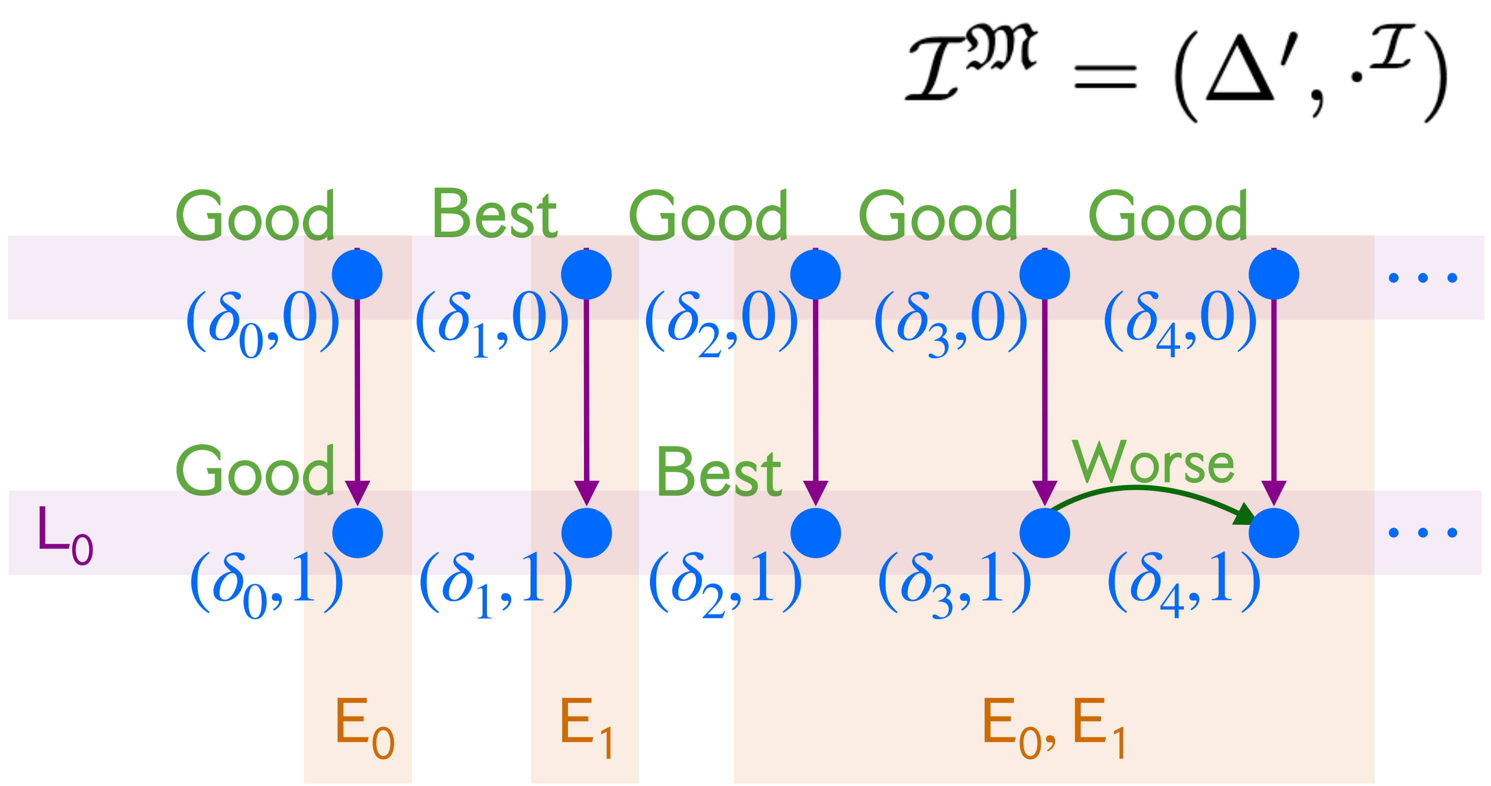

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An FO interpretation satisfies ϕ^m_{stack} *iff*
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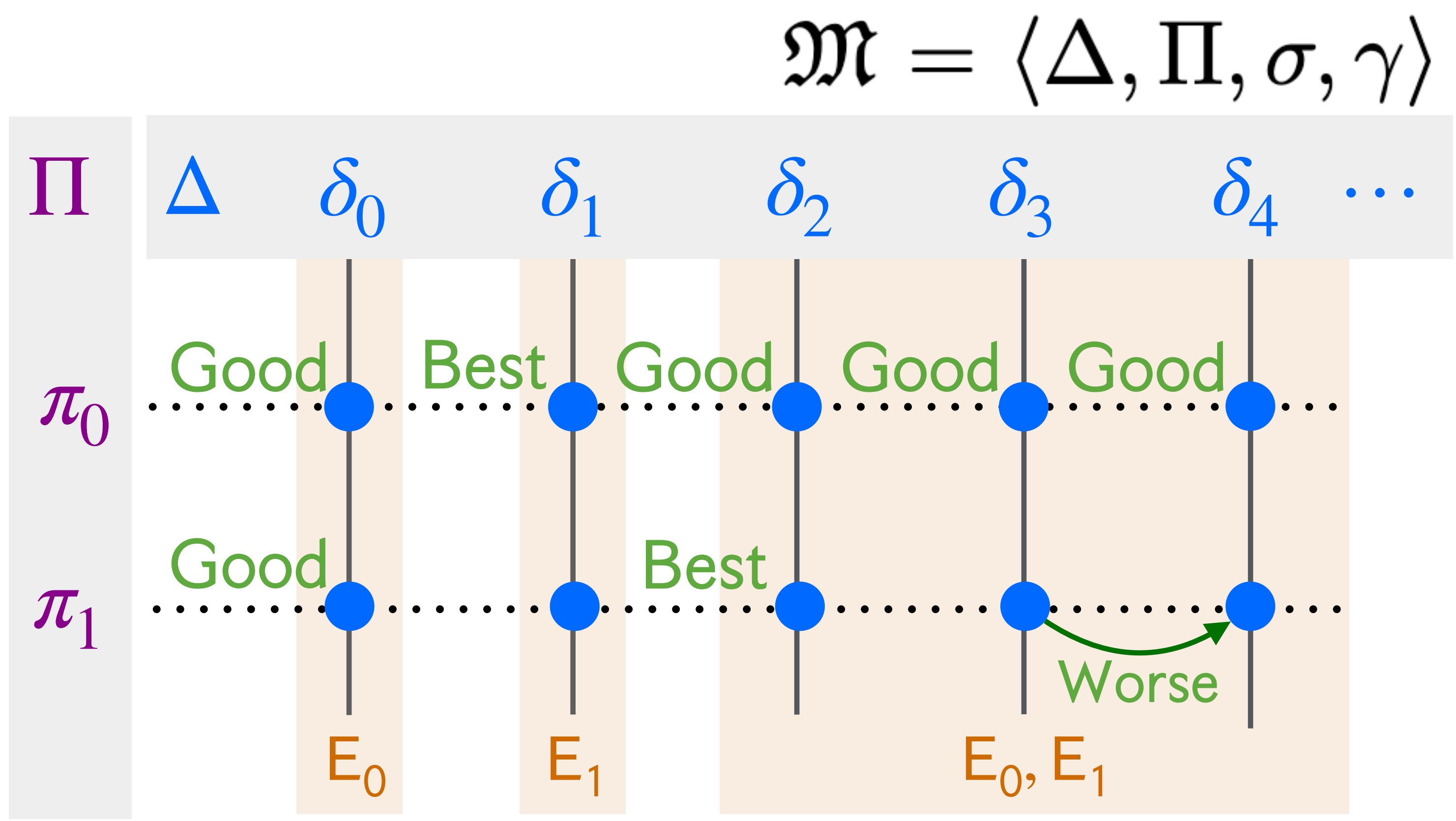

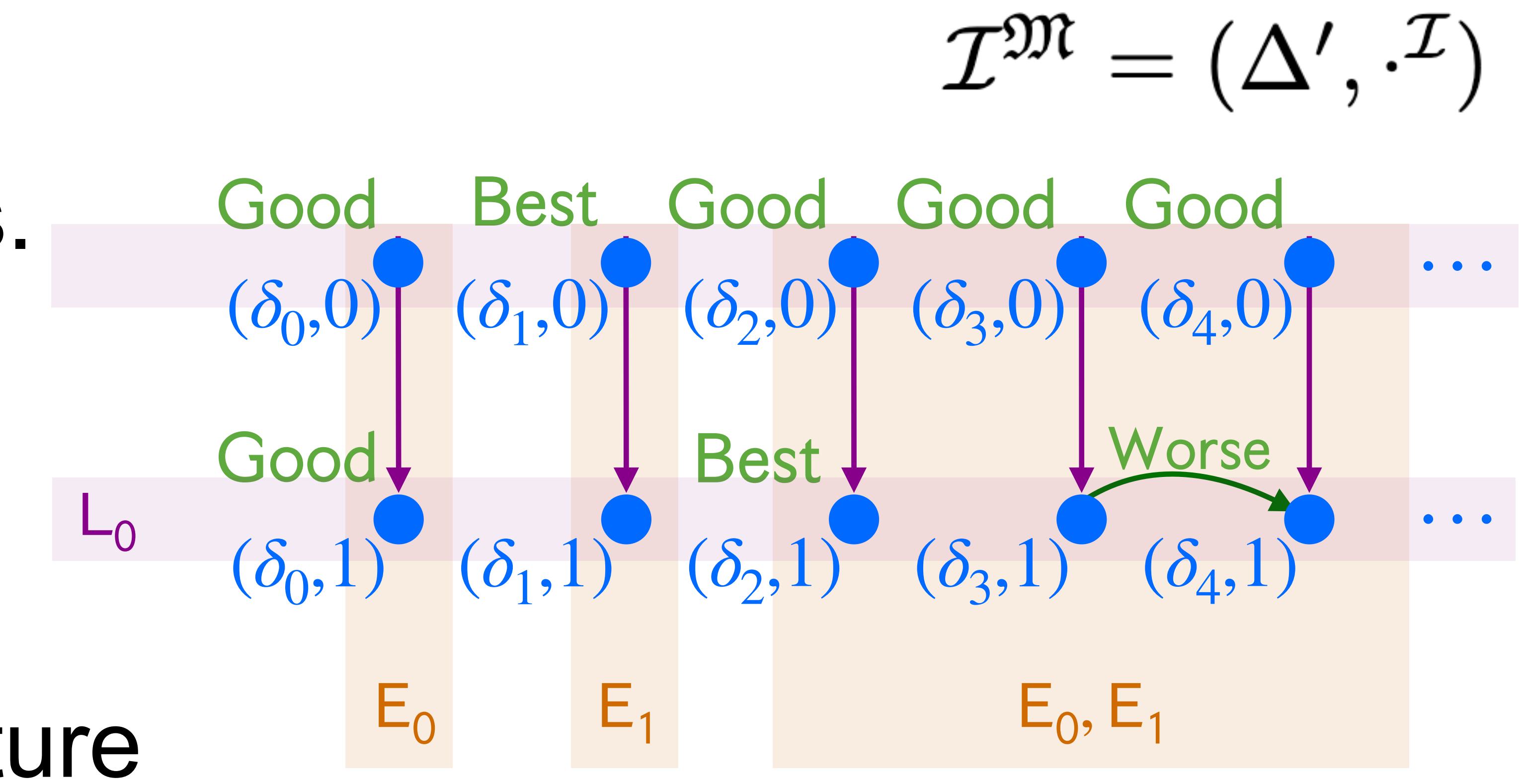

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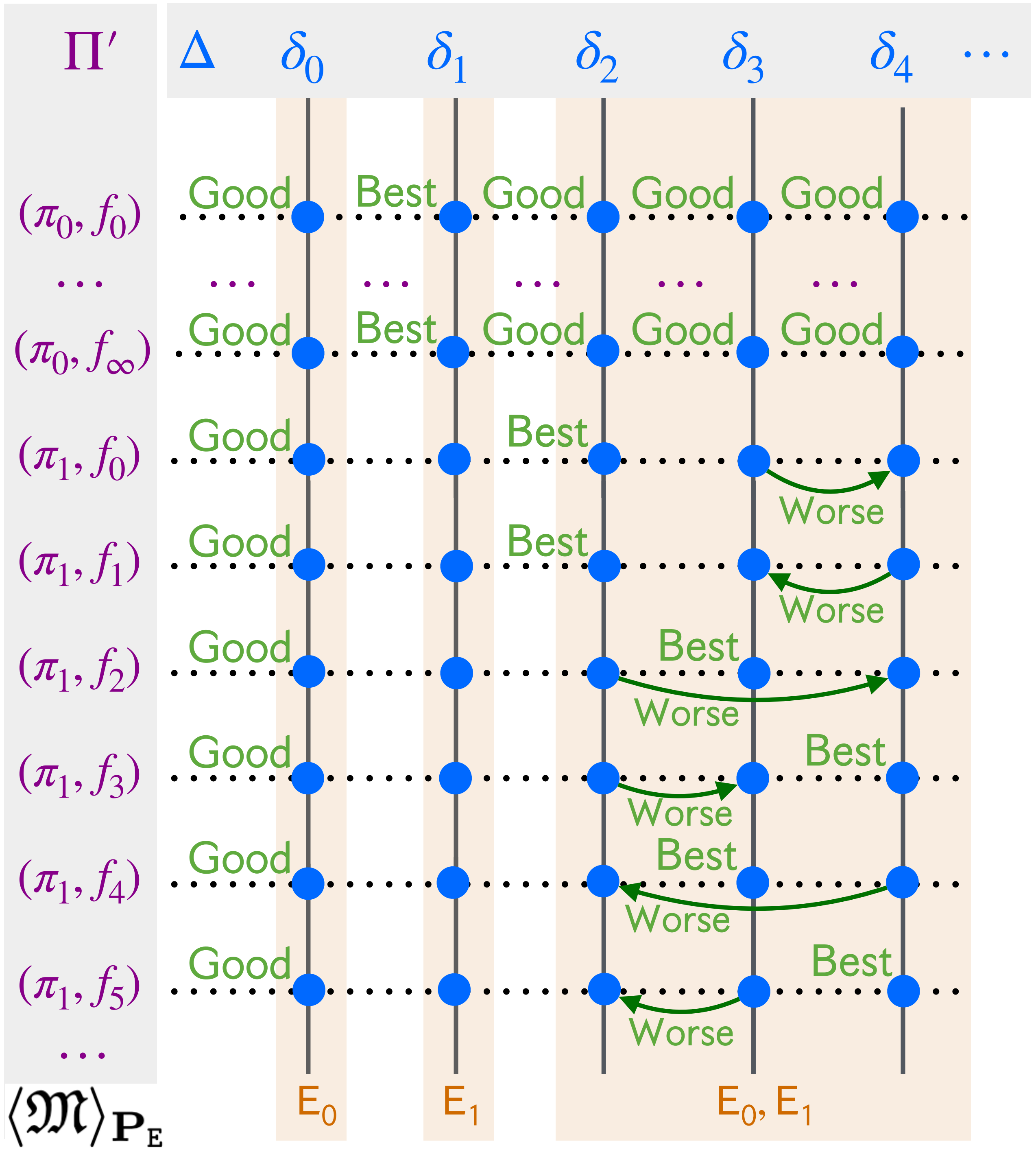

