Exercise Sheet 1: Relational Algebra

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Database Theory, 2025-04-15, Summer Term 2025

Exercise 1.1. Consider a cinema database with tables of the following form (adapted from a similar example in the textbook of Abiteboul, Hull and Vianu):

Films

Title	*Director*	*Actor*
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
…	…	…
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
…	…	…
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues

Cinema	*Address*	*Phone*
UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbräcker Str. 55	8032185
CinemaxX	Hüblerstr. 8	3158910
…	…	…

Program

Cinema	*Title*	*Time*
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Express the following queries in relational algebra:

1. Who is the director of "The Imitation Game"?
2. Which cinemas feature "The Imitation Game"?
3. What are the address and phone number of "Schauburg"?
4. Is a film directed by "Smith" playing in some cinema?
5. List the pairs of persons such that the first directed the second in a film and vice versa.
6. List the names of directors who have acted in a film they directed.
7. Always return $\{\mathit{Title} \mapsto \text{"Apocalypse Now"}, \mathit{Director} \mapsto \text{"Coppola"}\}$ as the answer.
8. Find the actors cast in at least one film by "Smith."
9. Find the actors cast in every film by "Smith."
10. Find the actors cast only in films by "Smith."
11. Find all pairs of actors who act together in at least one film.
12. Find all pairs of actors cast in exactly the same films.
13. Find the directors such that every actor is cast in one of his or her films.

Exercise 1.2. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.

Now for a table R, what are the results of the following expressions?

$$R \bowtie R \qquad\qquad R \bowtie \emptyset \qquad\qquad R \bowtie \{\varepsilon\}$$

Exercise 1.3. Express the following operations using other operations presented in the lecture:

- Intersection $R \cap S$.
- Cross product (Cartesian product) $R \times S$.
- Selection $\sigma_{n=a}(R)$ with a a constant.
- Arbitrary constant tables in queries (the constants in the lecture only had one single column and one single row; generalise this to any number of constants and rows)

Exercise 1.4. Consider the following identities and decide for each whether it is true or false. If true, prove your answer using the definitions from the lecture; if false, give a counterexample.

1. $R \bowtie S = S \bowtie R$
2. $R \bowtie (S \bowtie T) = (R \bowtie S) \bowtie T$.
3. $\pi_X(R \circ S) = \pi_X(R) \circ \pi_X(S)$ for all $\circ \in \{\cup, \cap, -, \bowtie\}$.
4. $\sigma_{n=m}(R \circ S) = \sigma_{n=m}(R) \circ \sigma_{n=m}(S)$ for all $\circ \in \{\cup, \cap, -\}$.
5. $\sigma_{n=m}(R \bowtie S) = \sigma_{n=m}(R) \bowtie S$, for n and m attributes of R only.

Why are these identities of interest?

Exercise 1.5. Let $R^{\mathcal{I}}$ and $S^{\mathcal{I}}$ be tables of schema $R[U]$ and $S[V]$, respectively. The *division* of $R^{\mathcal{I}}$ by $S^{\mathcal{I}}$, written as $(R^{\mathcal{I}} \div S^{\mathcal{I}})$, is defined to be the maximal table over the attributes $U \setminus V$ that satisfies $(R^{\mathcal{I}} \div S^{\mathcal{I}}) \bowtie S^{\mathcal{I}} \subseteq R^{\mathcal{I}}$. Note that the joined tables here do not have any attributes in common, so the natural join works as a cross product.

Consider the following table and use the division operator to express a query for the cities that have been visited by all people.

Visited

Person	**City**
Tomas	Berlin
Markus	Santiago
Markus	Berlin
Fred	New York
Fred	Berlin

Express division using the relational algebra operations introduced in the lecture.

Exercise 1.6. Suggest how to write the relational algebra operations for using the unnamed perspective. What changes?

Exercise 1.7. We have seen above that $\cap$ can be expressed in terms of the other standard operators of relational algebra. Indeed, the set of operations $\{\sigma, \pi, \cup, -, \bowtie, \delta\}$ can express all queries of relational algebra: it is complete. Try to show that it is not possible to reduce this set any further:

For each operator, first try to find an example query that cannot be expressed when using only the other operators. Then try to find a general argument that shows that the operator is really needed.