From Horn-$\mathcal{SRIQ}$ to Datalog: A Data-Independent Transformation that Preserves Assertion Entailment

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TECHNISCHE UNIVERSITÄT DRESDEN

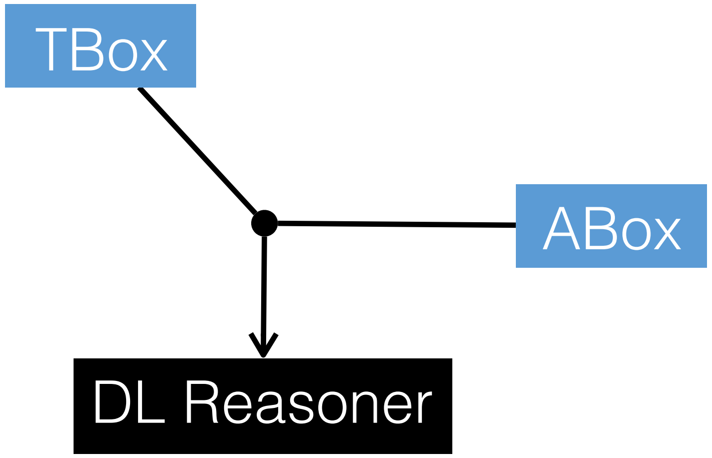
Introduction

Solving IQs with Datalog Rewritings

TBox

ABox

Solving IQs with Datalog Rewritings



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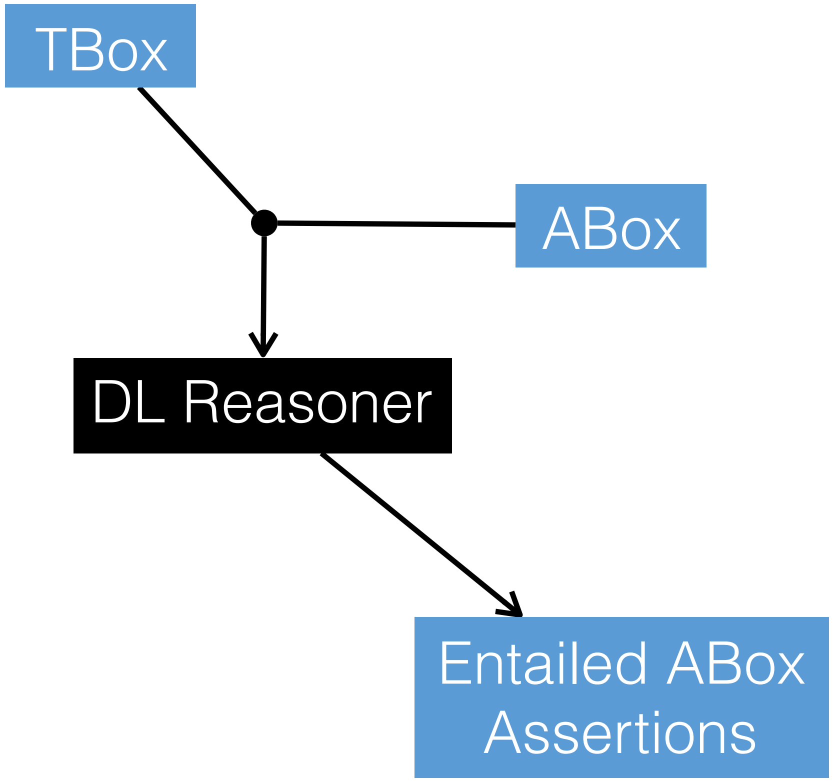

(i.e., facts over named individuals)

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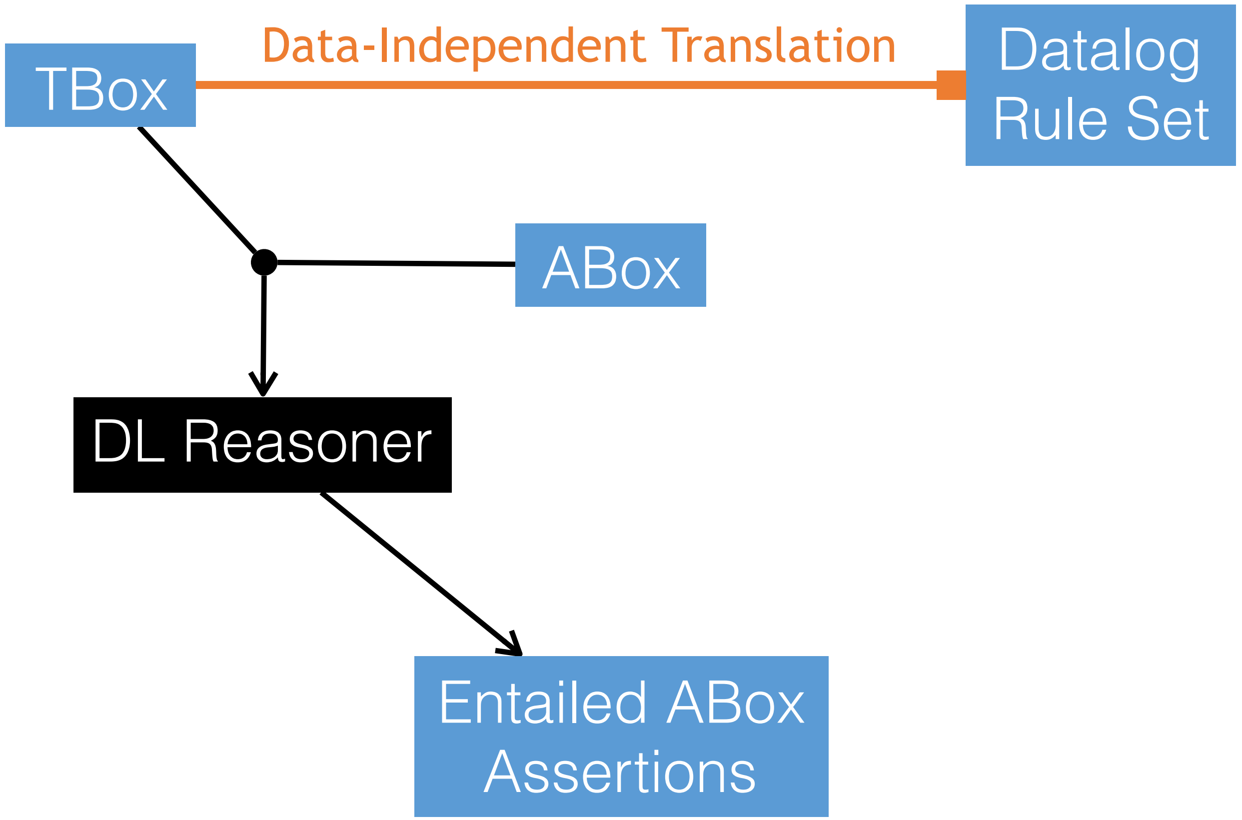