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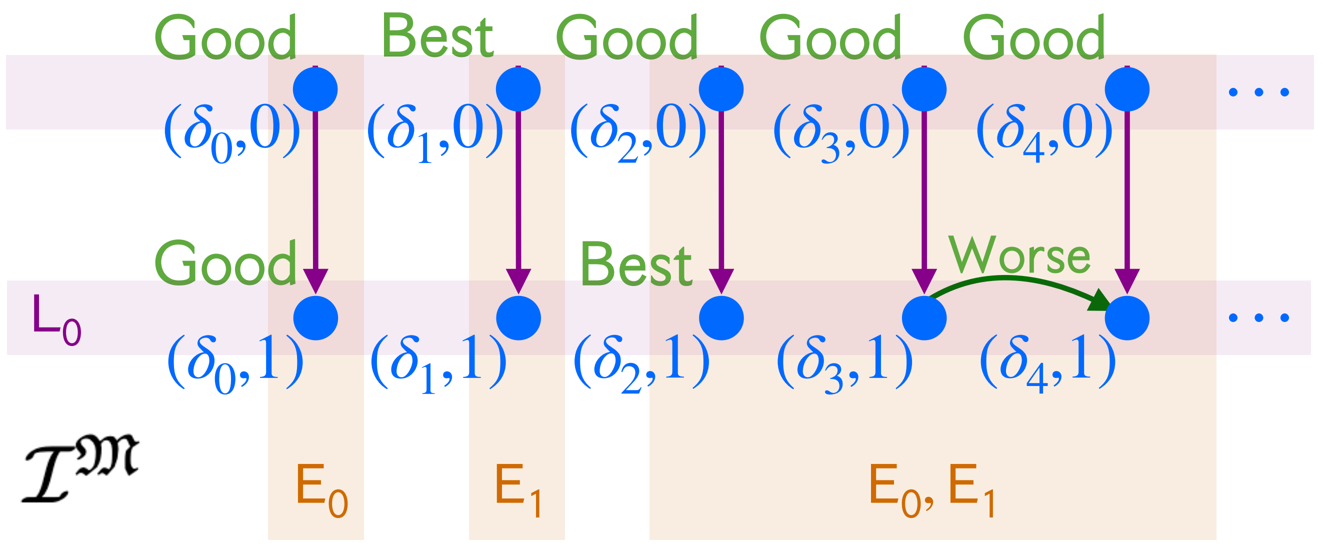

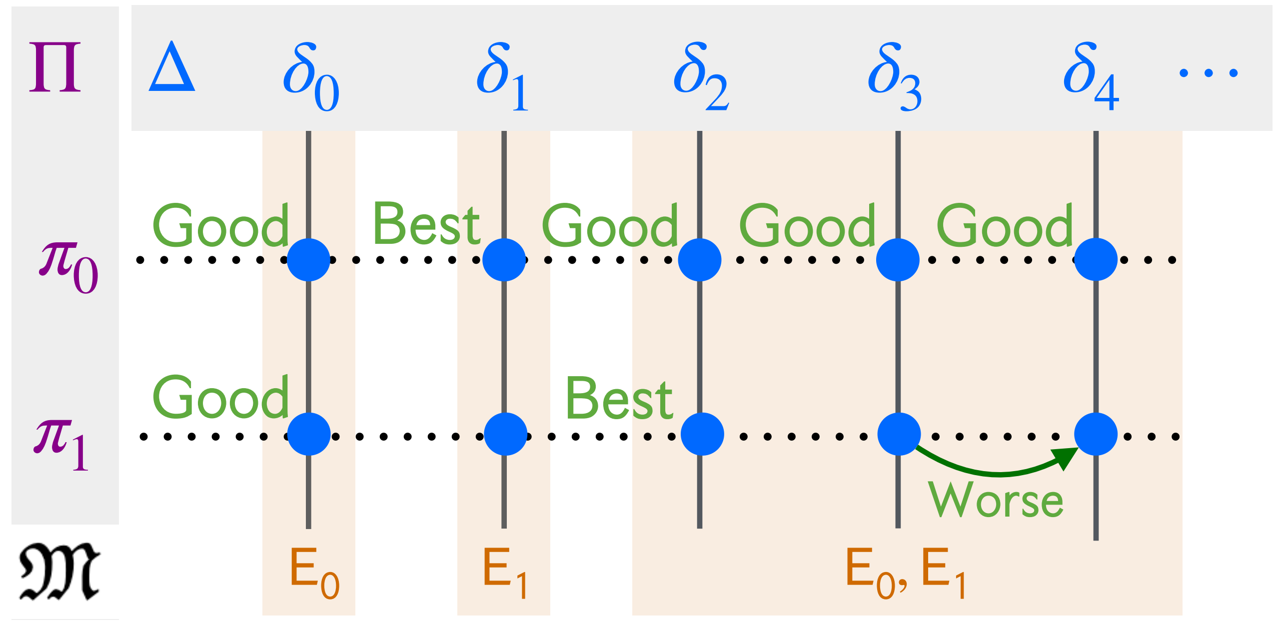

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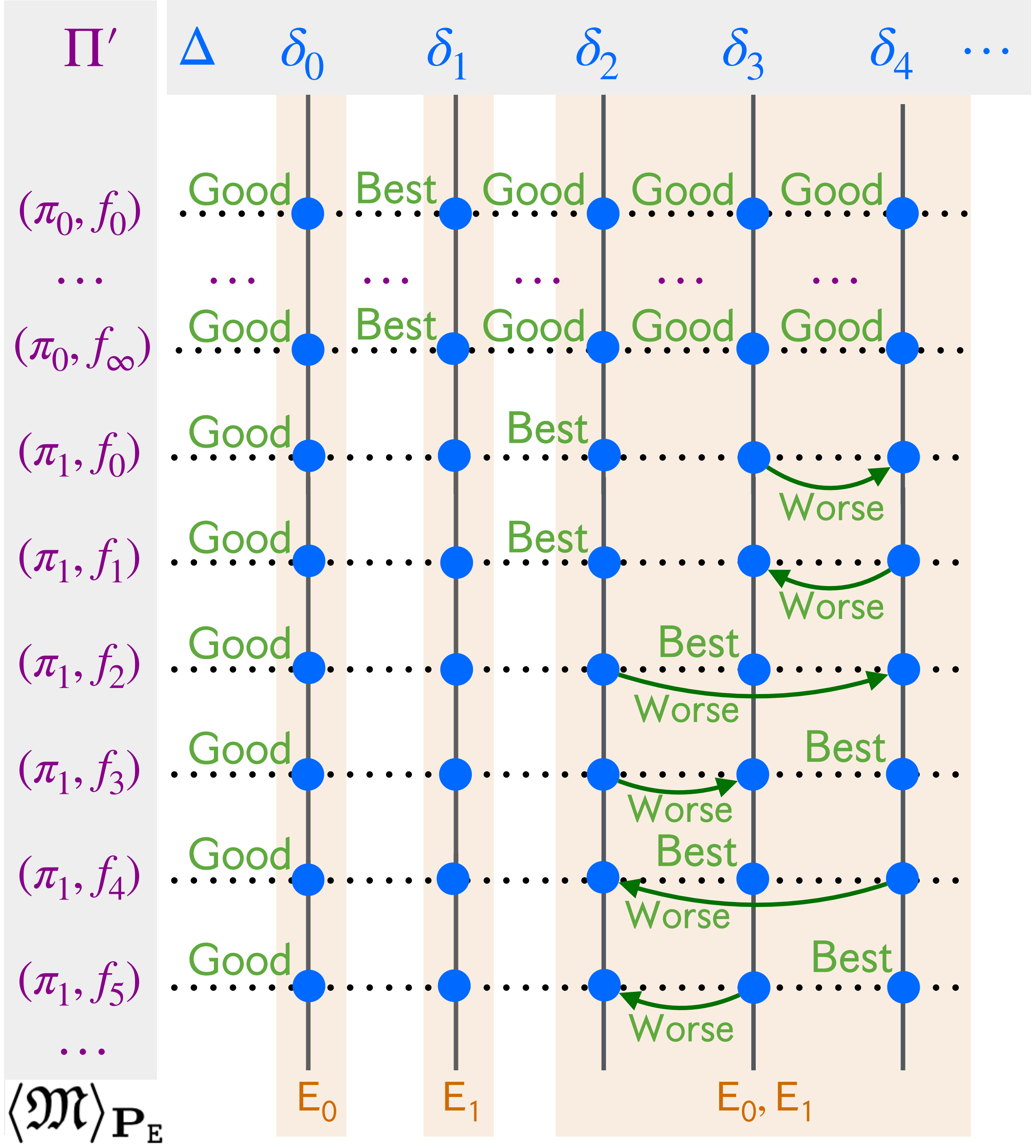

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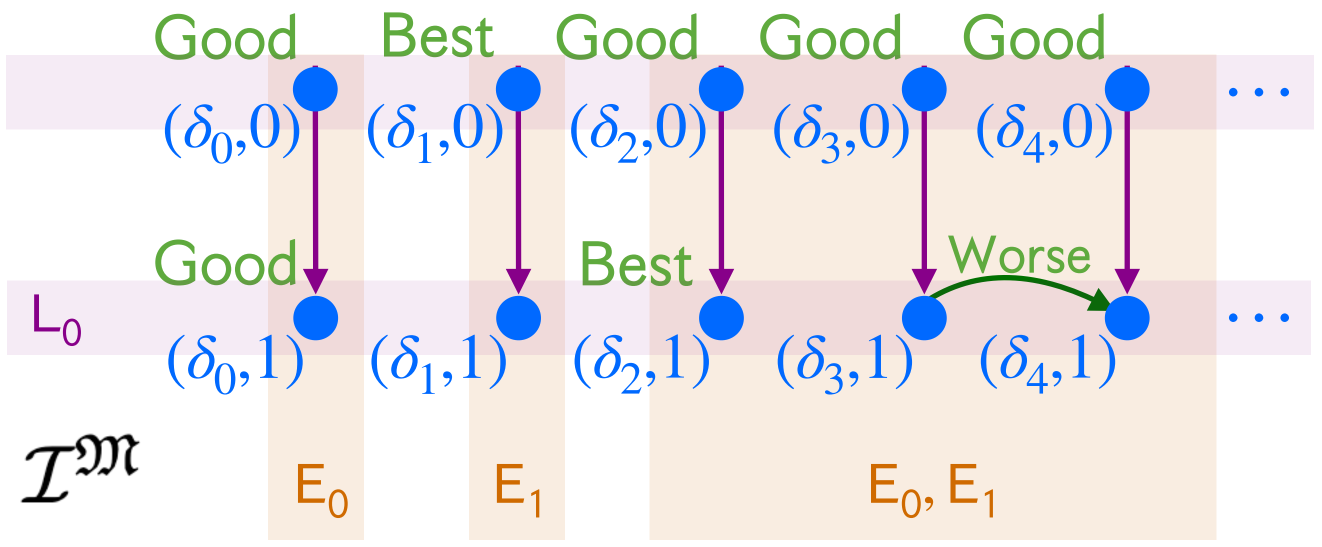

