
Exercise 1

Show that monadic fragment of FO has decidable sat (i.e. a fragment of FO where you can only use relational symbols of arity 1, no constants, no relations of higher arity, no functions). Hint: Translate to FO^1.

Exercise 2

Prove that the model-checking problem for FO^k for any fixed k is in PTime.

Exercise 3

Show that $\exists^*\forall^*\mathsf{FO}$ fragment of FO is decidable.

Exercise 4

In the lecture we presented a proof that C^1 is in NP but without constants. Provide a polynomial time translation from the satisfiability of C^1 with constants to C^1 without them.

Exercise 5

Consider an extension of C^1 in which we can express statements of the form $|P| \geq |Q|$, meaning that in every model of such formula the number of elements satisfying P is at least the number of elements satisfying Q. Prove that such an extension of C^1 is still in NP.

Exercise 6

We will soon see that FO^2 has the finite model property. Prove that C^2 (the two-variable fragment with counting quantifiers) doesn't have FMP. Hint: enforce infinite trees.

Exercise 7

We will soon see that FO^2 has the finite model property. Employ this fact to show that transitivity is not expressible in FO^2.