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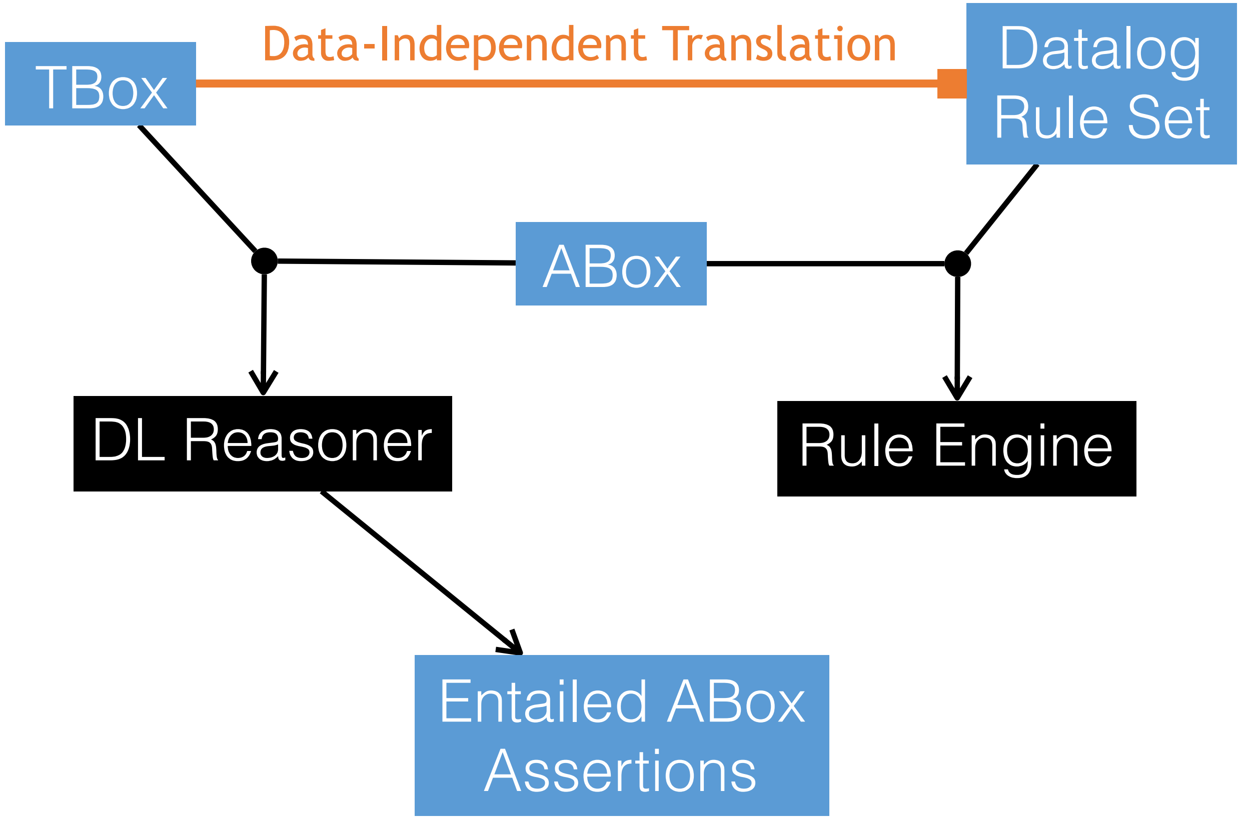