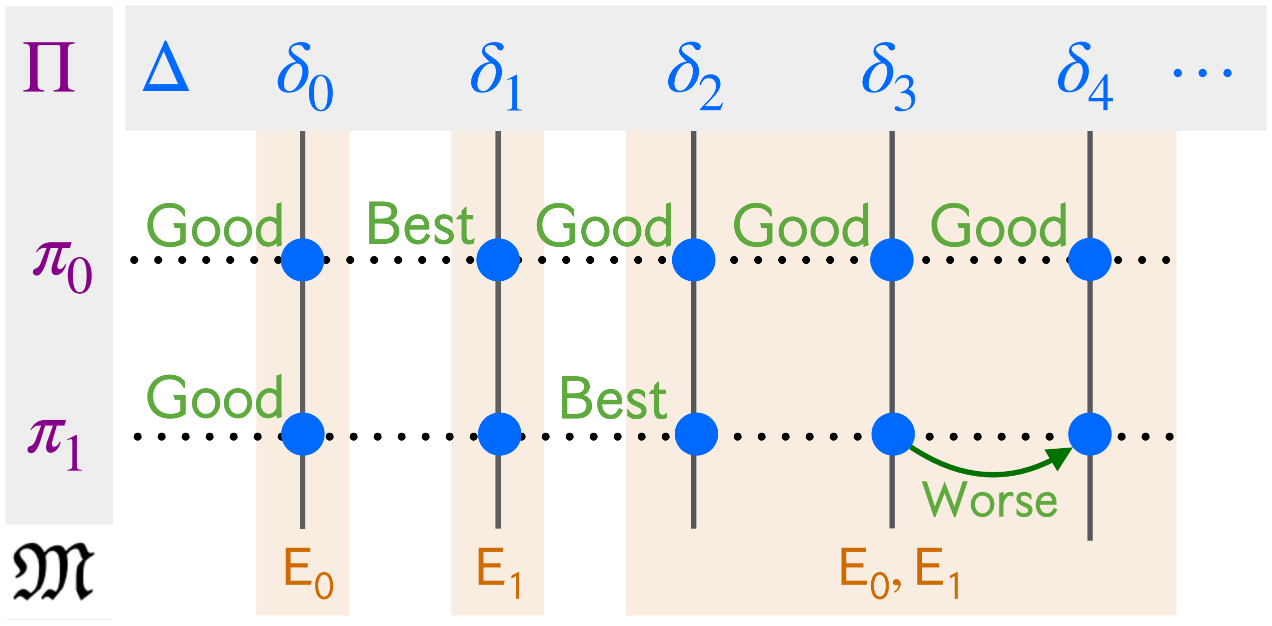

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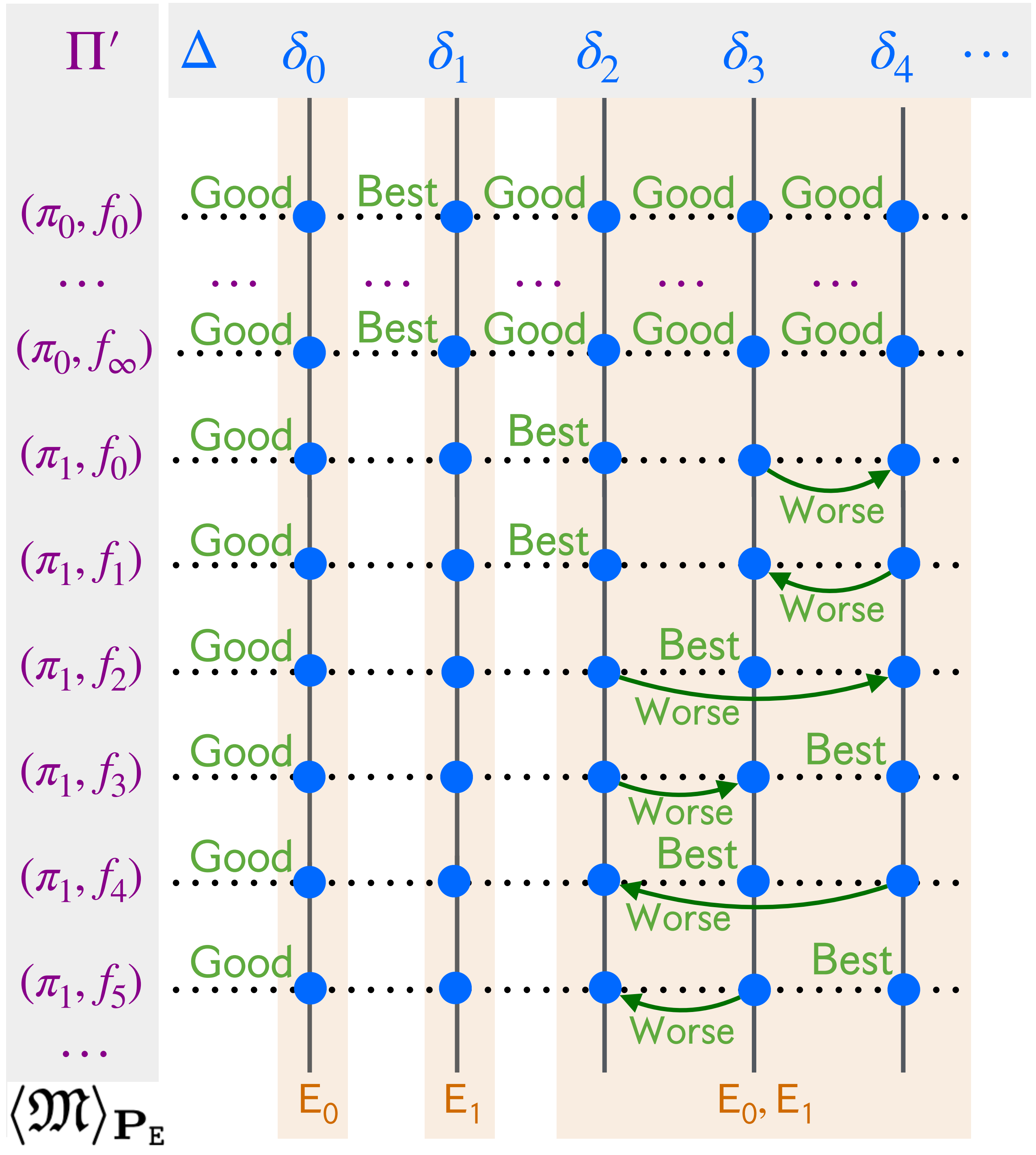
- $\exists^{\lhd n} x \,.\, \phi$ is evaluated on elements in the same L-layer
- $\Diamond\phi$ is evaluated on elements with the same E-type

  If an element of an E-type satisfies ϕ, there are permutations so all elements satisfy $\Diamond\phi$

  If $\Diamond\phi$ is sentential, the variable assignment does nothing



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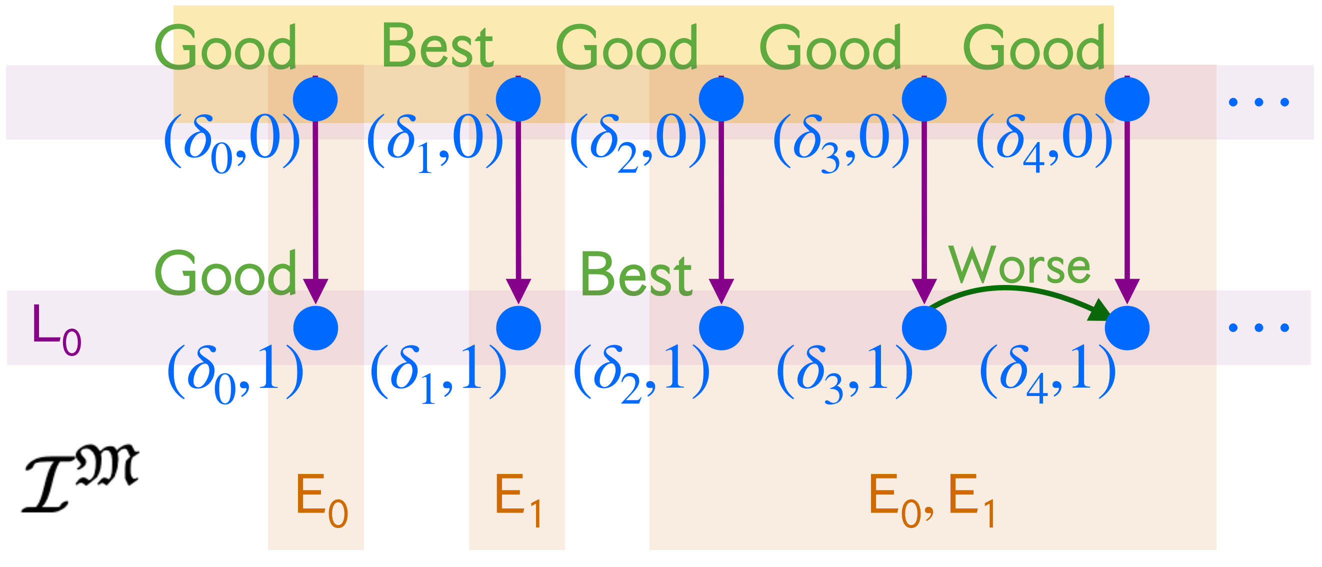

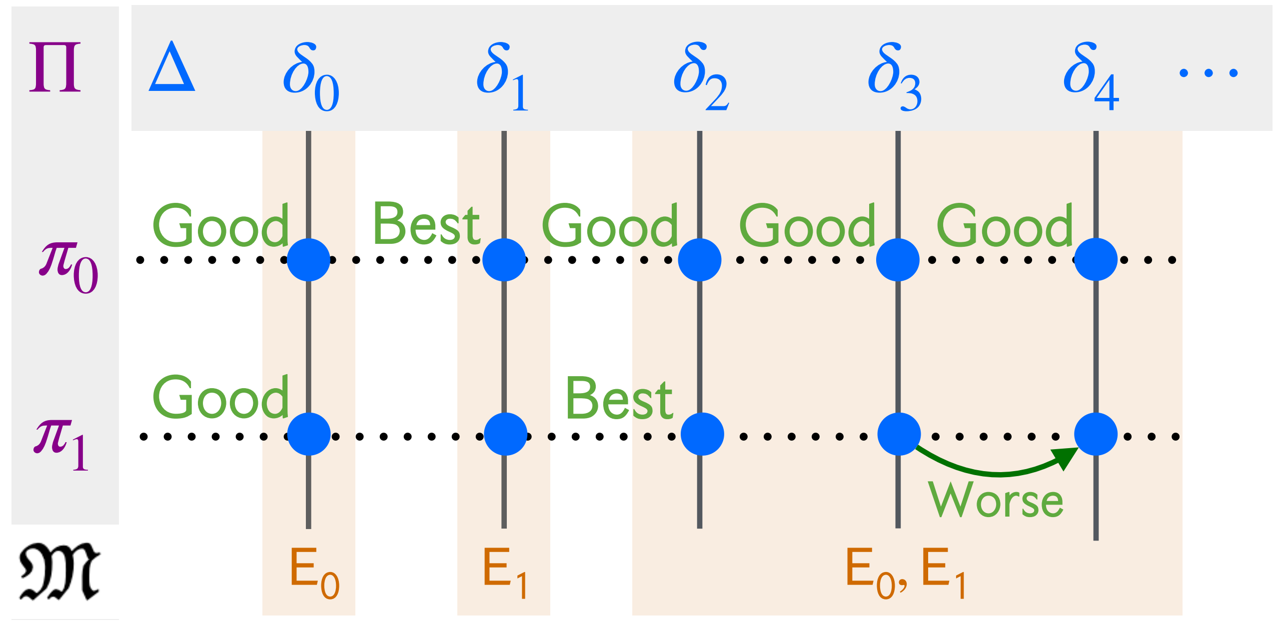

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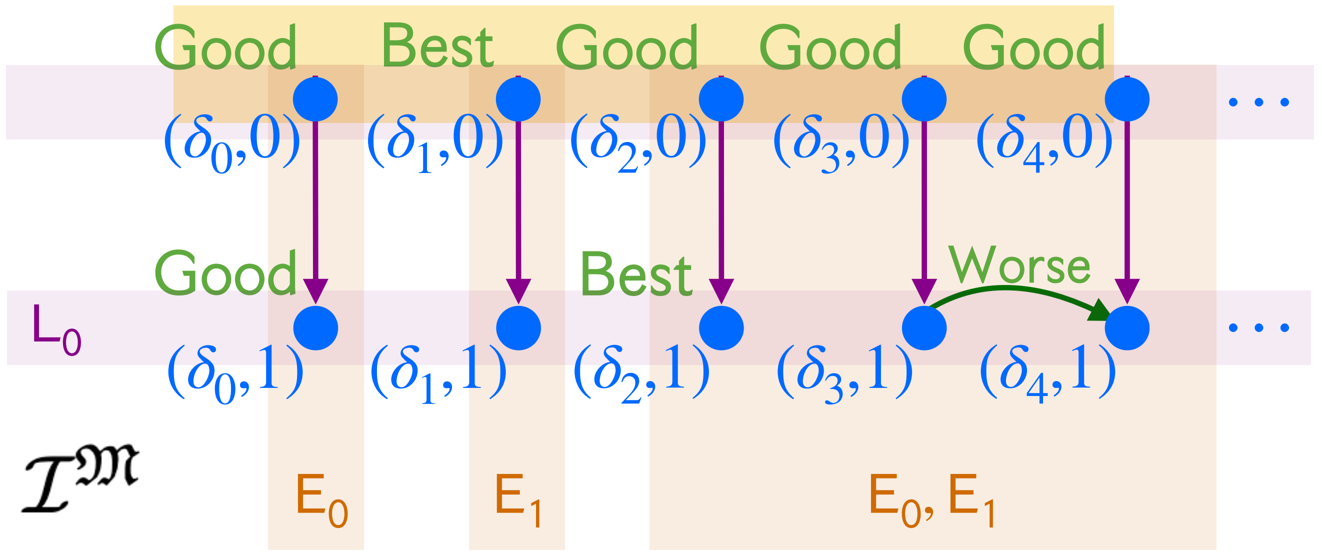

