

Exercise 1: Relational Algebra

Database Theory

2025-04-15

Lukas Gerlach, Maximilian Marx, Markus Krötzsch

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues

Cinema	Address	Phone
UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbrücker Str. 55	8032185
CinemaxX	Hübelerstr. 8	3158910
...

Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

1. Who is the director of "The Imitation Game"?

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The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
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1. Who is the director of "The Imitation Game"?

$$\pi_{Director}(\sigma_{Title="The Imitation Game"}(Films))$$

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Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
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The Internet's Own Boy	Knappenberger	Swartz
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1. Who is the director of "The Imitation Game"?

$$\pi_{Director}(\sigma_{Title="The Imitation Game"}(Films))$$

2. Which cinemas feature "The Imitation Game"?

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Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
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The Internet's Own Boy	Knappenberger	Swartz
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1. Who is the director of "The Imitation Game"?

$$\pi_{Director}(\sigma_{Title="The Imitation Game"}(Films))$$

2. Which cinemas feature "The Imitation Game"?

$$\pi_{Cinema}(\sigma_{Title="The Imitation Game"}(Program))$$

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Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
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3. What are the address and phone number of "Schauburg"?

Exercise 1

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Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
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The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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3. What are the address and phone number of "Schauburg"?

$$\pi_{Address, Phone}(\sigma_{Cinema="Schauburg"}(Venues))$$

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Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
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3. What are the address and phone number of "Schauburg"?

$$\pi_{Address,Phone}(\sigma_{Cinema="Schauburg"}(Venues))$$

4. *Boolean query*: Is a film directed by "Smith" playing in some cinema?

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Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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3. What are the address and phone number of "Schauburg"?

$$\pi_{Address,Phone}(\sigma_{Cinema="Schauburg"}(Venues))$$

4. Boolean query: Is a film directed by "Smith" playing in some cinema?

$$\pi_{\emptyset}(\sigma_{Director="Smith"}(Films) \bowtie Program)$$

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
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The Internet's Own Boy	Knappenberger	Swartz
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5. List the pairs of persons such that the first directed the second in a film, and vice versa.

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The Imitation Game	Tyldum	Knightley
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The Internet's Own Boy	Knappenberger	Swartz
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5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\pi_{Director,D}(\sigma_{Director=A}(\sigma_{Actor=D}(\delta_{Title,Director,Actor \rightarrow T,D,A}(Films) \bowtie Films)))$$

Exercise 1

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\pi_{Director,D}(\sigma_{Director=A}(\sigma_{Actor=D}(\delta_{Title,Director,Actor \rightarrow T,D,A}(Films) \bowtie Films)))$$

6. List the names of directors who have acted in a film they directed.

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Title	Director	Actor	Cinema	Address	Phone
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The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\pi_{Director,D}(\sigma_{Director=A}(\sigma_{Actor=D}(\delta_{Title,Director,Actor \rightarrow T,D,A}(Films) \bowtie Films)))$$

6. List the names of directors who have acted in a film they directed.

$$\pi_{Director}(\sigma_{Actor=Director}(Films))$$

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7. Always return {Title \mapsto "Apocalypse Now", Director \mapsto "Coppola"} as the answer.

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
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7. Always return {Title \mapsto "Apocalypse Now", Director \mapsto "Coppola"} as the answer.

$\{\{Title \mapsto "Apocalypse Now"\}\} \bowtie \{\{Director \mapsto "Coppola"\}\}$

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Films		
Title	Director	Actor
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The Imitation Game	Tyldum	Knightley
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7. Always return $\{\text{Title} \mapsto \text{"Apocalypse Now"}, \text{Director} \mapsto \text{"Coppola"}\}$ as the answer.

$\{\{\text{Title} \mapsto \text{"Apocalypse Now"}\}\} \bowtie \{\{\text{Director} \mapsto \text{"Coppola"}\}\}$

8. Find the actors cast in at least one film by "Smith".

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Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
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The Internet's Own Boy	Knappenberger	Swartz
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7. Always return $\{\text{Title} \mapsto \text{"Apocalypse Now"}, \text{Director} \mapsto \text{"Coppola"}\}$ as the answer.

$$\{\{\text{Title} \mapsto \text{"Apocalypse Now"}\}\} \bowtie \{\{\text{Director} \mapsto \text{"Coppola"}\}\}$$

8. Find the actors cast in at least one film by "Smith".

$$\pi_{\text{Actor}}(\sigma_{\text{Director}=\text{"Smith"}}(\text{Films}))$$

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9.1 Find the actors for which there is a movie directed by "Smith" that they are not cast in.