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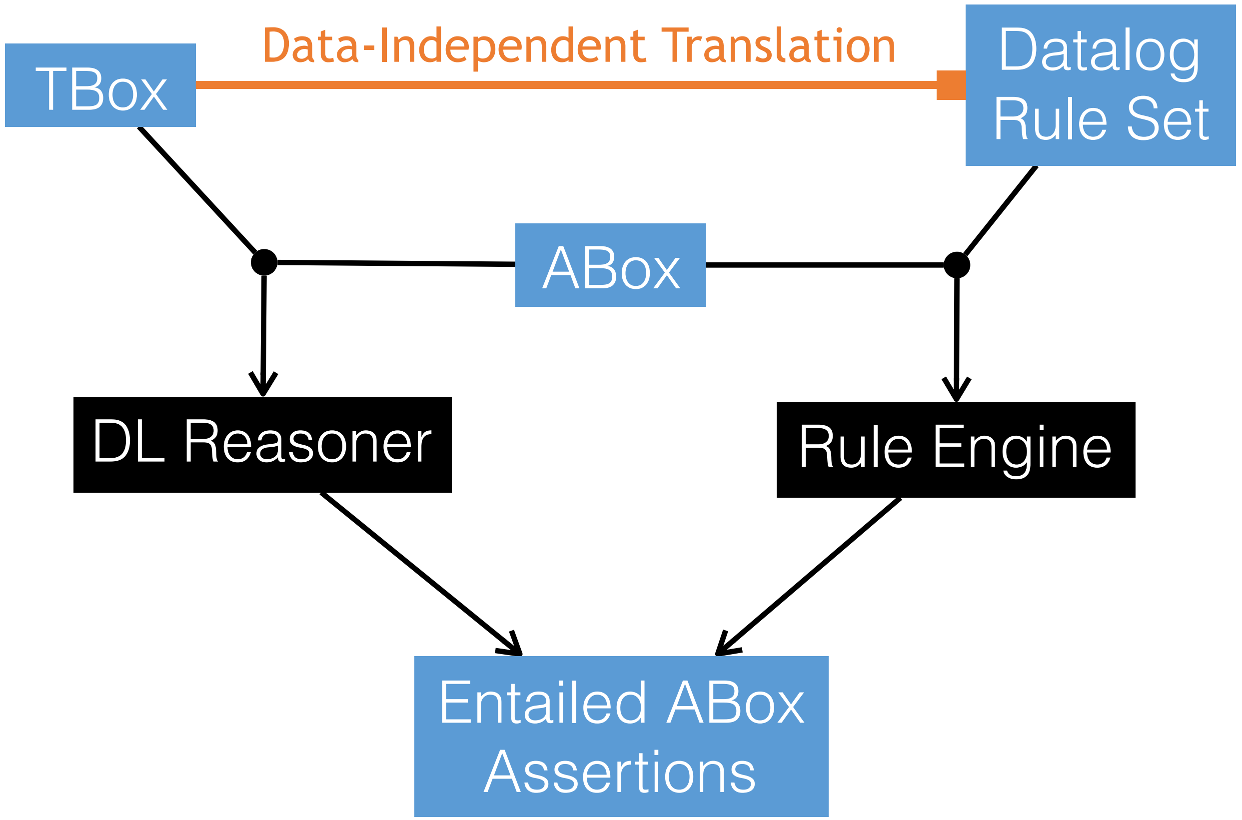

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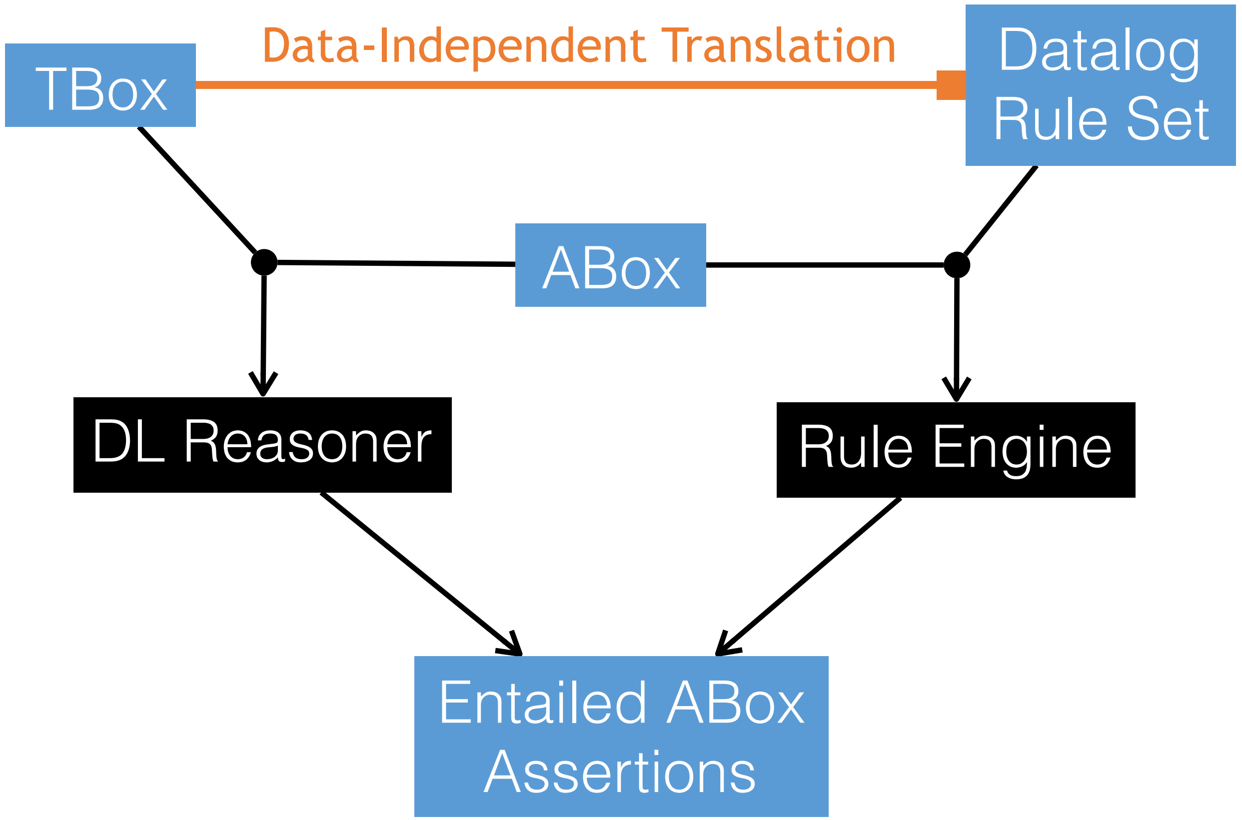

