Preserving Constraints with the Stable Chase

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Full paper: `https://iccl.inf.tu-dresden.de/web/Incidentals/en`

A bit of background

Motivation

- ontological modelling on knowledge graphs using *tuple-generating dependencies* (*TGDs*): $\varphi(\boldsymbol{x}, \boldsymbol{y}) \to \exists \boldsymbol{z}.\ \psi(\boldsymbol{x}, \boldsymbol{z})$
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- $\rightsquigarrow$ "*incidental functional dependencies*"
- Query rewriting becomes easier if (some) incidentals are known.

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Now

- What about *incidental TGDs*?

Incidental TGDs

Example

Consider $\Sigma = \{\exists x, y. R(x, y), \quad R(x, y) \to \exists z. R(y, z)\}$. Then

$$\bullet \xrightarrow{R} \bullet \xrightarrow{R} \bullet \xrightarrow{R} \bullet \xrightarrow{R} \dots \tag{1}$$

is the unique universal model.

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Definition

A TGD ρ is *incidental* for a set Σ of TGDs if $\mathsf{BCQ}(\Sigma) = \mathsf{BCQ}(\Sigma \cup \{\rho\})$.
$\mathsf{ICDT}(\Sigma)$ is the set of all TGDs incidental for Σ.

The finite case

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INCIDENTAL: Given Σ set of TGDs and ρ TGD,
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Then $\rho \in \mathsf{ICDT}(\Sigma)$ iff $\mathsf{core}\,\mathcal{I} \models \rho$.

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- Use the *core chase* to compute $\mathsf{core}\,\mathcal{I}$
- check $\mathsf{core}\,\mathcal{I} \models \rho$
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- $\rightsquigarrow$ INCIDENTAL is decidable if Σ has a finite universal model.
- But what can we say in general?

Deciding INCIDENTAL

Theorem

INCIDENTAL *is* Π^0_2*-complete, and thus neither in* RE *nor in* coRE.

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- Idea: if $\rho \notin \mathsf{ICDT}(\Sigma)$, there is a BCQ q with $\Sigma \not\models q$ and $\Sigma \cup \{\rho\} \models q$.
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- Can we do better?
- Unfortunately, no.

Undecidability of Incidental

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There is a class $\mathcal{C}$ of sets of TGDs and a full dependency ρ such that

- *BCQ entailment for $\Sigma \in \mathcal{C}$ is decidable,*
- *$\Sigma \cup \{\rho\} \in \mathcal{C}$ for any $\Sigma \in \mathcal{C}$, and*
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What about incidentals in general?

- Recall: in the finite case, $\rho \in \mathsf{ICDT}(\Sigma)$ iff core $\mathcal{I} \models \rho$
- Is there some universal model that entails all incidental TGDs?
- The core looks like a promising candidate.

Cores of infinite instances

Different definitions of *core* agree on finite instances, but differ in general.

Definition

An instance $\mathcal{I}$ is a *core* if every endomorphism $h : \mathcal{I} \to \mathcal{I}$ is an embedding.
A core $\mathcal{I}$ is a core of $\mathcal{J}$ if $\mathcal{I} = \mathcal{J}|_{h(\mathcal{J})}$ for an endomorphism $h : \mathcal{J} \to \mathcal{J}$.

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Unfortunately, there are sets Σ of TGDs with universal models $\mathcal{I}$ such that

- $\mathcal{I}$ doesn't have a core,
- $\mathcal{I}$ has two non-isomorphic cores, or
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- Can we generalise the core chase to this setting?

The stable chase by example

$$\Sigma = \{\exists x, y.\ R(x, y) \land S(x, y), \qquad R(y, z) \to \exists x.\ R(x, y),$$
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$\mathcal{Q}_1$

n_0 n_1

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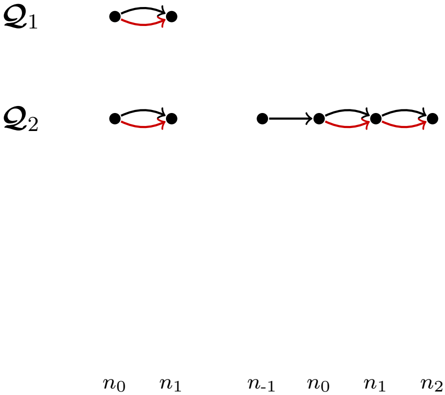

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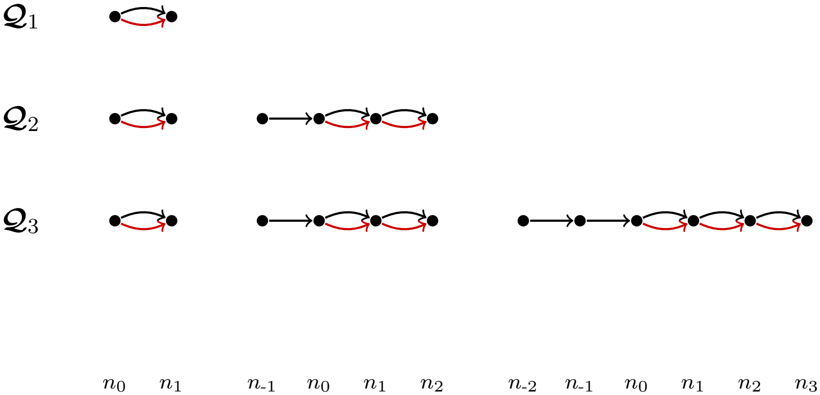