Exercise 1

Films		
Title	Director	Actor
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The Imitation Game	Tyldum	Knightley
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The Internet's Own Boy	Knappenberger	Swartz
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9.1 Find the actors for which there is a movie directed by "Smith" that they are not cast in.

$$q = \pi_{Actor}([\pi_{Actor}(Films) \bowtie \pi_{Title}(\sigma_{Director="Smith"}(Films))] - \pi_{Actor, Title}(\sigma_{Director="Smith"}(Films)))$$

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Films		
Title	Director	Actor
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The Internet's Own Boy	Knappenberger	Swartz
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9.1 Find the actors for which there is a movie directed by “Smith” that they are not cast in.

$$q = \pi_{Actor}[(\pi_{Actor}(Films) \bowtie \pi_{Title}(\sigma_{Director="Smith"}(Films))) - \pi_{Actor, Title}(\sigma_{Director="Smith"}(Films))]$$

9 Find the actors cast in every film by “Smith.”

Exercise 1

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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9.1 Find the actors for which there is a movie directed by "Smith" that they are not cast in.

$$q = \pi_{Actor}[(\pi_{Actor}(Films) \bowtie \pi_{Title}(\sigma_{Director="Smith"}(Films))) - \pi_{Actor, Title}(\sigma_{Director="Smith"}(Films))]$$

9 Find the actors cast in every film by "Smith."

$$\pi_{Actor}(Films) - q$$

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
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10 Find the actors cast only in films by "Smith."

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The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
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10 Find the actors cast only in films by "Smith."

$$\pi_{Actor}(Films) - \pi_{Actor}[Films - \sigma_{Director="Smith"}(Films)]$$

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Title	Director	Actor
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The Internet's Own Boy	Knappenberger	Swartz
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10 Find the actors cast only in films by "Smith."

$$\pi_{Actor}(Films) - \pi_{Actor}[Films - \sigma_{Director="Smith"}(Films)]$$

11 Find all pairs of actors who act together in at least one film.

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Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
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10 Find the actors cast only in films by "Smith."

$$\pi_{Actor}(Films) - \pi_{Actor}[Films - \sigma_{Director="Smith"}(Films)]$$

11 Find all pairs of actors who act together in at least one film.

$$\pi_{RA,Actor}[\delta_{Actor \rightarrow RA}(Films) \bowtie Films]$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues

Cinema	Address	Phone
UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbrücker Str. 55	8032185
CinemaxX	Hübelerstr. 8	3158910
...

Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a' .

Exercise 1

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
...	CinemaxX	Hübelerstr. 8	3158910
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
The Internet's Own Boy	Knappenberger	Berners-Lee			
...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a' .

$$q_1 = \pi_{Actor, RA}[(\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(Films)) - (Films \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films)))]$$

Exercise 1

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
...	CinemaxX	Hübelerstr. 8	3158910
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
The Internet's Own Boy	Knappenberger	Berners-Lee			
...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a' .

$$q_1 = \pi_{Actor, RA}[(\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(Films)) - (Films \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films)))]$$

If $\{\{Actor \mapsto a'\}, \{RA \mapsto a\}\} \in q_1(\mathcal{D})$, then a acts in a movie that does not feature a' .

Exercise 1

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
...	CinemaxX	Hübelerstr. 8	3158910
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
The Internet's Own Boy	Knappenberger	Berners-Lee			
...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a' .

$$q_1 = \pi_{Actor, RA}[(\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(Films)) - (Films \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films)))]$$

If $\{\{Actor \mapsto a'\}, \{RA \mapsto a\}\} \in q_1(\mathcal{D})$, then a acts in a movie that does not feature a' .

12.2 Find all pairs of actors a and a' such that a acts in all the movies that feature a' .

Exercise 1

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
...	CinemaxX	Hübelerstr. 8	3158910
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
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...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a' .

$$q_1 = \pi_{Actor, RA}[(\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(Films)) - (Films \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films)))]$$

If $\{\{Actor \mapsto a'\}, \{RA \mapsto a\}\} \in q_1(\mathcal{D})$, then a acts in a movie that does not feature a' .

12.2 Find all pairs of actors a and a' such that a acts in all the movies that feature a' .

$$q_2 = (\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films))) - q_1$$

Exercise 1

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
...	CinemaxX	Hübelerstr. 8	3158910
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
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...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
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12.1 Find all pairs of actors a and a' such that a acts in a movie that does not feature a' .

$$q_1 = \pi_{Actor, RA}[(\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(Films)) - (Films \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films)))]$$

If $\{\{Actor \mapsto a'\}, \{RA \mapsto a\}\} \in q_1(\mathcal{D})$, then a acts in a movie that does not feature a' .