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In **theory**:
* Correctness
* Complexity

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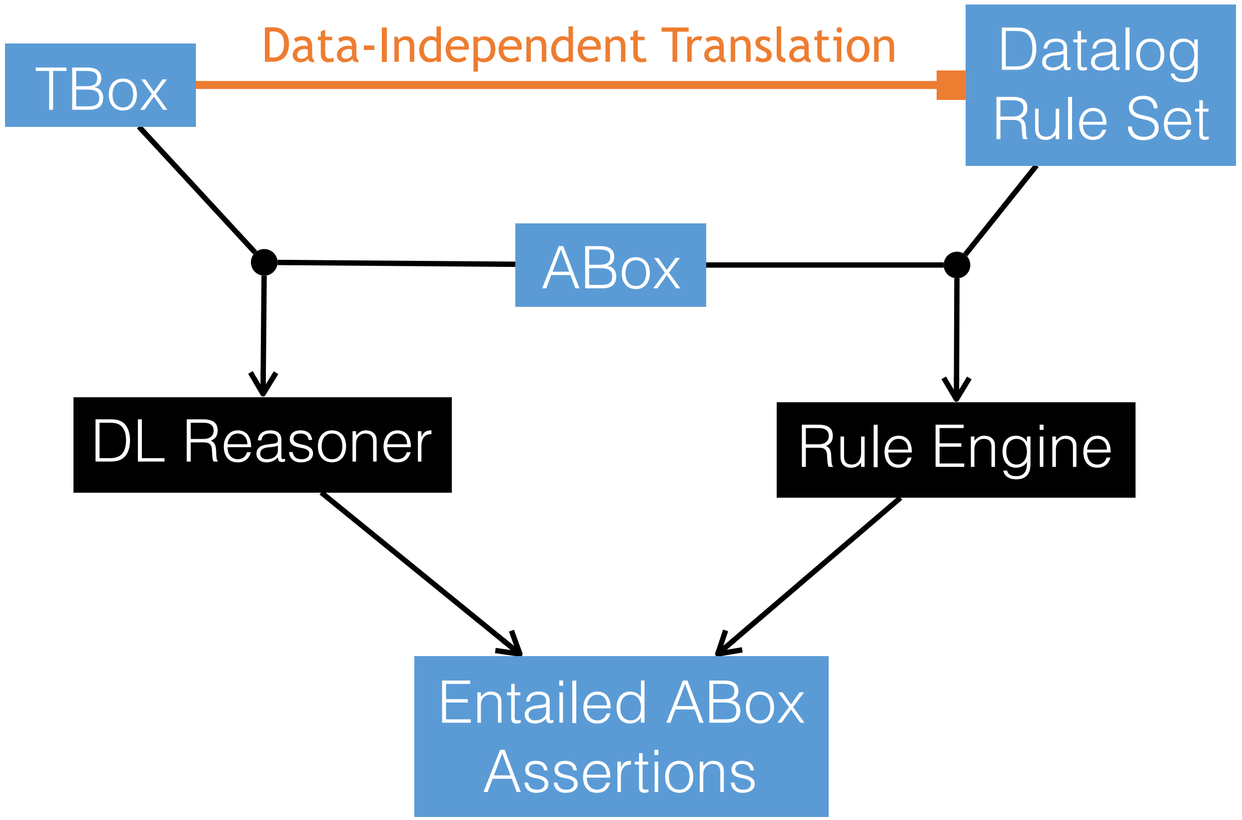

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In **theory**:
* Correctness
* Complexity

In **practice**:
* Implement mapping
* Evaluate performance

The DL Horn-SRIQ

$$C_1 \sqcap \ldots \sqcap C_n \sqsubseteq D \qquad C \sqsubseteq \leq 1R.D \qquad A(a)$$

$$\exists R.C \sqsubseteq D \qquad C \sqsubseteq \exists R.D \qquad R(a,b)$$

$$R_1 \circ \ldots \circ R_n \sqsubseteq S$$

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$\mathit{ResearchGroup} \sqsubseteq \forall \mathit{HasMember} . \mathit{Researcher}$

$\mathit{Researcher} \sqsubseteq \exists \mathit{HasMember}^{-} . \mathit{ResearchGroup}$

