Deduction Systems

Tutorial 2

Sebastian Rudolph, Lukas Schweizer

SS 2016

Exercise 2.1. Transform the following concepts into negation normal form:

(a) $\neg(A \sqcap \forall r.B)$

(b) $\neg\forall r.\exists s.(\neg B \sqcup \exists r.A)$

(c) $\neg((\neg A \sqcap \exists r.\top) \sqcup \geqslant 3\, s.(A \sqcup \neg B))$

Exercise 2.2. Apply the tableau algorithm in order to check if the axiom $A \sqsubseteq B$ is a logical consequence of the TBox $\{\neg C \sqsubseteq B, A \sqcap C \sqsubseteq \bot\}$.

Exercise 2.3. Aply the tableau algorithm in order to check satisfiability of the concept $A \sqcap \forall r.B$ w.r.t. the TBox $\{A \sqsubseteq \exists r.A, B \sqsubseteq \exists r^-.C, C \sqsubseteq \forall r.\forall r.B\}$.

Exercise 2.4. Markus wants to apply the tableau algorithm for checking the satisfiability of the concept $B \sqcap \exists r^-.A$ w.r.t. the TBox $\{A \sqsubseteq \exists r^-.A \sqcap \exists r.B, \top \sqsubseteq \leqslant 1\, r\}$. He arrives at the situation depicted below and concludes that no further rules are applicable, since v_2 is blocked by v_1. What is Markus' error? Continue the algorithm until its termination. (You don't have to illustrate all intermediate steps, just provide the final state.)

$$\begin{tikzcd} v_0 \arrow[d, "r^-"'] \\ v_1 \arrow[d, "r^-"'] \\ v_2 \end{tikzcd}$$

$$\begin{aligned} L(v_0) &= \{B \sqcap \exists r^-.A, B, \exists r^-.A, C_{\mathcal{T}}, \neg A, \leqslant 1\, r\} \\ L(v_1) &= \{A, C_{\mathcal{T}}, \exists r^-.A, \exists r.B, \leqslant 1\, r\} \\ L(v_2) &= \{A, C_{\mathcal{T}}, \exists r^-.A, \exists r.B, \leqslant 1\, r\}. \end{aligned}$$

Exercise 2.5. Extend the $\leqslant 1$ rule in a way that also qualified functionality axioms of the form $\top \sqsubseteq \leqslant 1\, r.A$ can be treated correctly, where A is an atomic concept.
Can you also treat arbitrary axioms of the form $C \sqsubseteq \leqslant 1\, r.D$?