12.2 Find all pairs of actors a and a' such that a acts in all the movies that feature a' .

$$q_2 = (\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films))) - q_1$$

If $\{\{Actor \mapsto a\}, \{RA \mapsto a'\}\} \in q_2(\mathcal{D})$, then a acts in all the movies that feature a' .

Exercise 1

Films		
Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...
Dogma	Smith	Damon
Dogma	Smith	Affleck
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Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

12 Find all pairs of actors cast in exactly the same films.

$$q_1 = \pi_{Actor, RA}[(\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(Films)) - (Films \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films)))]$$

$$q_2 = (\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films))) - q_1$$

Exercise 1

Films		
Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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UFA	St. Petersburger Str. 24	4825825
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Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

12 Find all pairs of actors cast in exactly the same films.

$$q_1 = \pi_{Actor, RA}[(\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(Films)) - (Films \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films)))]$$

$$q_2 = (\pi_{Actor}(Films) \bowtie \delta_{Actor \rightarrow RA}(\pi_{Actor}(Films))) - q_1$$

$$q_2 \bowtie \delta_{Actor, RA \rightarrow RA, Actor}(q_2)$$

Exercise 1

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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Cinema	Address	Phone
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...

Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

13.1 Find all pairs of directors d and actors a such that d directs some movie that features a .

Exercise 1

Films		
Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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CinemaxX	The Imitation Game	19:30

13.1 Find all pairs of directors d and actors a such that d directs some movie that features a .

$$q_1 = \pi_{Director, Actor}(Films)$$

Exercise 1

Films		
Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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CinemaxX	The Imitation Game	19:30

13.1 Find all pairs of directors d and actors a such that d directs some movie that features a .

$$q_1 = \pi_{Director, Actor}(Films)$$

13.2 Find the directors who do not direct all actors.

Exercise 1

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Cinema	Address	Phone
UFA	St. Petersburger Str. 24	4825825
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Cinema	Title	Time
Schauburg	The Imitation Game	19:30
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CinemaxX	The Imitation Game	19:30

13.1 Find all pairs of directors d and actors a such that d directs some movie that features a .

$$q_1 = \pi_{Director, Actor}(Films)$$

13.2 Find the directors who do not direct all actors.

$$q_2 = \pi_{Director}((\pi_{Director}(Film) \bowtie \pi_{Actor}(Film)) - q_1)$$

Exercise 1

Films		
Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues		
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UFA	St. Petersburger Str. 24	4825825
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...

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

13 Find the directors such that every actor is cast in one of their films.

$$q_1 = \pi_{Director, Actor}(Films)$$

$$q_2 = \pi_{Director}((\pi_{Director}(Film) \bowtie \pi_{Actor}(Film)) - q_1)$$

Exercise 1

Films		
Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues		
Cinema	Address	Phone
UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbrücker Str. 55	8032185
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...

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Schauburg	The Imitation Game	19:30
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13 Find the directors such that every actor is cast in one of their films.

$$q_1 = \pi_{Director, Actor}(Films)$$

$$q_2 = \pi_{Director}((\pi_{Director}(Film) \bowtie \pi_{Actor}(Film)) - q_1)$$

$$\pi_{Director}(Film) - q_2$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$(1) R \bowtie R$$

$$(2) R \bowtie \emptyset$$

$$(3) R \bowtie \{\varepsilon\}$$

Solution.

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$(1) R \bowtie R$$

$$(2) R \bowtie \emptyset$$

$$(3) R \bowtie \{\varepsilon\}$$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{ f : U \cup V \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S \},$$

where f_U and f_V are the restriction of f to elements in U and V , respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

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Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{f : U \cup V \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S\},$$

where f_U and f_V are the restriction of f to elements in U and V , respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(1)

$$R \bowtie R = \{f : U \cup U \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_U \in R\}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$(1) R \bowtie R$$

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Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{f : U \cup V \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S\},$$

where f_U and f_V are the restriction of f to elements in U and V , respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(1)

$$\begin{aligned} R \bowtie R &= \{f : U \cup U \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_U \in R\} \\ &= \{f : U \rightarrow \mathbf{dom} \mid f_U \in R\} \end{aligned}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$(1) R \bowtie R$$