$\mathit{Collaborates} \circ \mathit{HasMember}^{-} \circ \mathit{HasMember} \sqsubseteq \mathit{HasConflict}$

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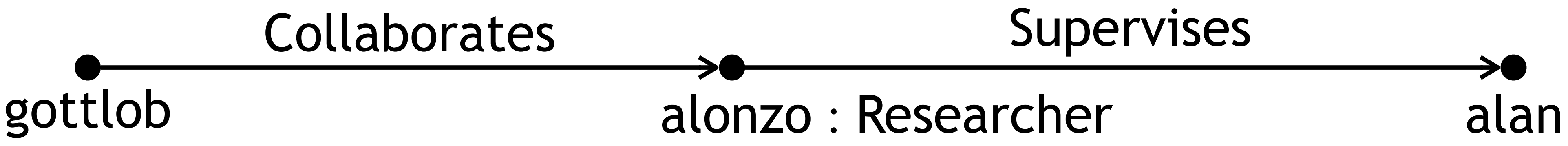

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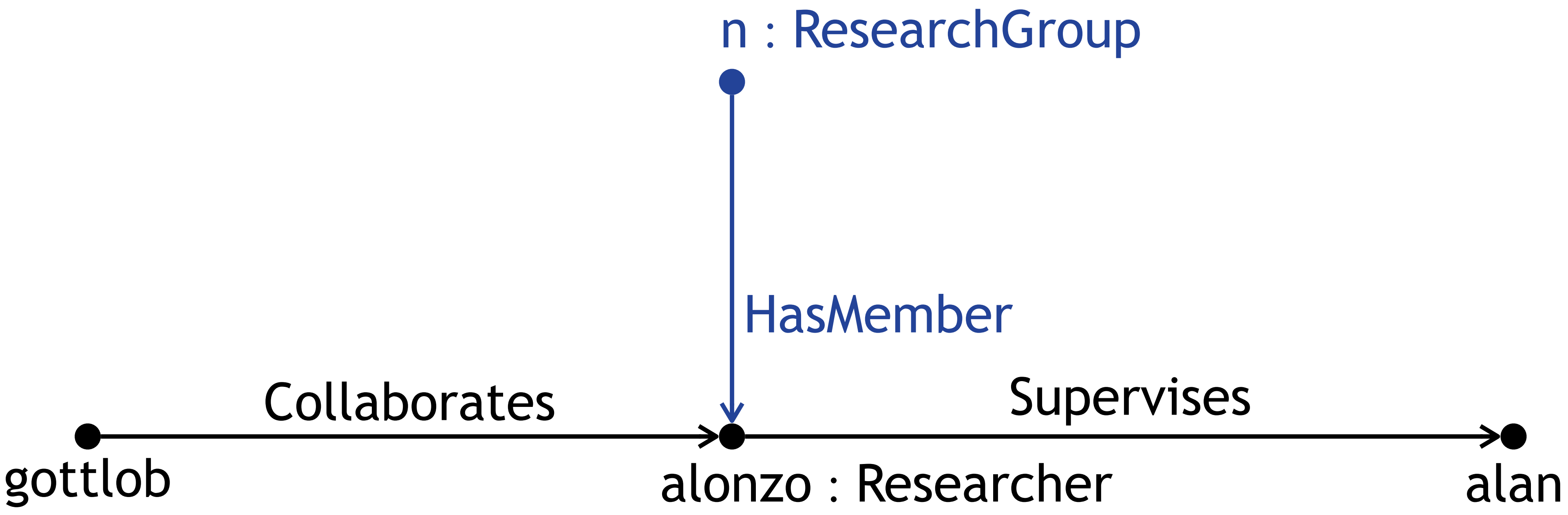

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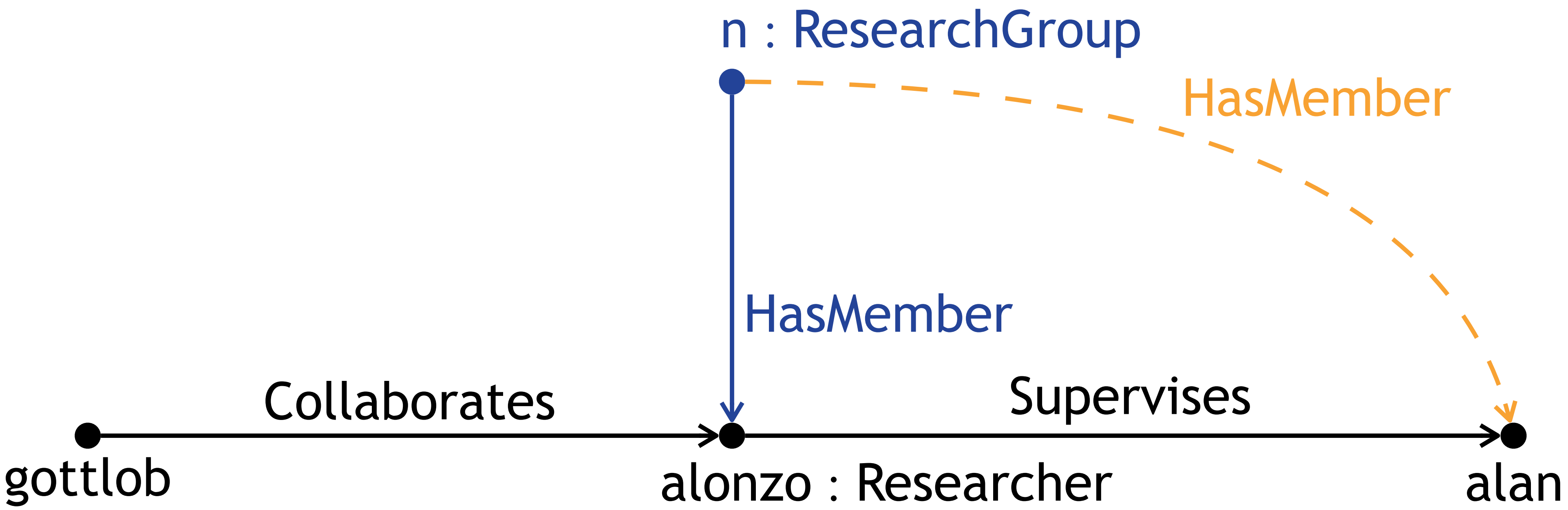