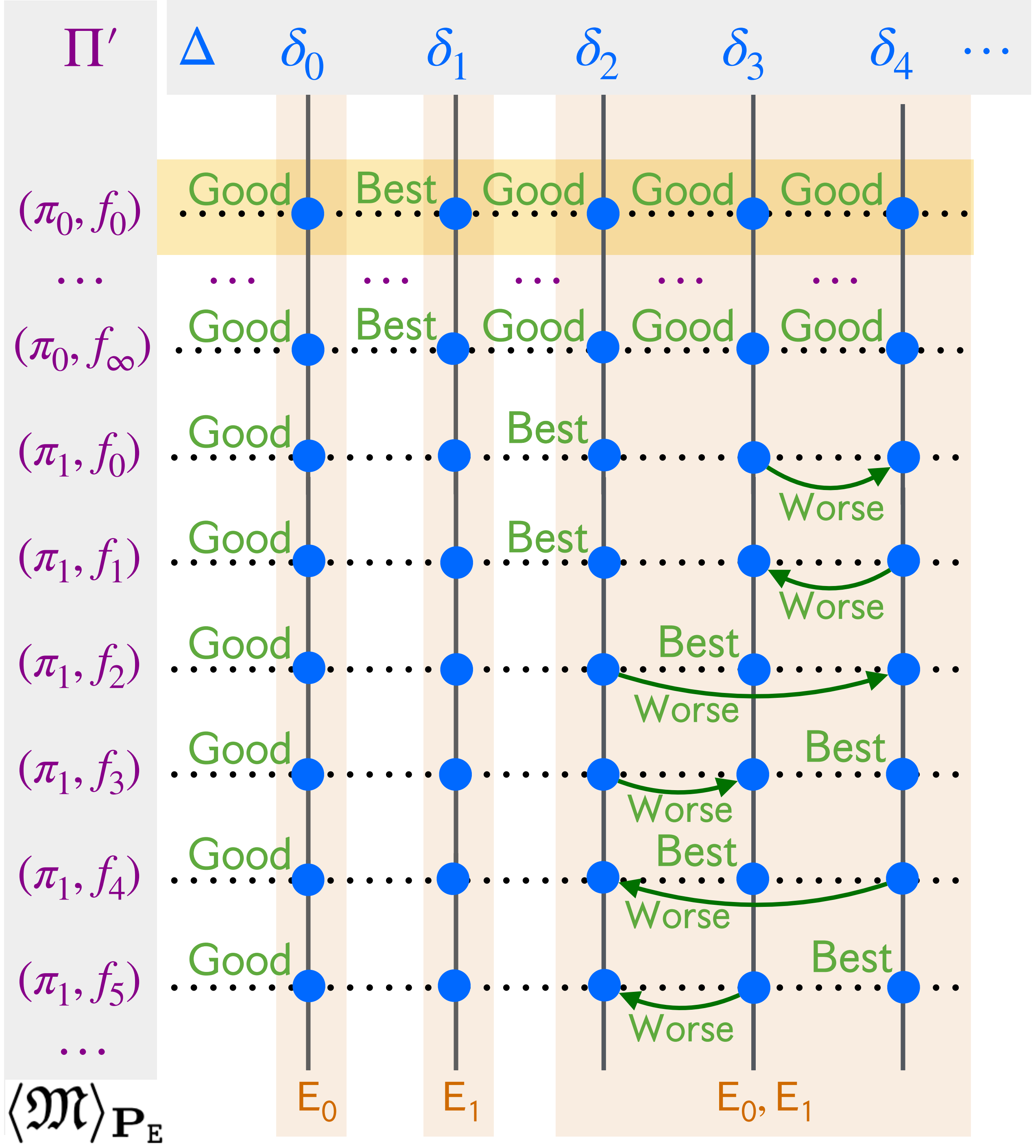

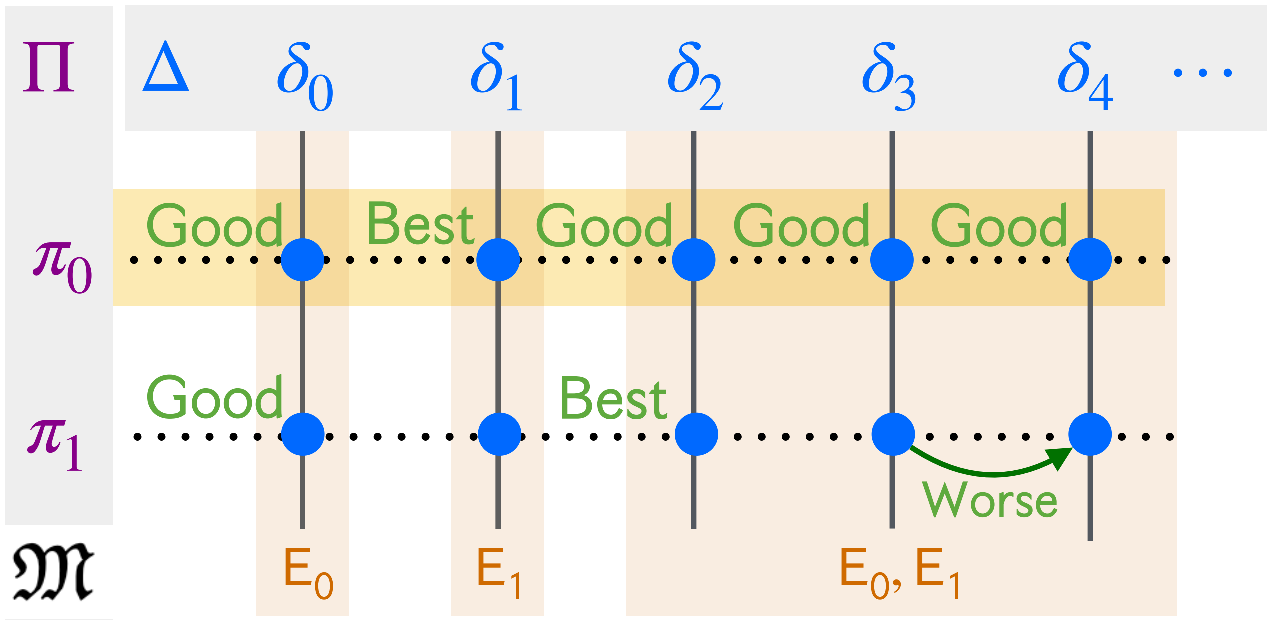

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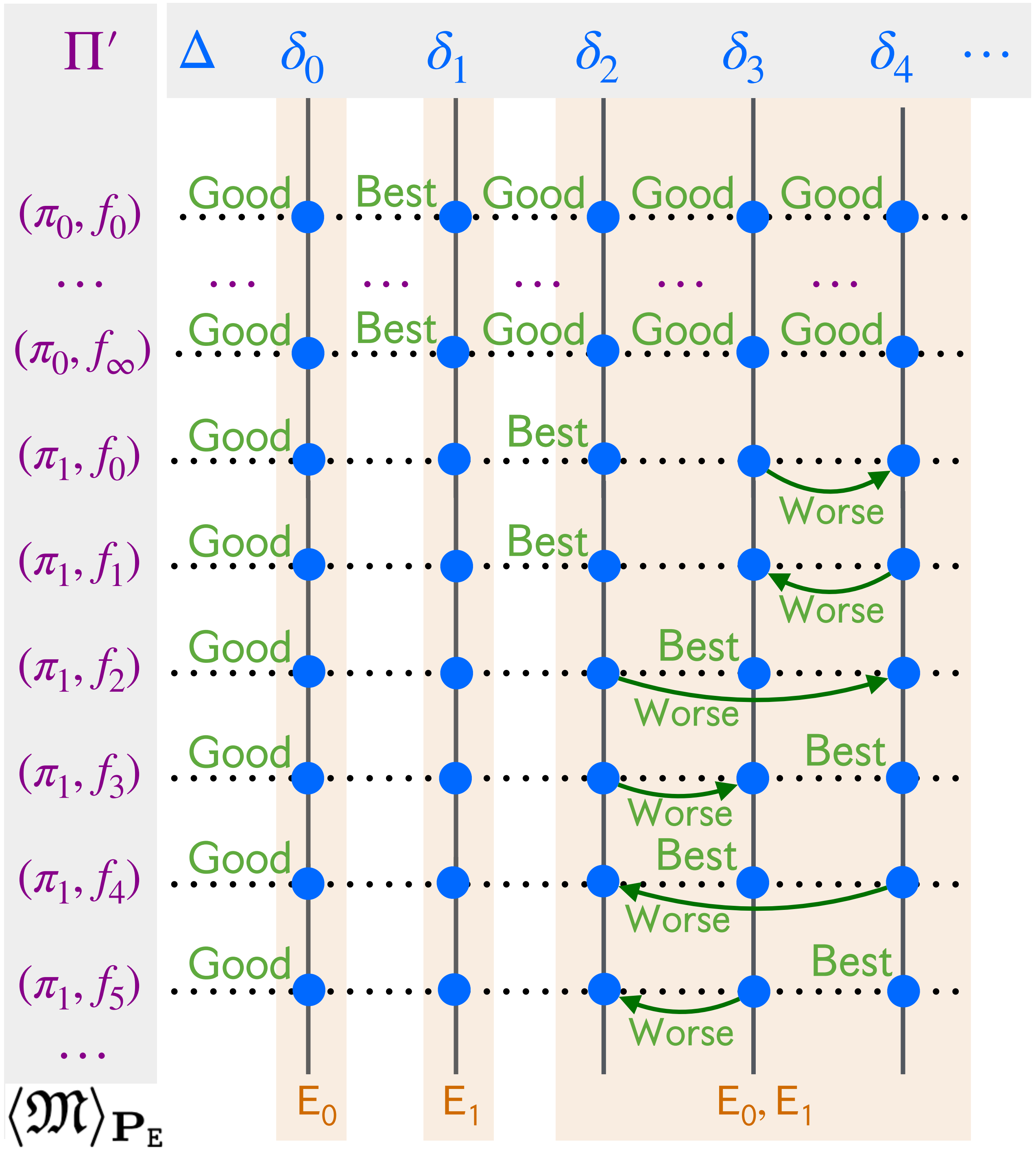

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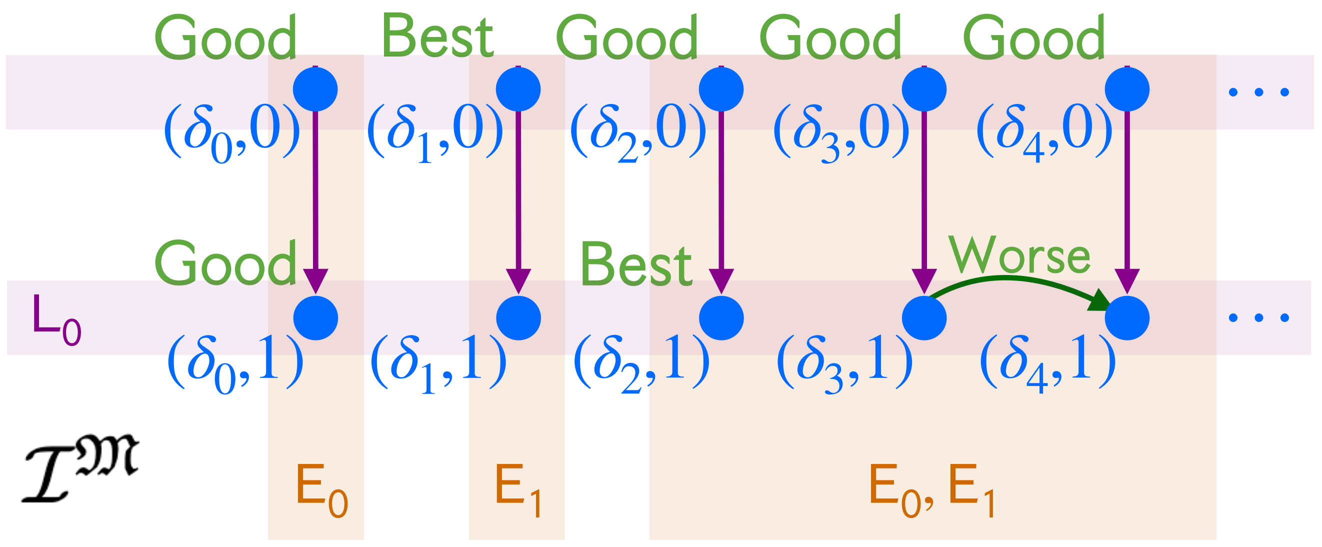

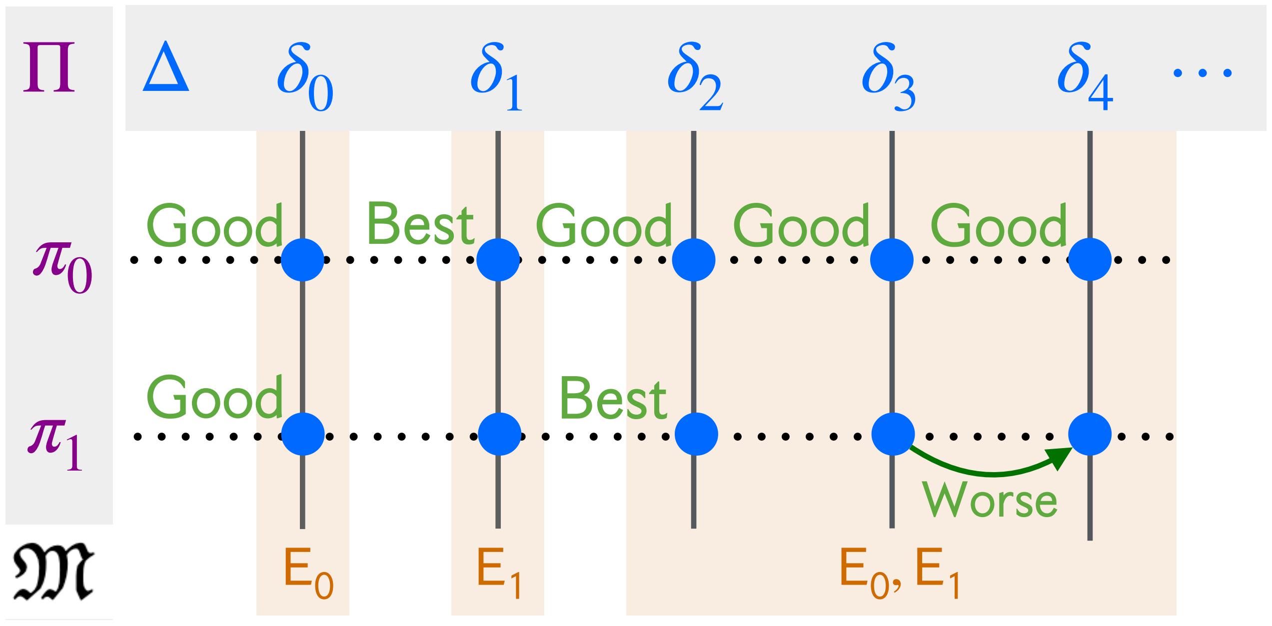

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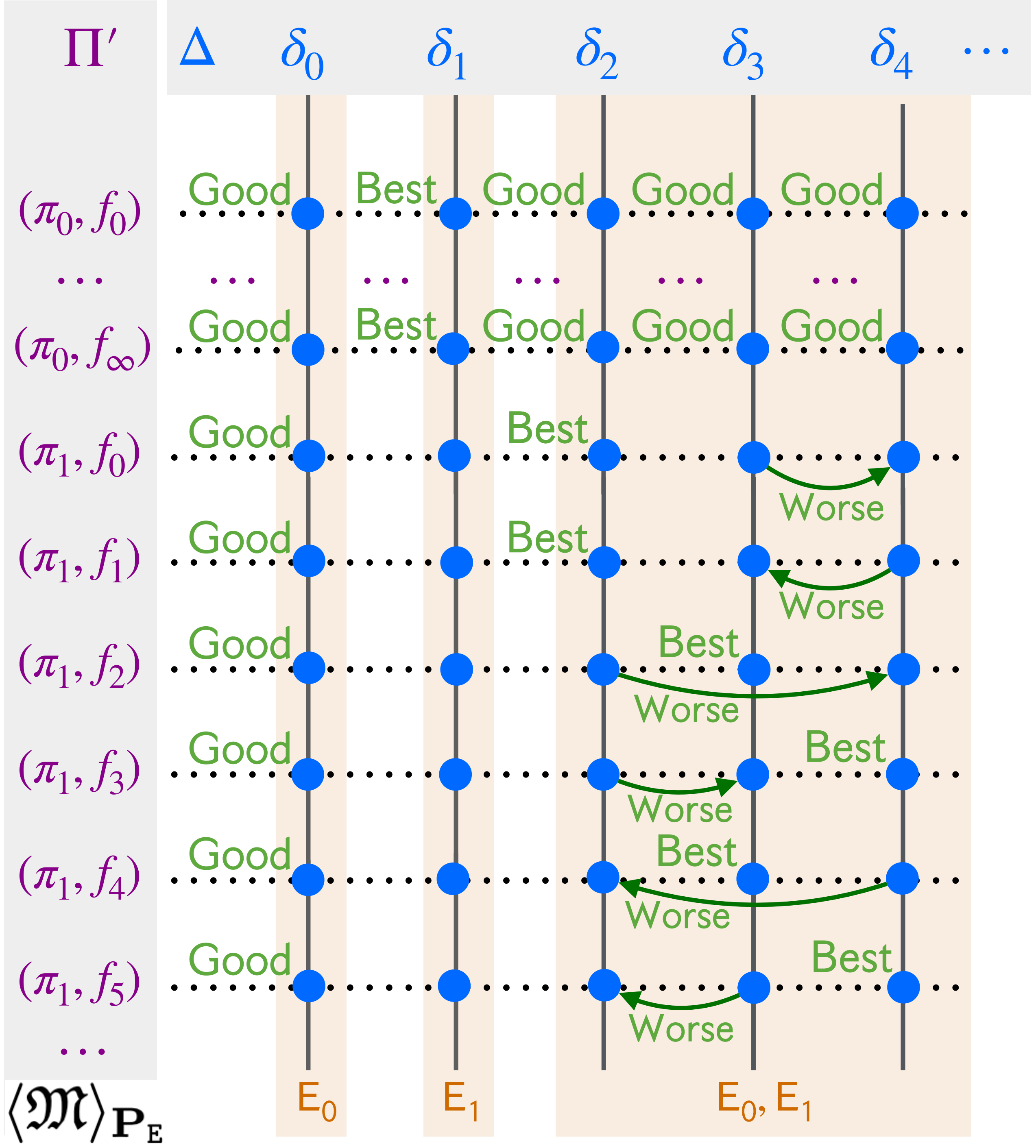