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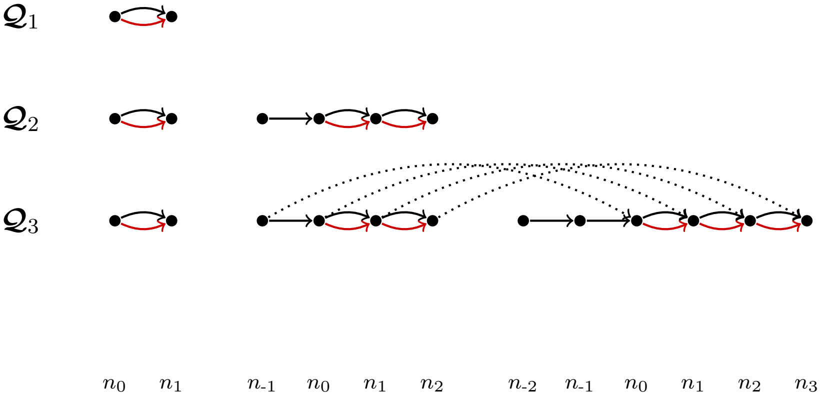

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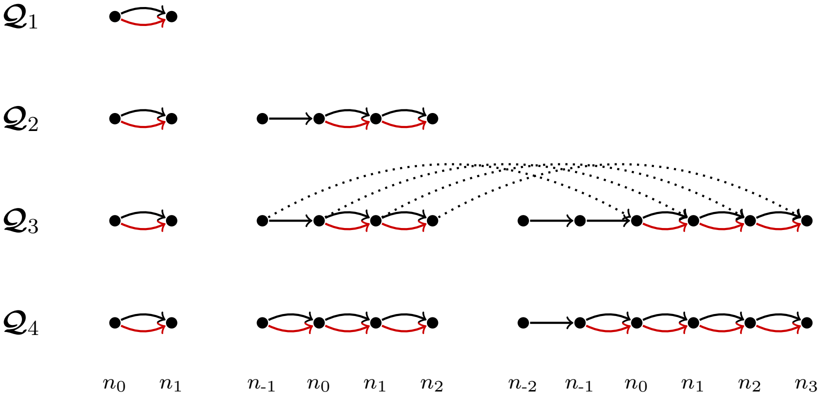

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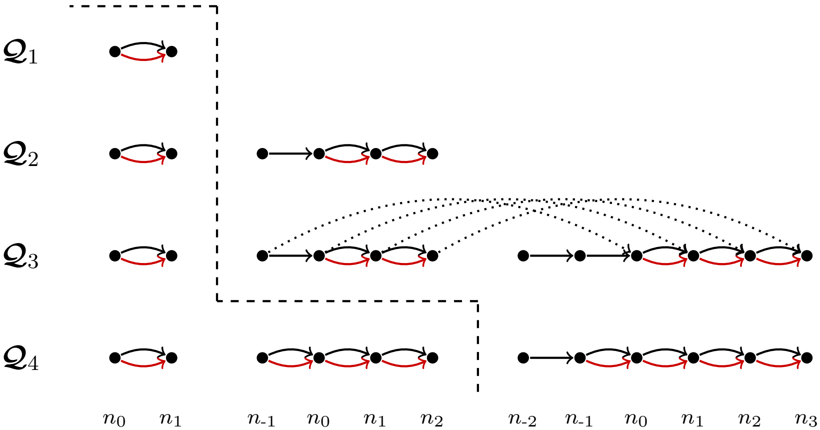

The stable chase

The idea

- Consider a sequence of initial segments of chase sequences (prefixes):
- apply TGDs to last instance and obtain a longer prefix, or
- *rewrite* prefix according to some non-embedding endomorphism between instances $\rightsquigarrow$ enforce an embedding
- instances can only be rewritten finitely often and stabilise at some point
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Theorem

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- If Σ has a finite universal model $\mathcal{J}$, then core $\mathcal{J} = \mathcal{I}$.
- Beware: Stability of an instance is undecidable.

Conclusion & Outlook

Results

- Incidentality is Π_2^0-complete, and still not in RE even when BCQ entailment is decidable.
- Each set Σ of TGDs has a BCQ-equivalent core $\mathcal{I}$ with $\mathcal{I} \models \mathsf{ICDT}(\Sigma)$.
- The stable chase generalises the core chase to classes that don't admit finite universal models.
- The stable chase yields a core that characterises the full incidental dependencies.

Future work

- Further generalise the stable chase to characterise all incidentals.
- Investigate complexity of INCIDENTAL for decidable classes.
- Design (incomplete) algorithms that compute incidentals.