Exercise 2.1. Transform the following concepts into negation normal form:
(a) \(\neg (A \cap \forall r. B)\)
(b) \(\neg \forall r. \exists s. (\neg B \sqcup \exists r. A)\)
(c) \(\neg ((\neg A \cap \exists r. \top) \sqcup \exists s. (A \sqcup \neg B))\)

Exercise 2.2. Apply the tableau algorithm in order to check if the axiom \(A \sqsubseteq B\) is a logical consequence of the TBox \(\{\neg C \sqsubseteq B, A \cap C \sqsubseteq \bot\}\).

Exercise 2.3. Apply the tableau algorithm in order to check satisfiability of the concept \(A \cap \forall r. B\) w.r.t. the TBox \(\{A \sqsubseteq \exists r. A, B \sqsubseteq \exists r. C, C \sqsubseteq \forall r. \forall r. B\}\).

Exercise 2.4. Markus wants to apply the tableau algorithm for checking the satisfiability of the concept \(B \cap \exists r. A\) w.r.t. the TBox \(\{A \sqsubseteq \exists r. A, A \cap B \sqsubseteq \exists r. C, C \sqsubseteq \forall r. \forall r. B\}\). He arrives at the situation depicted below and concludes that no further rules are applicable, since \(v_2\) is blocked by \(v_1\). What is Markus’ error? Continue the algorithm until its termination. (You don’t have to illustrate all intermediate steps, just provide the final state.)

\[
\begin{align*}
\text{v}_0 & \quad \text{L}(\text{v}_0) = \{B \cap \exists r. A, B, \exists r. A, C, \neg A, \leq 1 r\} \\
& \quad r^- \quad \text{v}_1 \\
\text{v}_1 & \quad \text{L}(\text{v}_1) = \{A, C, \exists r. A, \exists r. B, \leq 1 r\} \\
& \quad r^- \quad \text{v}_2 \\
\text{v}_2 & \quad \text{L}(\text{v}_2) = \{A, C, \exists r. A, \exists r. B, \leq 1 r\}.
\end{align*}
\]

Exercise 2.5. Extend the \(\leq 1\) rule in a way that also qualified functionality axioms of the form \(\top \sqsubseteq \leq 1 r. A\) can be treated correctly, where \(A\) is an atomic concept. Can you also treat arbitrary axioms of the form \(C \sqsubseteq \leq 1 r. D\)?