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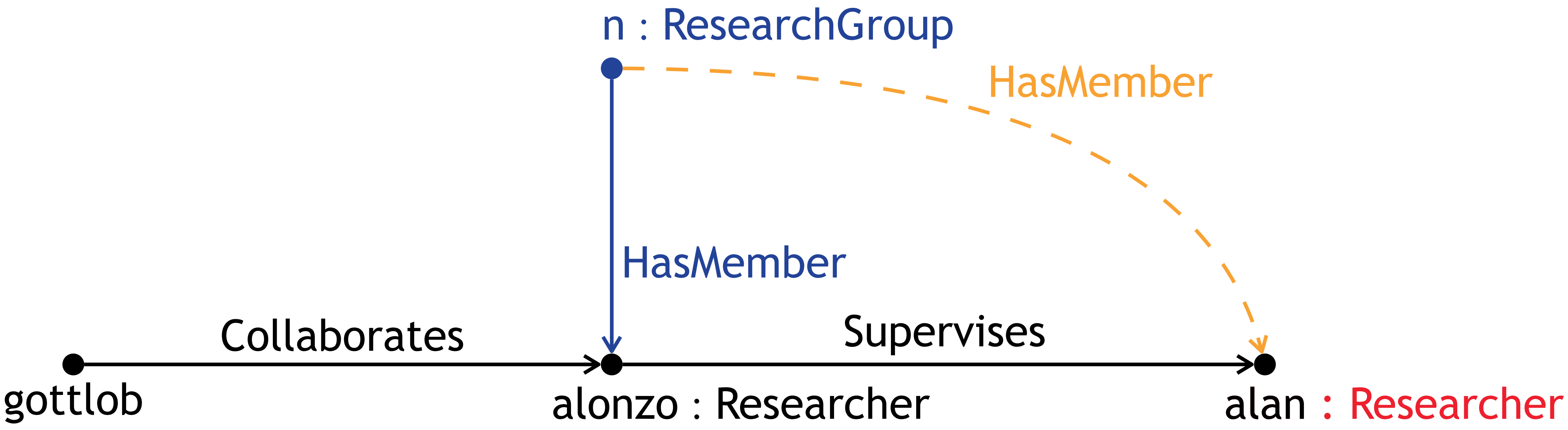

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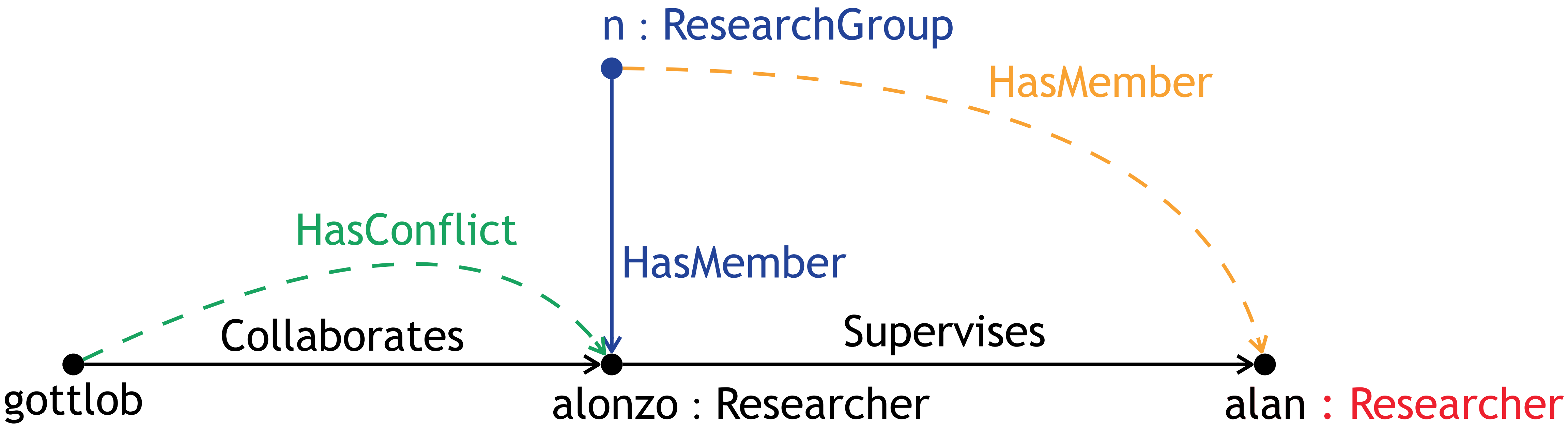