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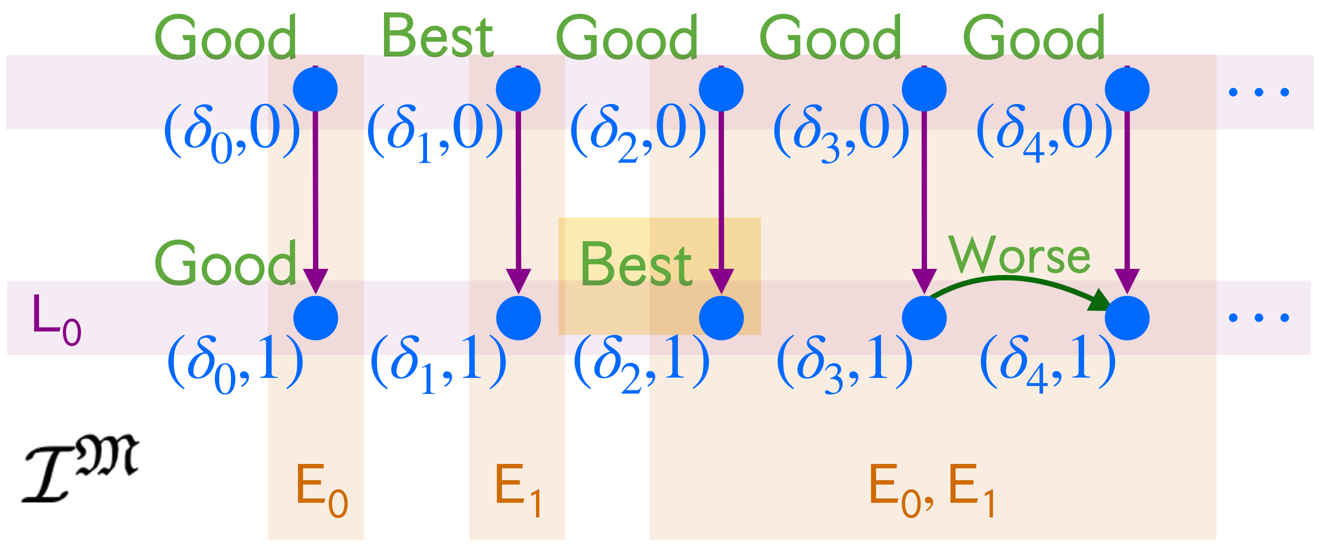

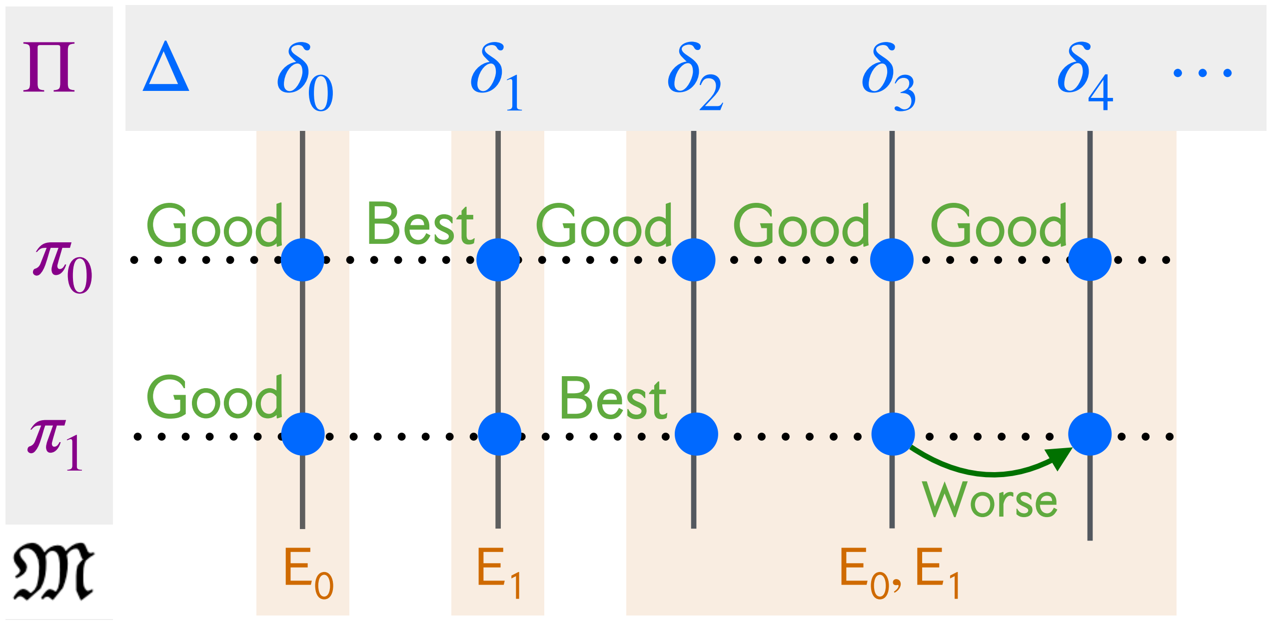

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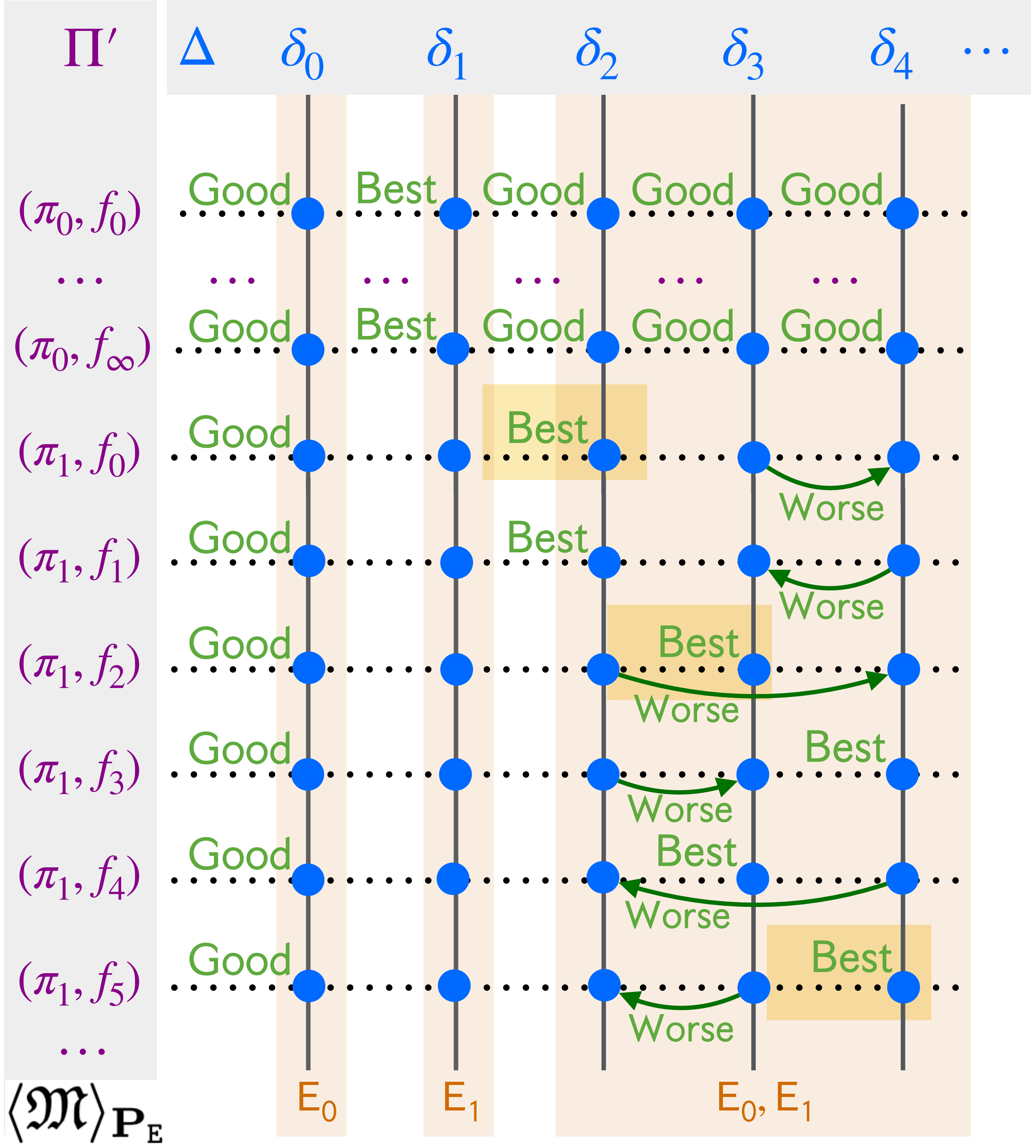

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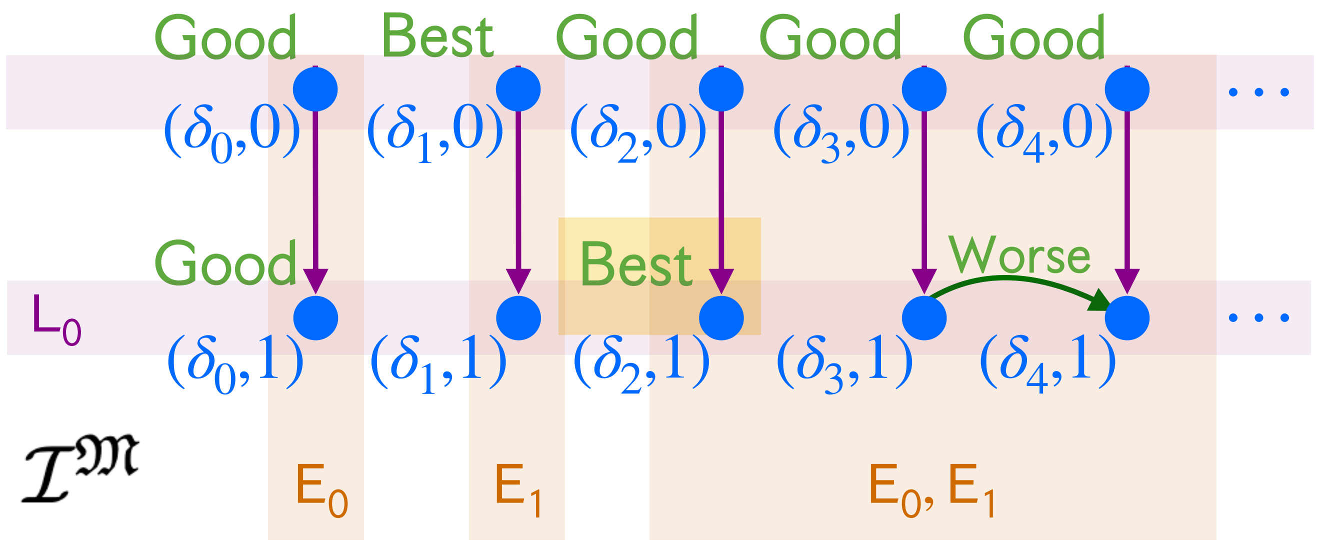

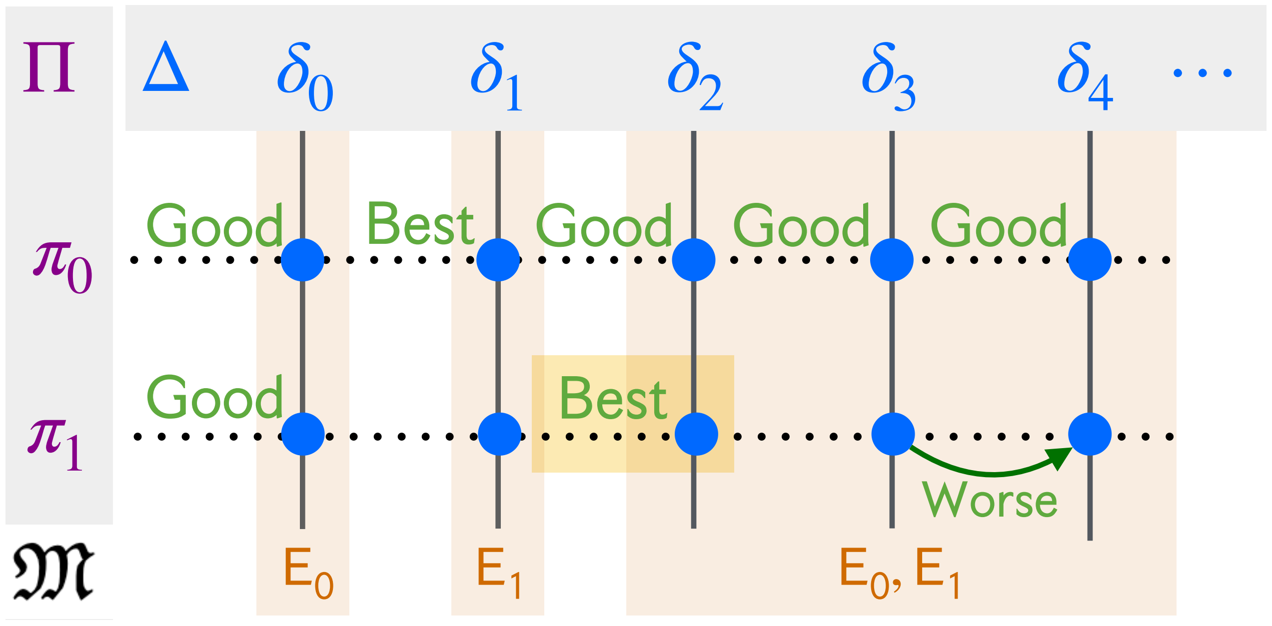

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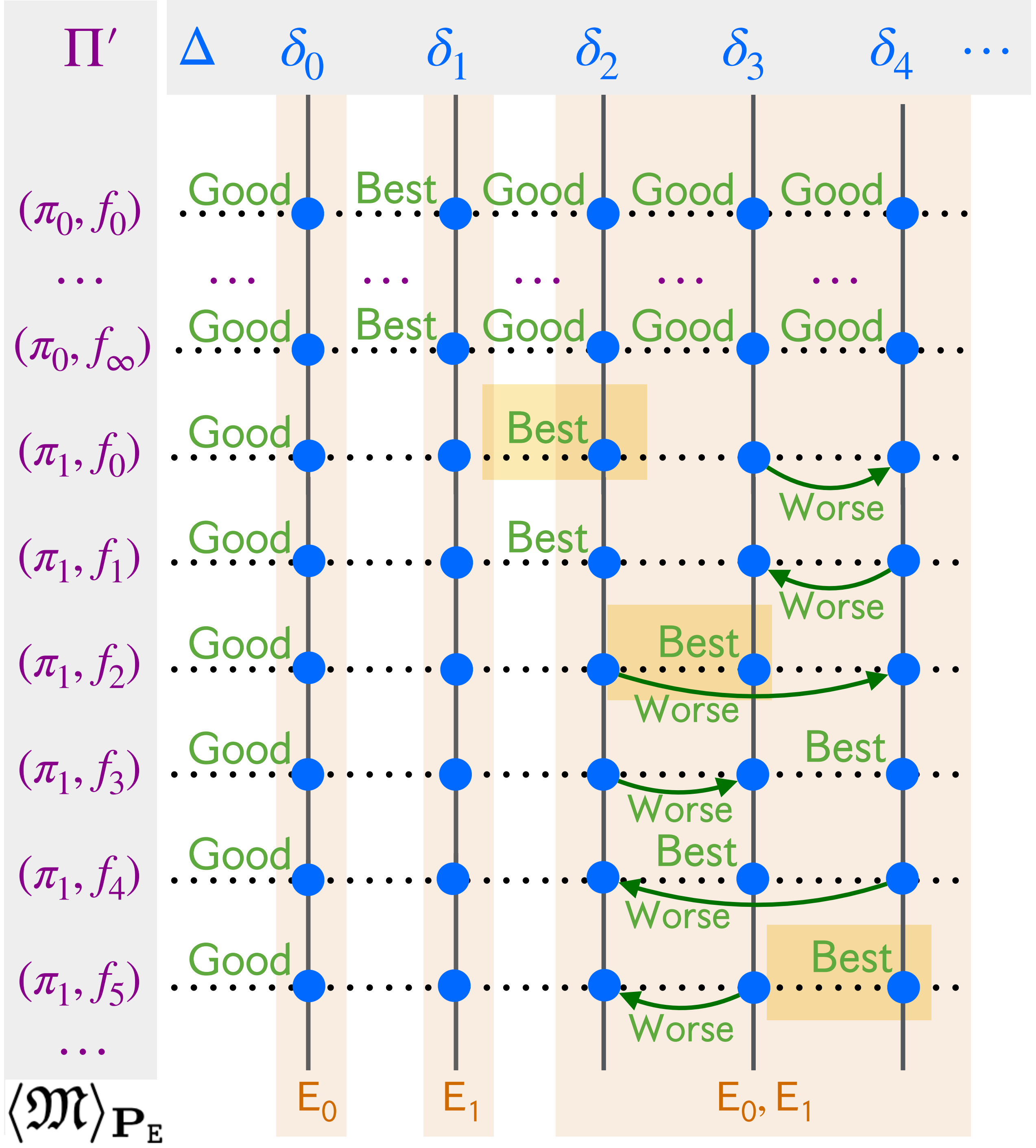

