Exercise 12.1. Let $\mathcal{L}$ be a fragment of first-order logic for which finite model entailment and arbitrary model entailment coincide, i.e., for every $\mathcal{L}$-theory $\mathcal{T}$ and every $\mathcal{L}$-formula $\varphi$, we find that $\varphi$ is true in all models of $\mathcal{T}$ if and only if $\varphi$ is true in all finite models of $\mathcal{T}$.

(a) Give an example for a proper fragment of first-order logic with this property.

(b) Give an example for a proper fragment of first-order logic without this property.

(c) Show that entailment is decidable in any fragment with this property.

Exercise 12.2. Consider the following set of tgds $\Sigma$:

\[
\begin{align*}
A(x) & \rightarrow \exists y. R(x, y) \land B(y) \\
B(x) & \rightarrow \exists y. S(x, y) \land A(y) \\
R(x, y) & \rightarrow S(y, x) \\
S(x, y) & \rightarrow R(y, x)
\end{align*}
\]

Does the oblivious chase universally terminate for $\Sigma$? What about the restricted chase?

Exercise 12.3. Is the following set of tgds weakly acyclic?

\[
\begin{align*}
B(x) & \rightarrow \exists y. S(x, y) \land A(x) \\
A(x) \land C(x) & \rightarrow \exists y. R(x, y) \land B(y)
\end{align*}
\]

Does the oblivious chase universally terminate over this set of tgds?

Exercise 12.4. Termination of the oblivious (resp. restricted) chase over a set of tgds $\Sigma$ implies the existence of a finite universal model for $\Sigma$. Is the converse true? That is, does the existence of a finite universal model for $\Sigma$ imply termination of the oblivious (resp. restricted) chase?

Exercise 12.5. A term is cyclic if it is of the form $f(t_1, \ldots, t_n)$ and, for some $i \in \{1, \ldots, n\}$, the function symbol $f$ syntactically occurs in $t_i$. A set $\Sigma$ of tgds that does not contain any constants is model-faithful acyclic (MFA) iff no cyclic term occurs in the skolem chase of $\Sigma \cup I_*$ (with $I_*$ the critical instance). Show the following:

- Checking MFA membership is decidable.
- If $\Sigma$ is MFA, then the skolem chase universally terminates for $\Sigma$. 