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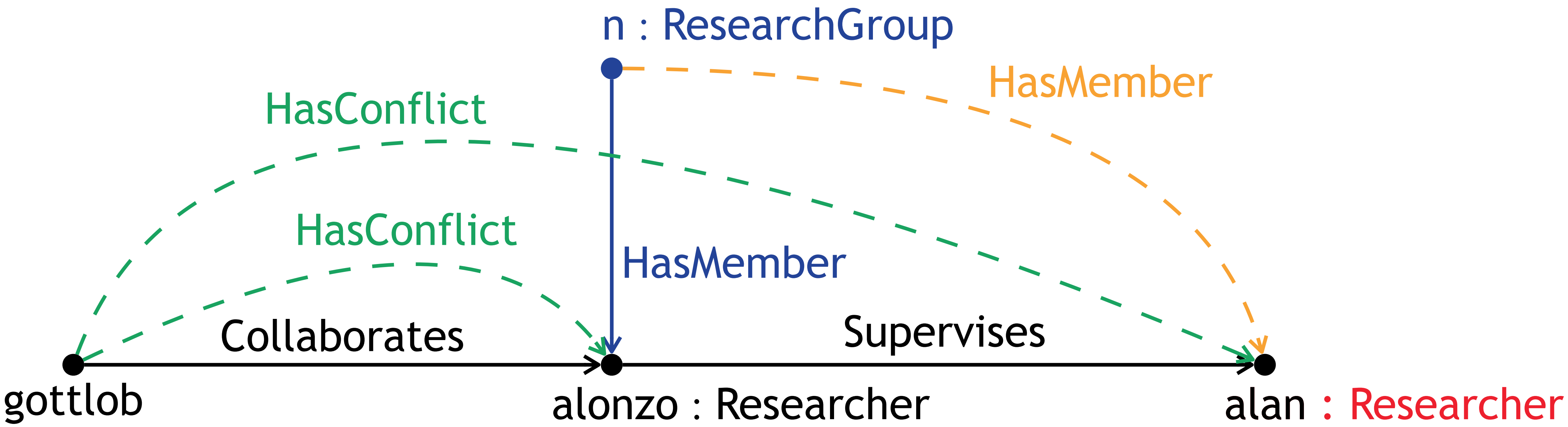
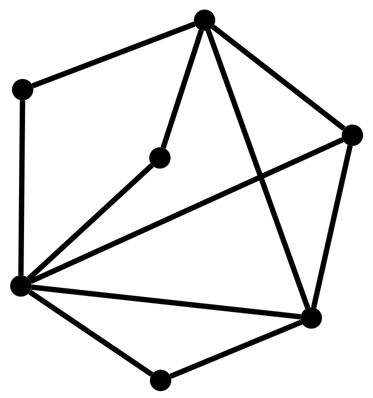


From Horn-ALCHIQ to Datalog

Forest Model Property



$C_1 \sqcap \ldots \sqcap C_n \sqsubseteq D$

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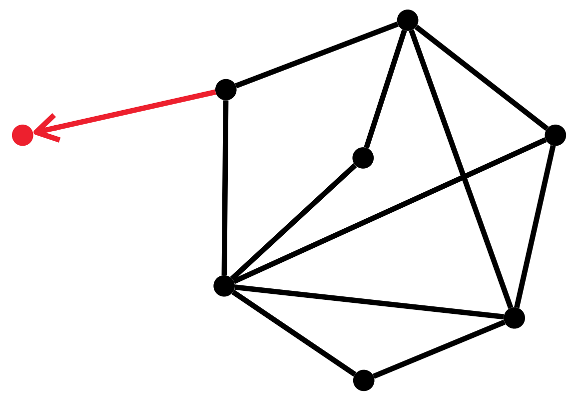

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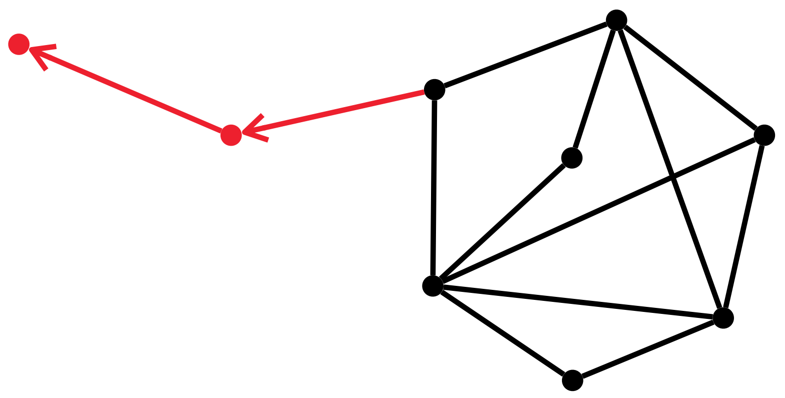