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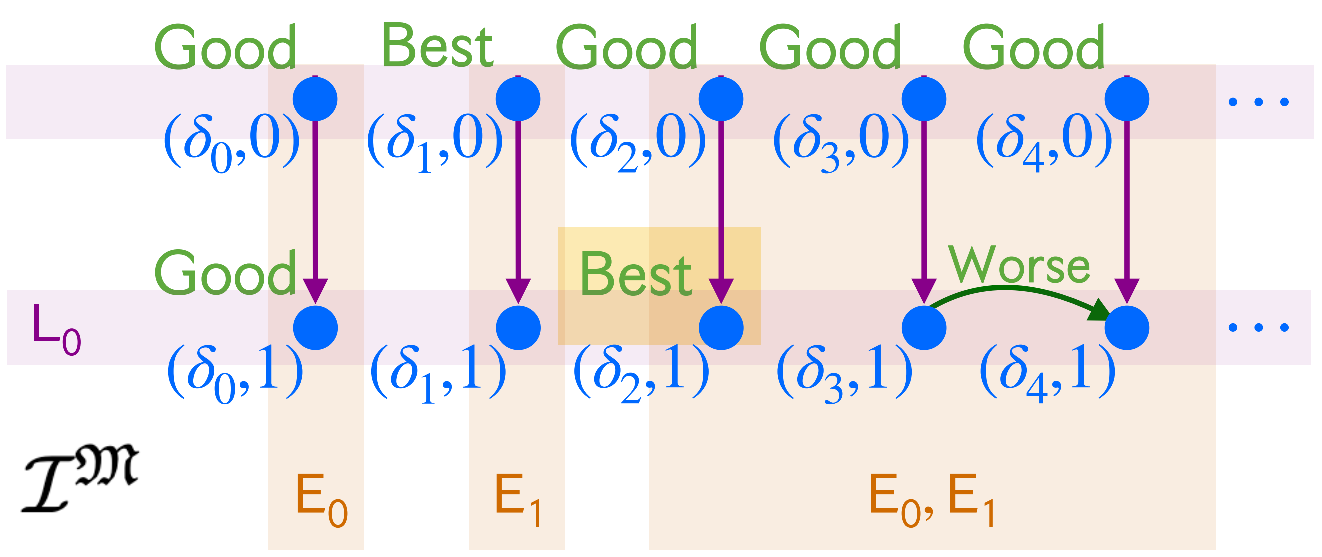

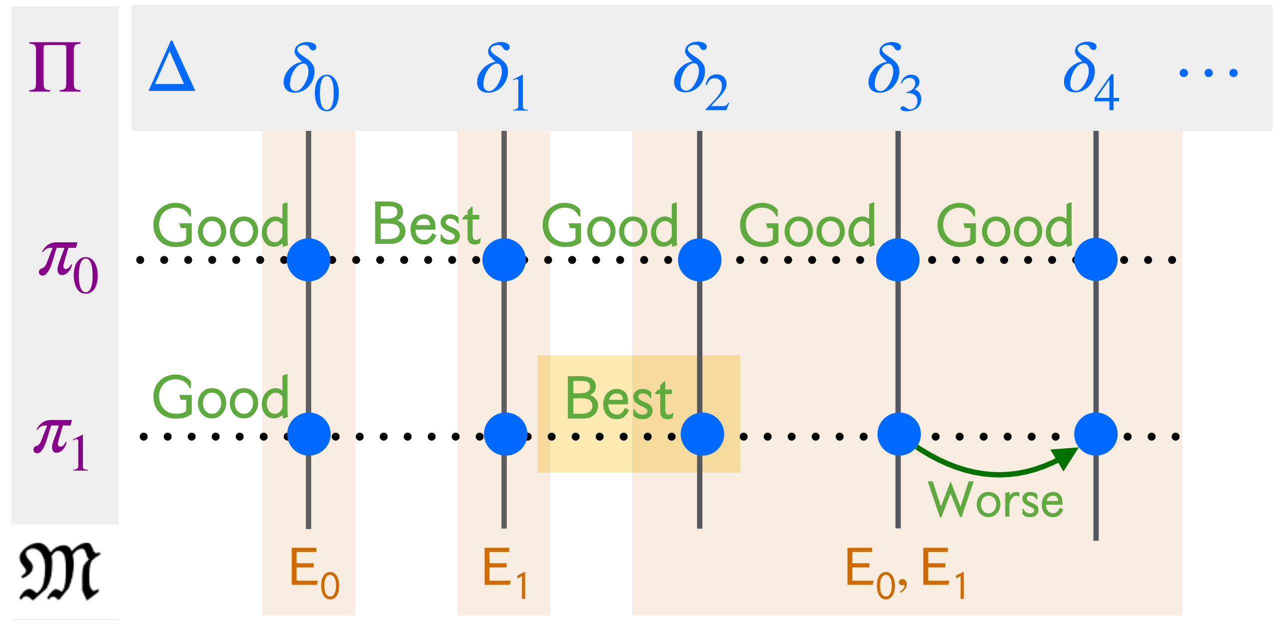

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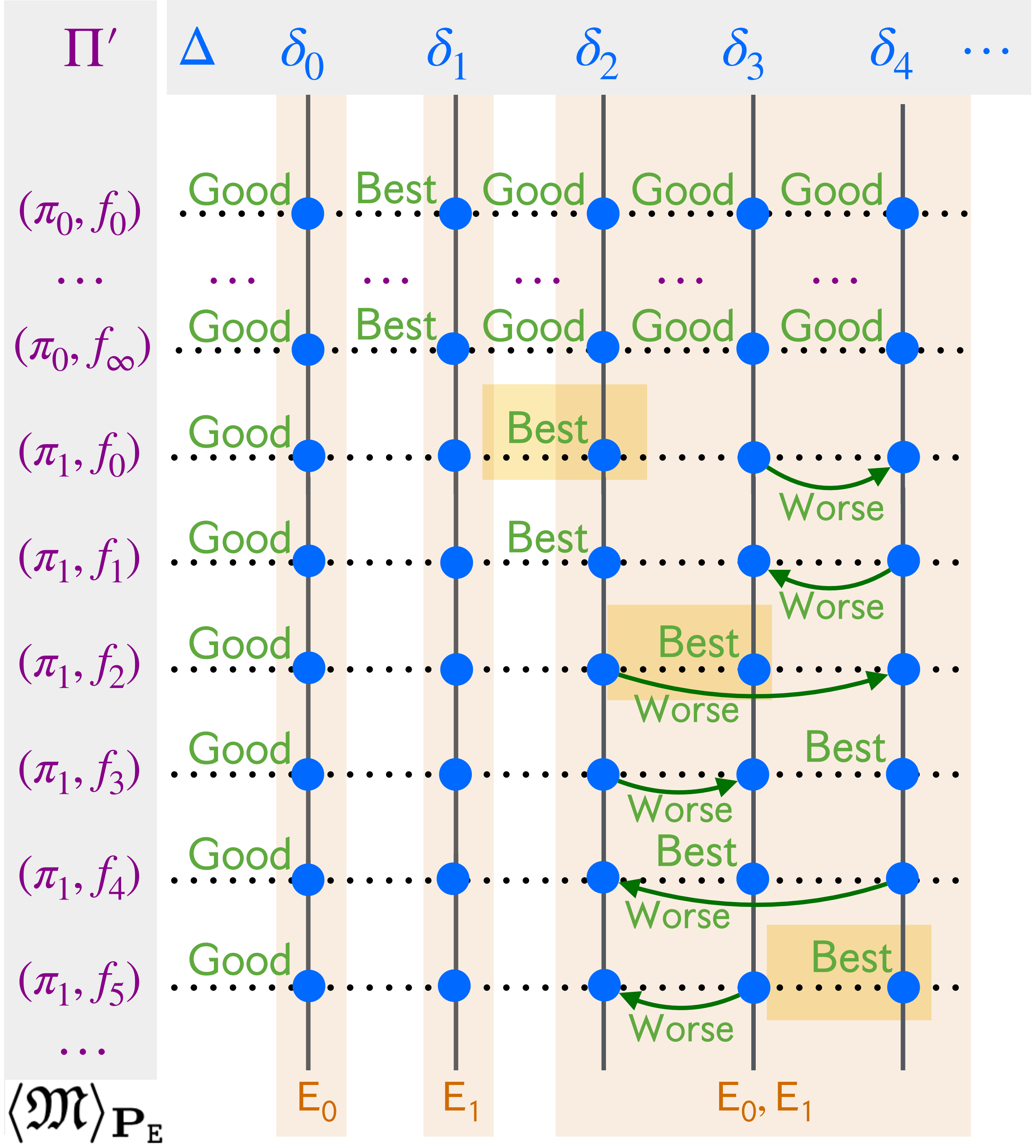

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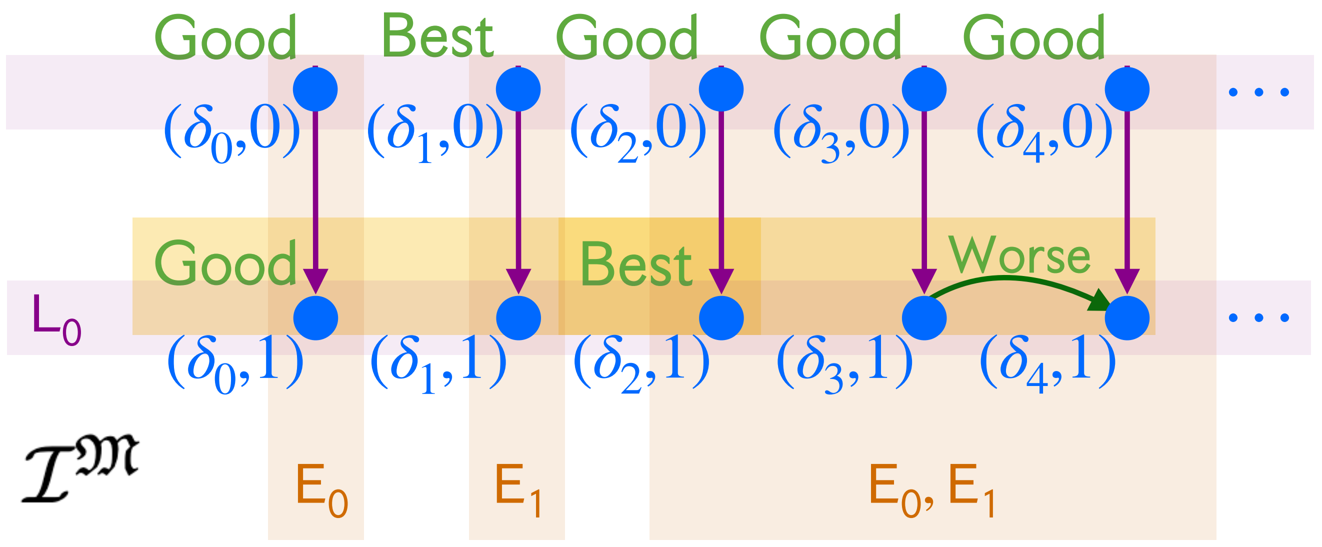