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Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{f : U \cup V \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S\},$$

where f_U and f_V are the restriction of f to elements in U and V , respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(1)

$$\begin{aligned} R \bowtie R &= \{f : U \cup U \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_U \in R\} \\ &= \{f : U \rightarrow \mathbf{dom} \mid f_U \in R\} \\ &= \{f_U \mid f_U \in R\} \end{aligned}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$(1) R \bowtie R$$

$$(2) R \bowtie \emptyset$$

$$(3) R \bowtie \{\varepsilon\}$$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{f : U \cup V \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S\},$$

where f_U and f_V are the restriction of f to elements in U and V , respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(1)

$$\begin{aligned} R \bowtie R &= \{f : U \cup U \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_U \in R\} \\ &= \{f : U \rightarrow \mathbf{dom} \mid f_U \in R\} \\ &= \{f_U \mid f_U \in R\} \\ &= R \end{aligned}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$(1) R \bowtie R$$

$$(2) R \bowtie \emptyset$$

$$(3) R \bowtie \{\varepsilon\}$$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{f : U \cup V \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S\},$$

where f_U and f_V are the restriction of f to elements in U and V , respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(2)

$$R \bowtie \emptyset = \{f : U \cup \emptyset \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_\emptyset \in \emptyset\}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$(1) R \bowtie R$$

$$(2) R \bowtie \emptyset$$

$$(3) R \bowtie \{\varepsilon\}$$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{ f : U \cup V \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S \},$$

where f_U and f_V are the restriction of f to elements in U and V , respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(2)

$$\begin{aligned} R \bowtie \emptyset &= \{ f : U \cup \emptyset \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_\emptyset \in \emptyset \} \\ &= \emptyset \end{aligned}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$(1) R \bowtie R$$

$$(2) R \bowtie \emptyset$$

$$(3) R \bowtie \{\varepsilon\}$$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{ f : U \cup V \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S \},$$

where f_U and f_V are the restriction of f to elements in U and V , respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(3)

$$R \bowtie \{\varepsilon\} = \{ f : U \cup \emptyset \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_\emptyset \in \{\varepsilon\} \}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$(1) R \bowtie R$$

$$(2) R \bowtie \emptyset$$

$$(3) R \bowtie \{\varepsilon\}$$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{ f : U \cup V \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S \},$$

where f_U and f_V are the restriction of f to elements in U and V , respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

(3)

$$\begin{aligned} R \bowtie \{\varepsilon\} &= \{ f : U \cup \emptyset \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_\emptyset \in \{\varepsilon\} \} \\ &= \{ f : U \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_\emptyset \in \{\varepsilon\} \} \end{aligned}$$

Exercise 2

Exercise. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$(1) R \bowtie R$$

$$(2) R \bowtie \emptyset$$

$$(3) R \bowtie \{\varepsilon\}$$

Solution. Recall the definition of the natural join (Lecture 1, Slide 22):

$$R \bowtie S = \{ f : U \cup V \rightarrow \mathbf{dom} \mid f_U \in R \text{ and } f_V \in S \},$$

where f_U and f_V are the restriction of f to elements in U and V , respectively, i.e., $f(u) = f_U(u)$ for all $u \in U$ and $f(v) = f_V(v)$ for all $v \in V$.

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Exercise 3

Exercise. Express the following operations using other operations presented in the lecture:

1. Intersection $R \cap S$.
2. Cartesian product $R \times S$.
3. Selection $\sigma_{n=a}(R)$ with a a constant.
4. Arbitrary constant tables in queries.

Solution.

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Solution.

1. Note that $R \cap S$ is well-defined only if the attributes of R and S coincide. Suppose that the common set of attributes is U . Then we have

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2. Suppose R has attributes U and S has attributes V . Let W be a set of fresh attributes with $|W| = |V|$ and $W \cap U = \emptyset$. Then, $R \times S = R \bowtie \delta_{\vec{V} \rightarrow \vec{W}}(S)$.

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3. $\sigma_{n=a}(R) = R \bowtie \{ \{n \mapsto a\} \}$
4. To create a constant table with a single row and many attribute-value pairs, simply join several single attribute-value pair constant tables (cf. query 7 in Exercise 1). Then use union to create a table with several rows.

Exercise 4

Exercise. Consider the following identities and decide for each whether it is true or false. If true, prove your answer using the definitions from the lecture; if false, give a counterexample.

1. $R \bowtie S = S \bowtie R$
2. $R \bowtie (S \bowtie T) = (R \bowtie S) \bowtie T$
3. $\pi_X(R \circ S) = \pi_X(R) \circ \pi_X(S)$ for all $\circ \in \{\cup, \cap, -, \bowtie\}$
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$$\pi_X(R \cup S) = \pi_X(R) \cup \pi_X(S)$$

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Let $f \in \pi_X(R \cup S)$. Then there is some $f' \in R \cup S$ with $f'_X = f$ and hence $f \in \pi_X(R) \cup \pi_X(S)$.

