Prof. Dr. Sebastian Rudolph   M.A. Jonas Karge

Knowledge Representation and Reasoning
Winter 2025 Term
Exercises 2
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Problem 1.
Consider the following set of formulae:

1. $A \lor \neg A$

2. $(\neg A \lor B) \land A \land (\neg C \lor A \lor B) \land \neg B$

3. $(A \lor B) \land (A \lor \neg B \lor \neg C) \land (\neg A \lor B \land \neg C)$

Decide which of these formulae are Horn-formulae.

Problem 2.
Consider the following knowledge bases (1) in propositional logic and (2) in first-order logic:

Knowledge Base (1):

- Cinema $\lor$ Swimming Pool

- Cinema $\rightarrow$ Fun

- Swimming Pool $\lor$ Sleeping_In $\rightarrow$ Fun

Knowledge Base (2):

- $\forall x(\text{GradStudent}(x) \rightarrow \text{Student}(x))$

- $\forall x(\text{Student}(x) \rightarrow \text{HardWorker}(x))$

- GradStudent(Lisa)

Use Resolution to:

1. Show KB_1 $\models$ Fun.

2. Show KB_1 $\not\models$ Swimming Pool.

3. Decide whether KB_2 $\models$ HardWorker(Lisa) holds.

Problem 3.
Assume that Parent(x, y) is true iff (if and only if) x is a parent of y, Male(x) is true iff x is male and Female(x) is true iff x is female. Write down Datalog rules that specify membership conditions

for the binary familial relationship predicates Father, Mother, Son, Daughter, Sibling, Brother, Sister, Grandparent, Uncle, and Cousin. For example:

$$\text{Parent}(x, y) \wedge \text{Male}(x) \rightarrow \text{Father}(x, y)$$

Make the specification as precise as possible; e.g., in the case of Father, also add:

$$\text{Father}(x, y) \rightarrow \text{Parent}(x, y)$$
$$\text{Father}(x, y) \rightarrow \text{Male}(x)$$

Try to be more succinct by reusing the specified predicates; e.g., use Sibling in the specification of Brother and Sister. Explain the nature of the problem in any case where expressivity limitations of Datalog make exact specification impossible.

Problem 4.
Consider the Datalog knowledge base $\mathcal{K} = \langle \mathcal{R}, \mathcal{F} \rangle$ where $\mathcal{F}$ contains the following facts:

$$\mathcal{F} = \{\text{Father}(john, mary), \text{Mother}(mary, peter), \text{Father}(john, david)\}$$

and $\mathcal{R}$ contains the following rules:

$$
\begin{aligned}
\text{Parent}(x, y) \wedge \text{Parent}(y, z) &\rightarrow \text{GrandParent}(x, z) & (1)\\
\text{Parent}(x, y) \wedge \text{Parent}(x, z) &\rightarrow \text{Sibling}(y, z) & (2)\\
\text{Father}(x, y) &\rightarrow \text{Parent}(x, y) & (3)\\
\text{Mother}(x, y) &\rightarrow \text{Parent}(x, y) & (4)
\end{aligned}
$$

Do the following:

1. Using backward-chaining show that John is a grand parent of Peter.

2. Using forward-chaining show that Mary and David are siblings.