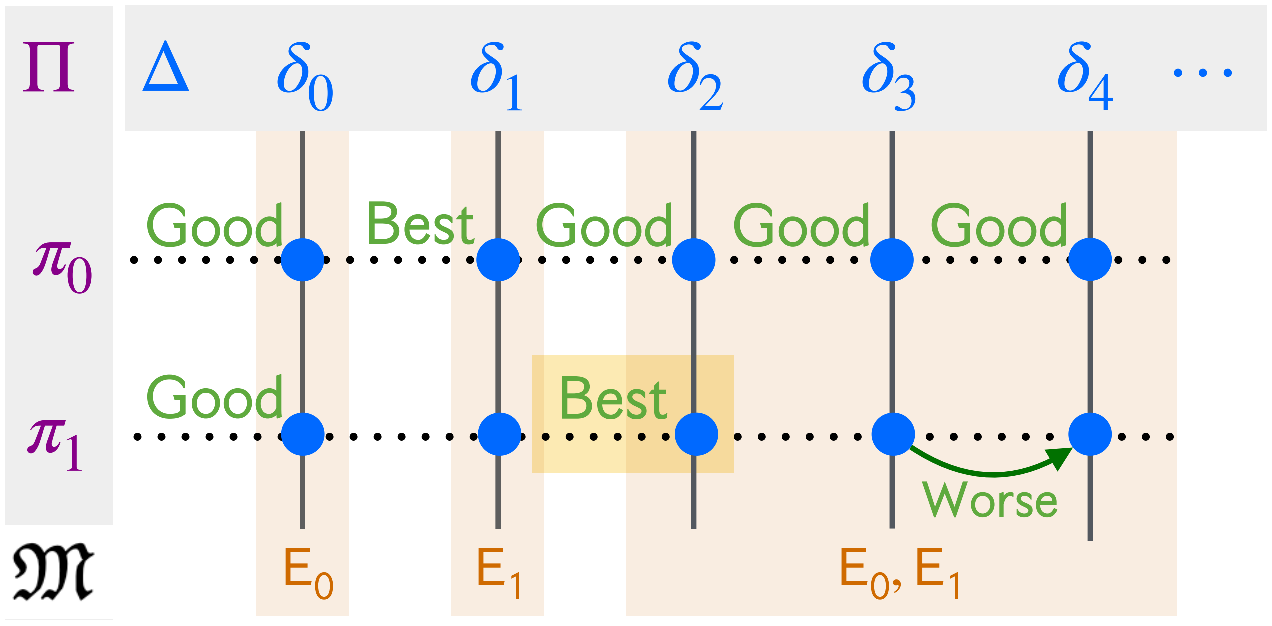

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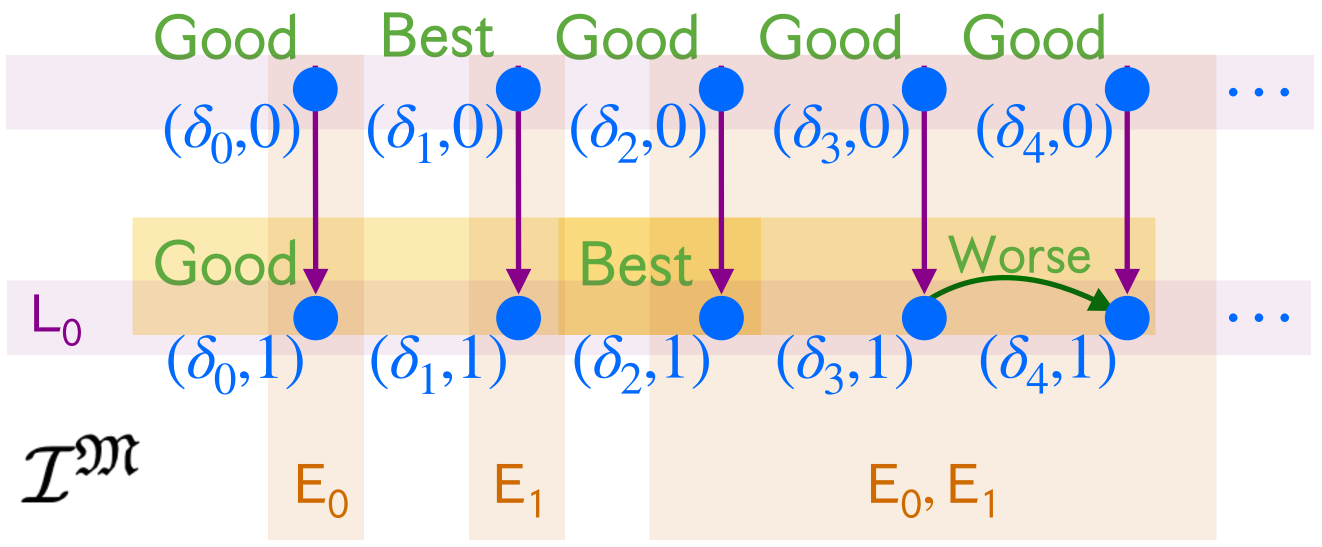

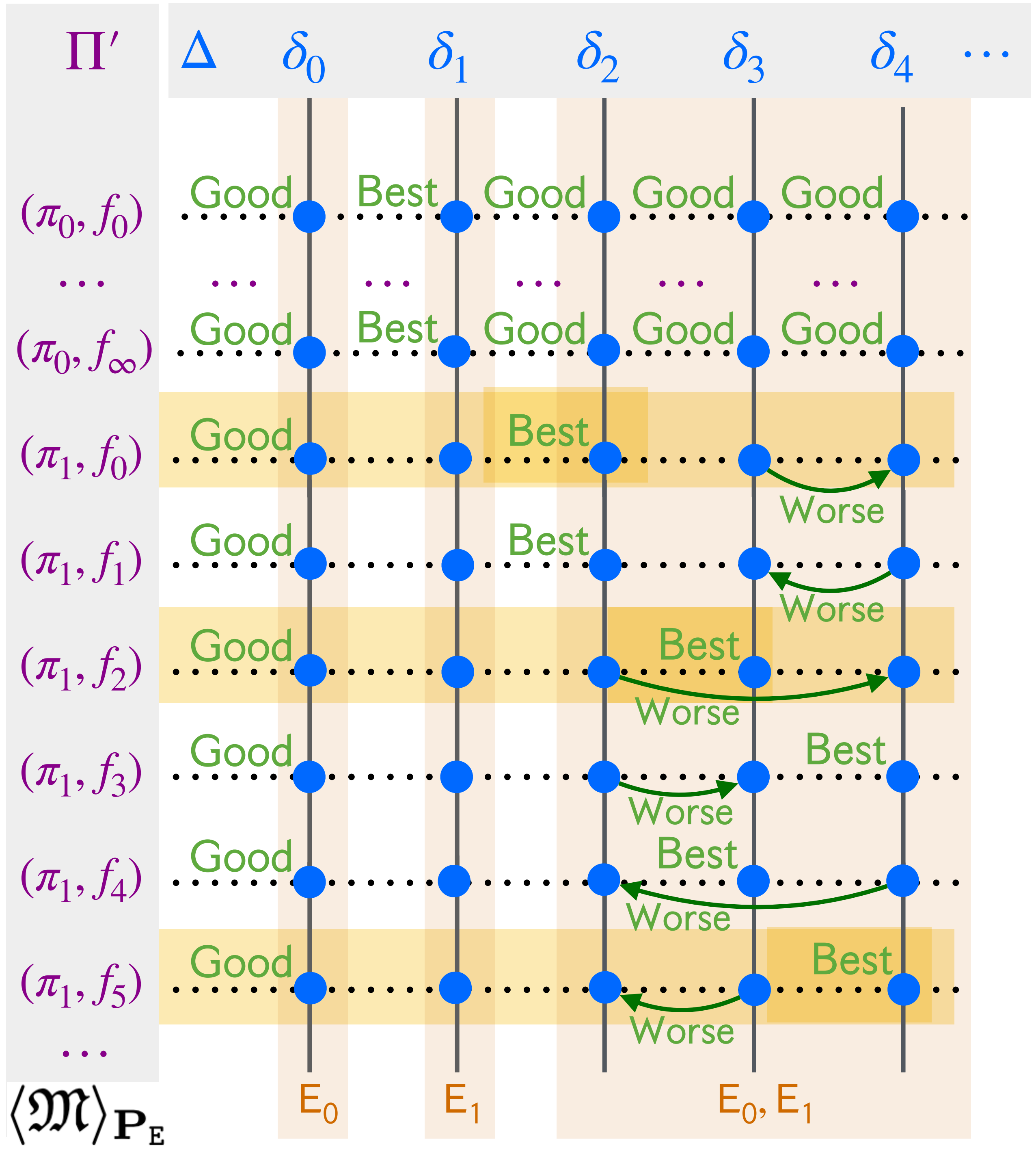

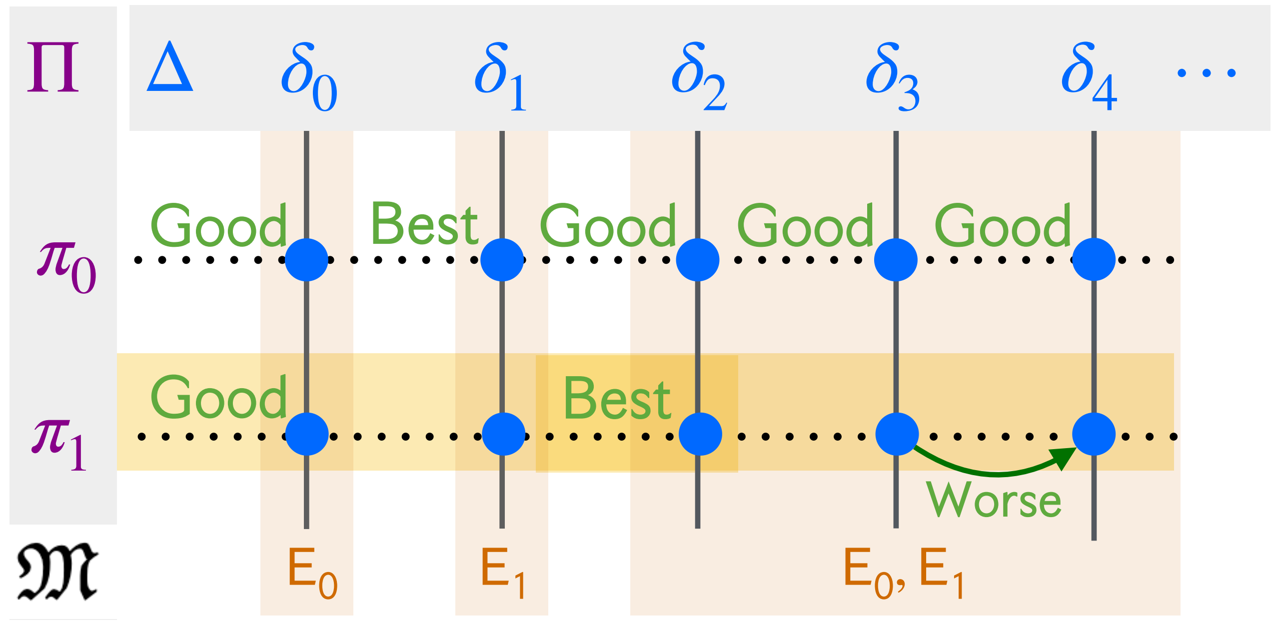

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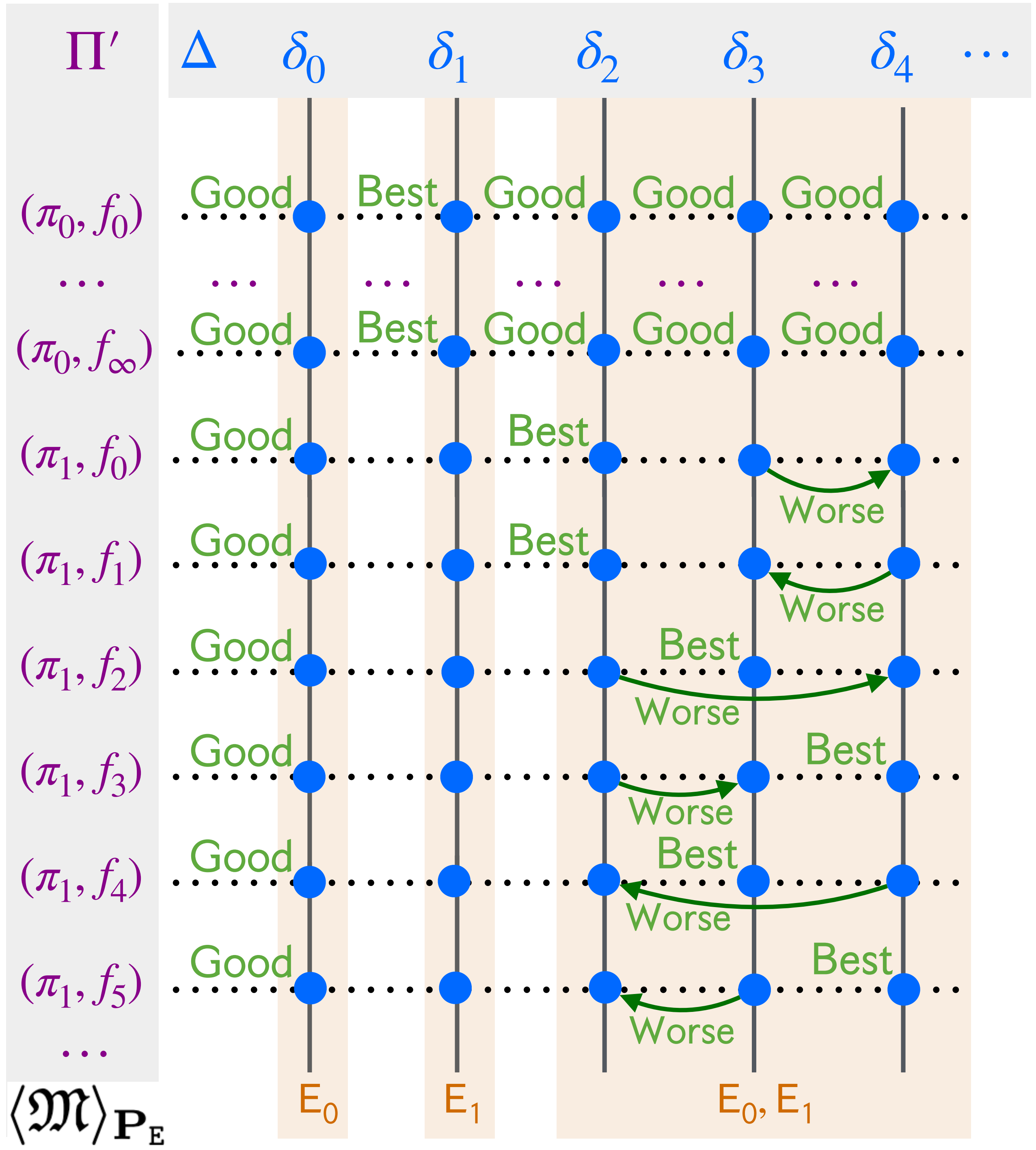

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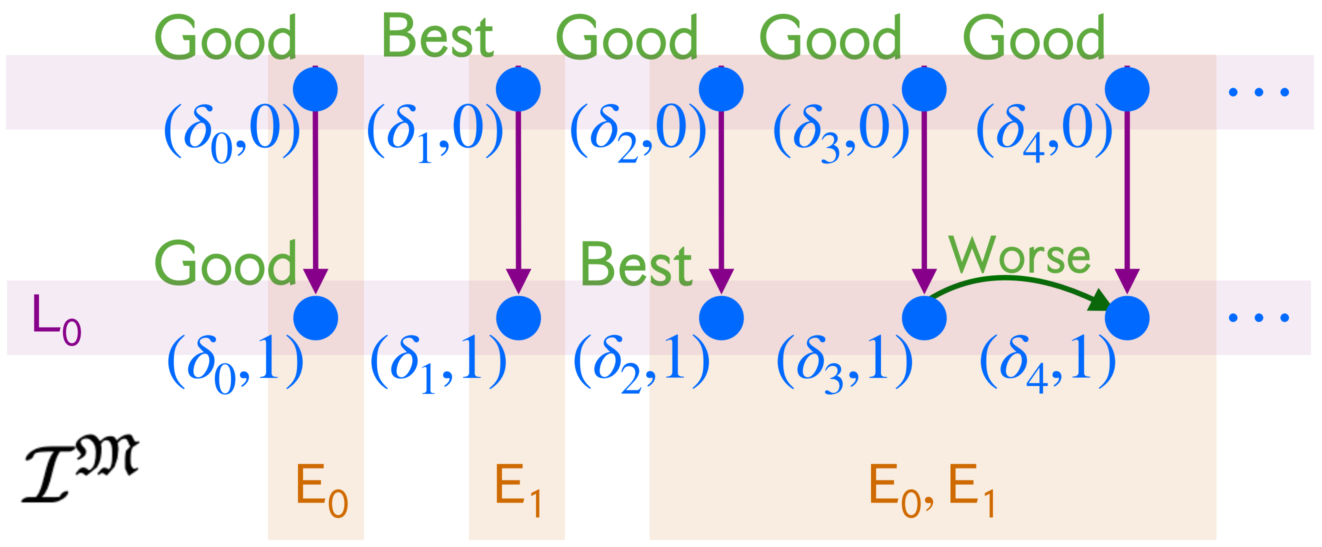