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Exercise. Consider the following identities and decide for each whether it is true or false. If true, prove your answer using the definitions from the lecture; if false, give a counterexample.

1. $R \bowtie S = S \bowtie R$
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Let $f \in \pi_X(R \cup S)$. Then there is some $f' \in R \cup S$ with $f'_X = f$ and hence $f \in \pi_X(R) \cup \pi_X(S)$.

Conversely, let $f \in \pi_X(R) \cup \pi_X(S)$. Then $f \in \pi_X(R)$ or $f \in \pi_X(S)$, and there is some $f' \in R \cup S$ such that $f'_X = f$.

Thus $f \in \pi_X(R \cup S)$.

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3.2

$$\pi_X(R \cap S) = \pi_X(R) \cap \pi_X(S)$$

Exercise 4

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4. $\sigma_{n=m}(R \circ S) = \sigma_{n=m}(R) \circ \sigma_{n=m}(S)$ for all $\circ \in \{\cup, \cap, -\}$.
5. $\sigma_{n=m}(R \bowtie S) = \sigma_{n=m}(R) \bowtie S$, for n and m attributes of R only.

Why are these identities of interest?

Solution. These identities can be used to optimise queries, e.g., by pushing selection inwards so that joins receive smaller inputs.

3.2

$$\pi_X(R \cap S) = \pi_X(R) \cap \pi_X(S)$$

Consider tables $R = \{ \{ A \mapsto 1, B \mapsto 2 \} \}$ and $S = \{ \{ A \mapsto 1, B \mapsto 3 \} \}$.

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Exercise. Consider the following identities and decide for each whether it is true or false. If true, prove your answer using the definitions from the lecture; if false, give a counterexample.

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Then $\pi_A(R - S) = \{ \{ A \mapsto 1 \} \} \supsetneq \pi_A(R) - \pi_A(S) = \emptyset$.

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Exercise 5

Exercise. Let R^I and S^I be tables of schema $R[U]$ and $S[V]$, respectively. The *division* of R^I by S^I , written as $(R^I \div S^I)$, is defined to be the maximal table over the attributes $U \setminus V$ that satisfies $(R^I \div S^I) \bowtie S^I \subseteq R^I$. Note that the joined tables here do not have any attributes in common, so the natural join works as a cross product. Consider the following table and use the division operator to (1) express a query for the cities that have been visited by all people.

Visited

<i>Person</i>	<i>City</i>
Tomas	Berlin
Markus	Santiago
Markus	Berlin
Fred	New York
Fred	Berlin

Then, (2) express division using the standard relational algebra operators.

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Solution.

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$$Visited \div \pi_{Person}(Visited)$$

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Solution.

(1)

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(2) Let X be the set of all attributes of R that are not attributes of S (i.e., $X = U \setminus V$).

$$R \div S = \pi_X(R) - \pi_X[(\pi_X(R) \bowtie S) - R]$$

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Exercise. Suggest how to write the relational algebra operations for using the unnamed perspective. What changes?

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Solution.

- ▶ Natural join becomes cartesian product \times .
- ▶ No renaming.
- ▶ Order matters in projections.
- ▶ New set of operators: $\{\sigma, \pi, \cup, -, \times\}$.

Exercise 7

Exercise. The set of operations $\{\sigma, \pi, \cup, -, \bowtie, \delta\}$ can express all queries of relational algebra. Show that it is not possible to reduce this set any further.

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 6. The query language $\{\sigma, \pi, -, \bowtie, \delta\}$ is less expressive than $\{\sigma, \pi, \cup, -, \bowtie, \delta\}$.

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 5. Then, $q(\mathcal{D}) \neq \{\{A \mapsto 1, B \mapsto 1\}, \{A \mapsto 2, B \mapsto 2\}\}$.

Exercise 7

Exercise. The set of operations $\{\sigma, \pi, \cup, -, \bowtie, \delta\}$ can express all queries of relational algebra. Show that it is not possible to reduce this set any further.

Solution.

- ▶ Operator δ is the only one that can rename attributes in tables.
- ▶ Operator π is the only one that can produce tables with less attributes.
- ▶ Operator \bowtie is the only one that can produce tables with more attributes.
- ▶ Operator \cup cannot be removed.
- ▶ Operator σ cannot be removed:
 1. Let \mathcal{D} be the database containing the table $R = \{A \mapsto 1, A \mapsto 2\}$.
 2. Then, $\sigma_{A=B}(R \bowtie \delta_{A \rightarrow B}(R))(\mathcal{D}) = \{A \mapsto 1, B \mapsto 1, A \mapsto 2, B \mapsto 2\}$.
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