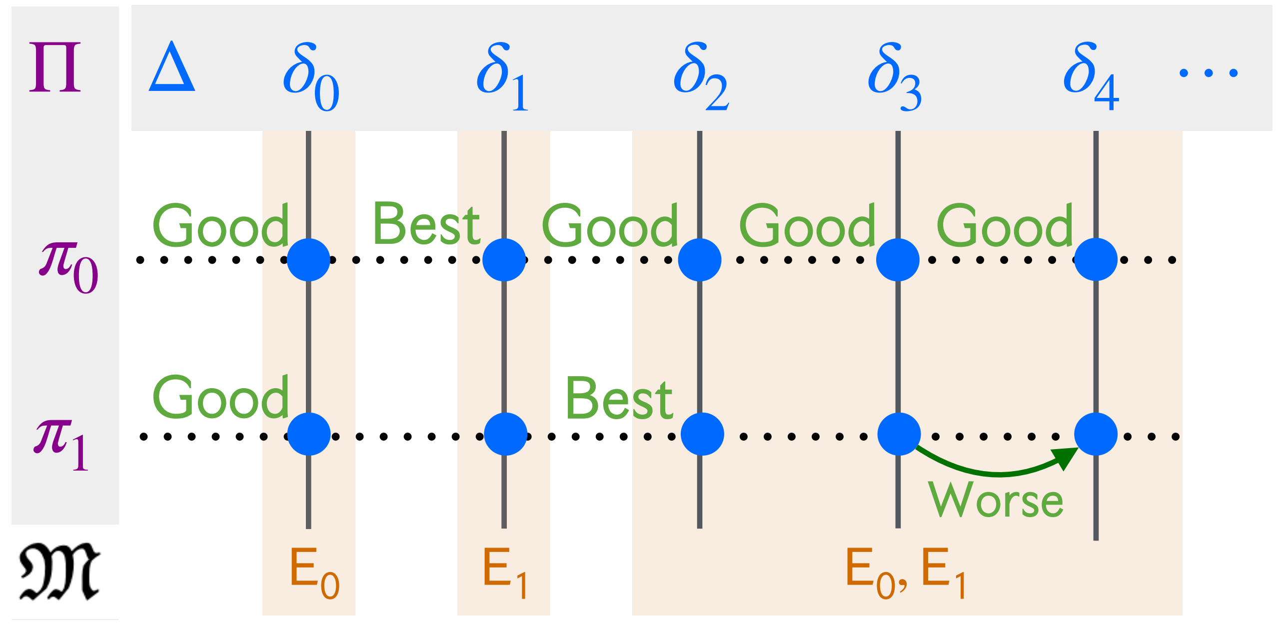

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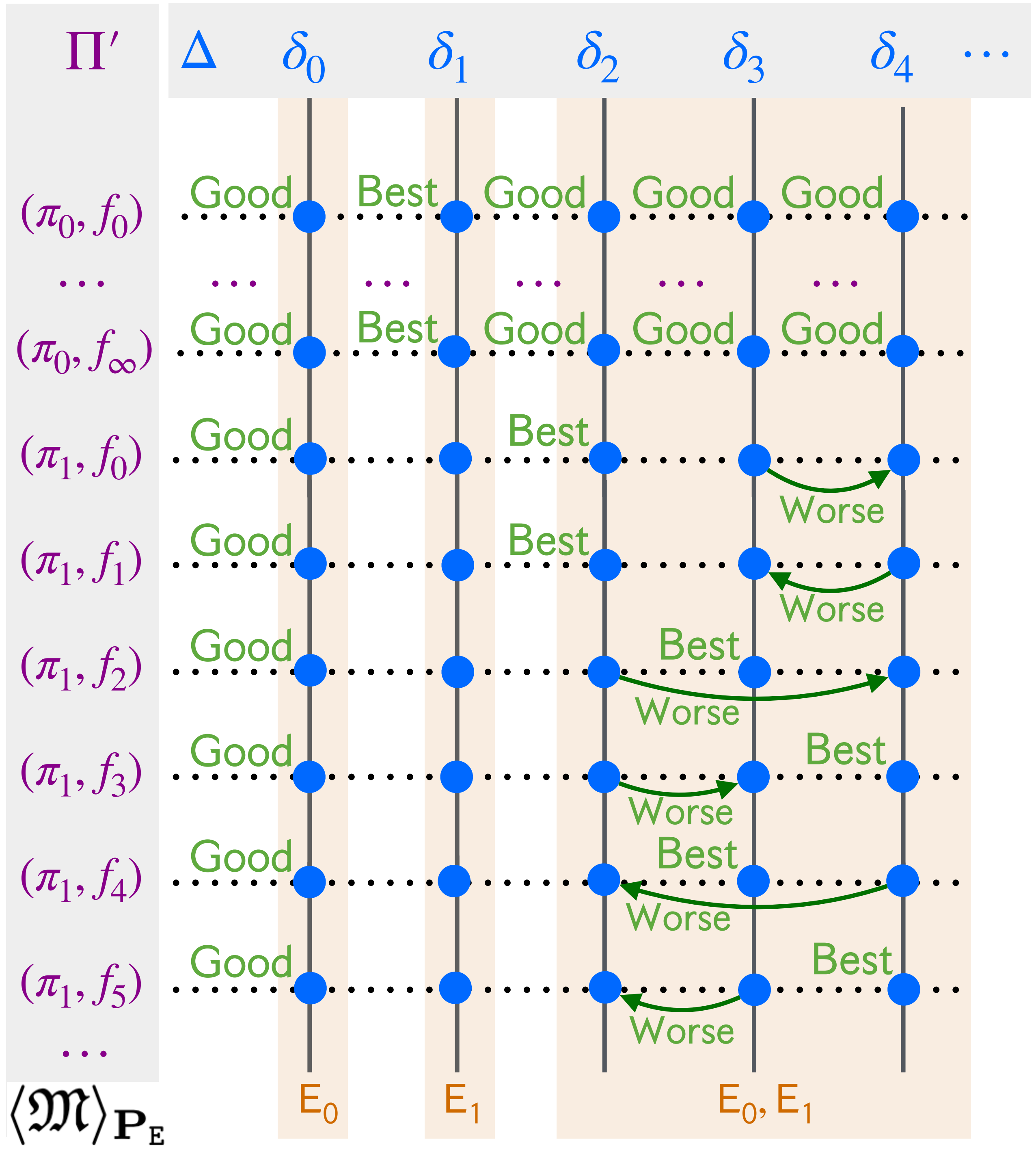

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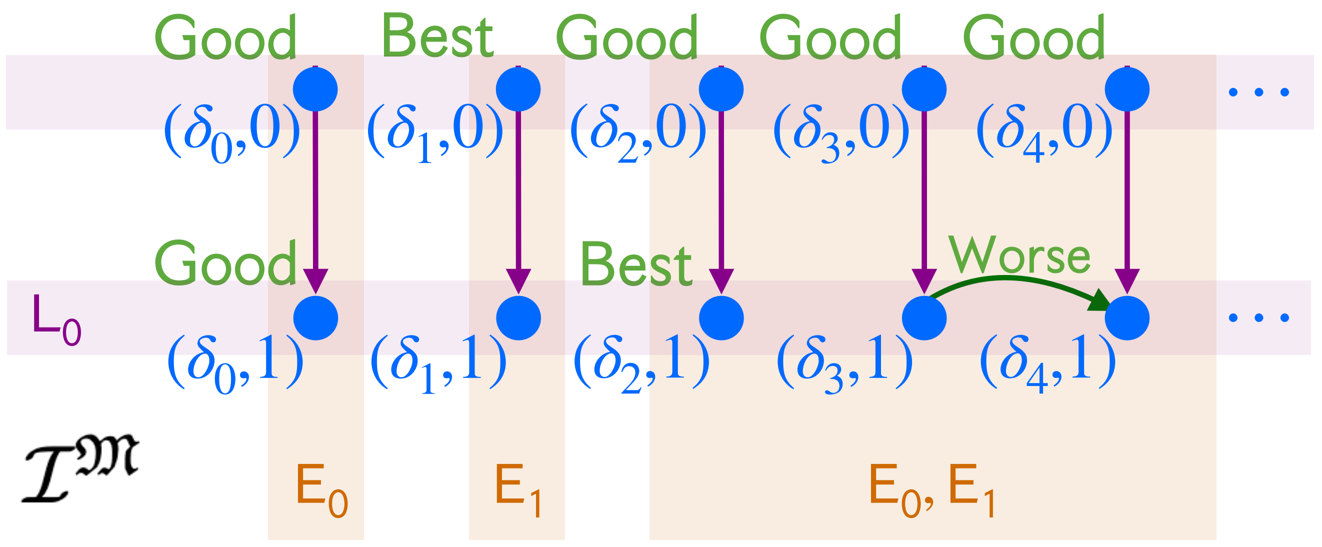

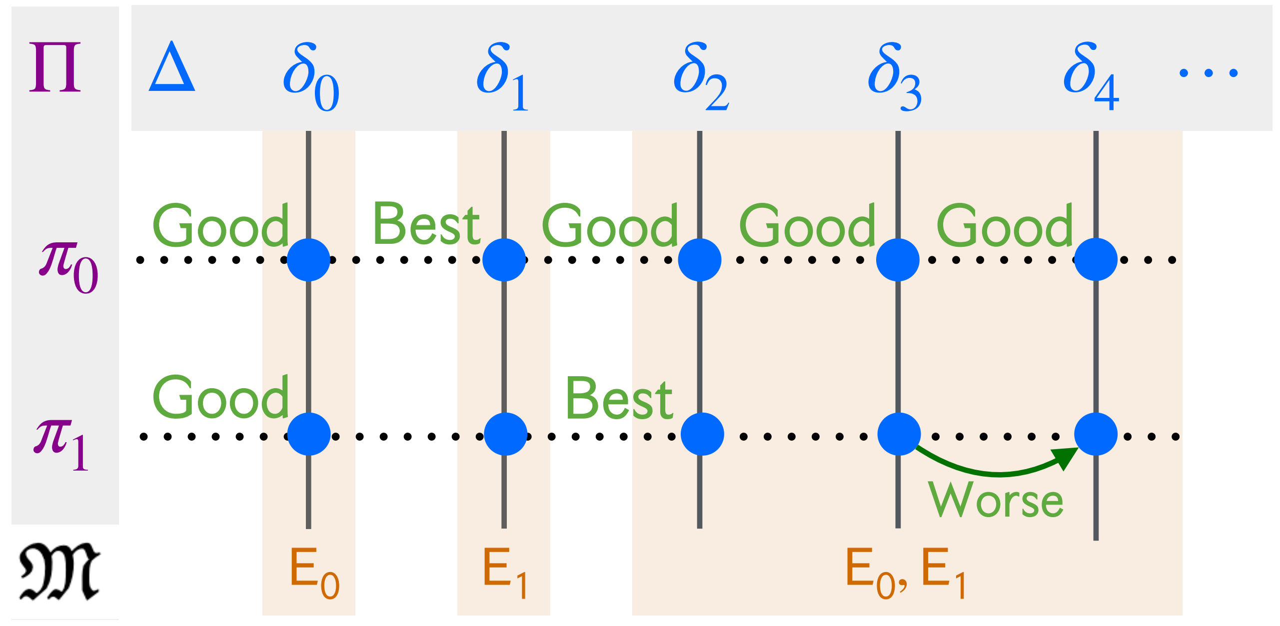

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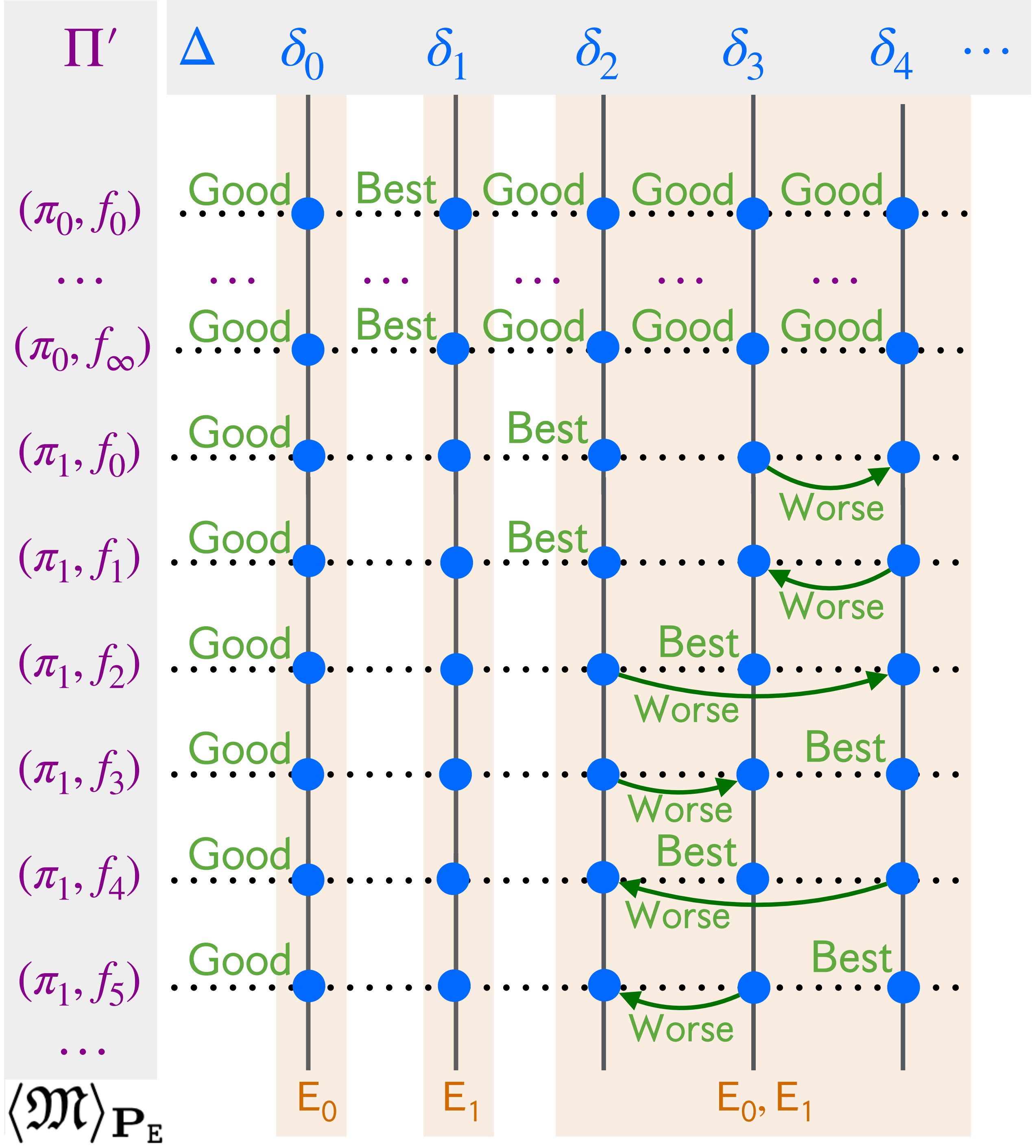

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Corollary: ***Satisfiability in monodic standpoint C^2 is*** NExpTime-***complete***