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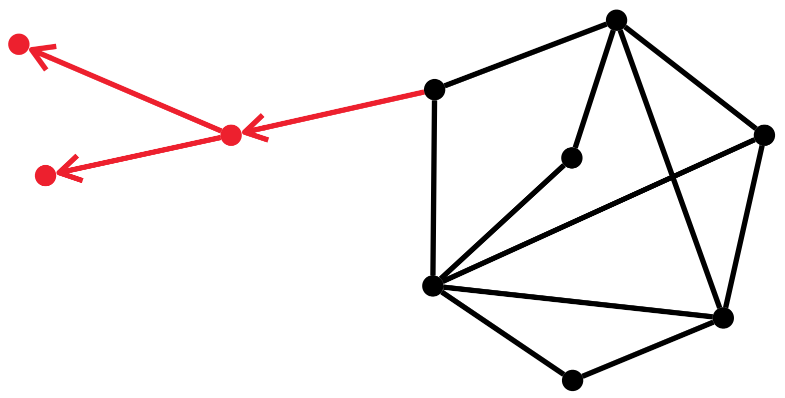

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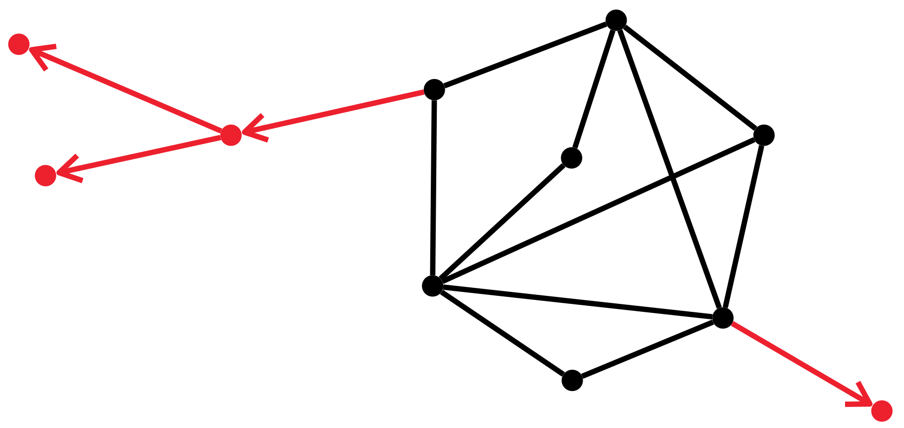

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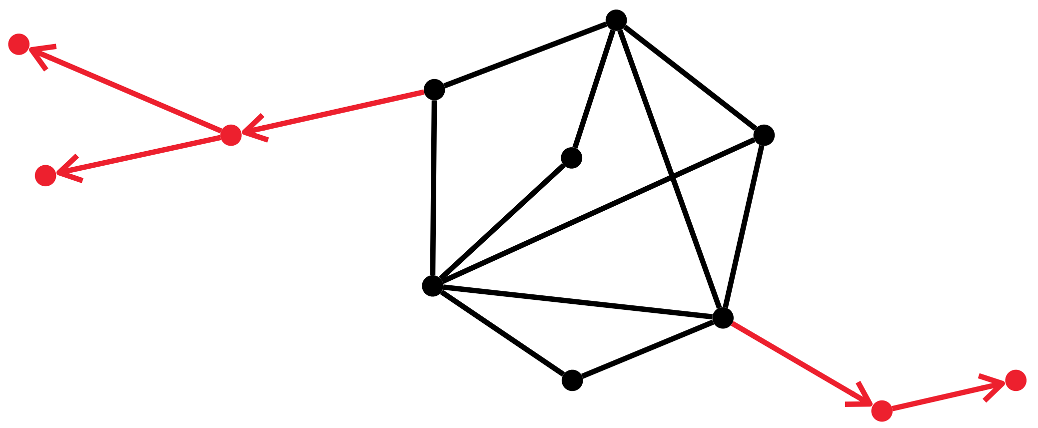

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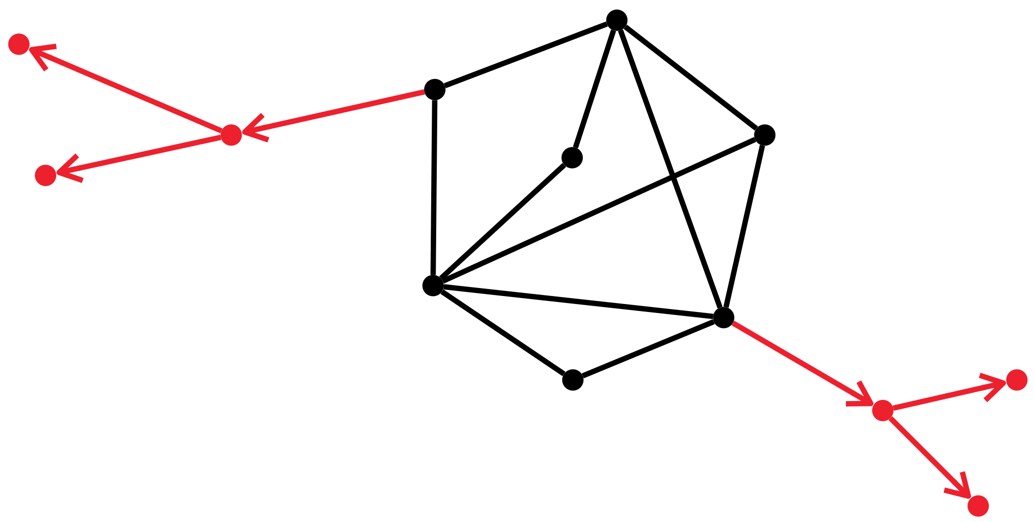

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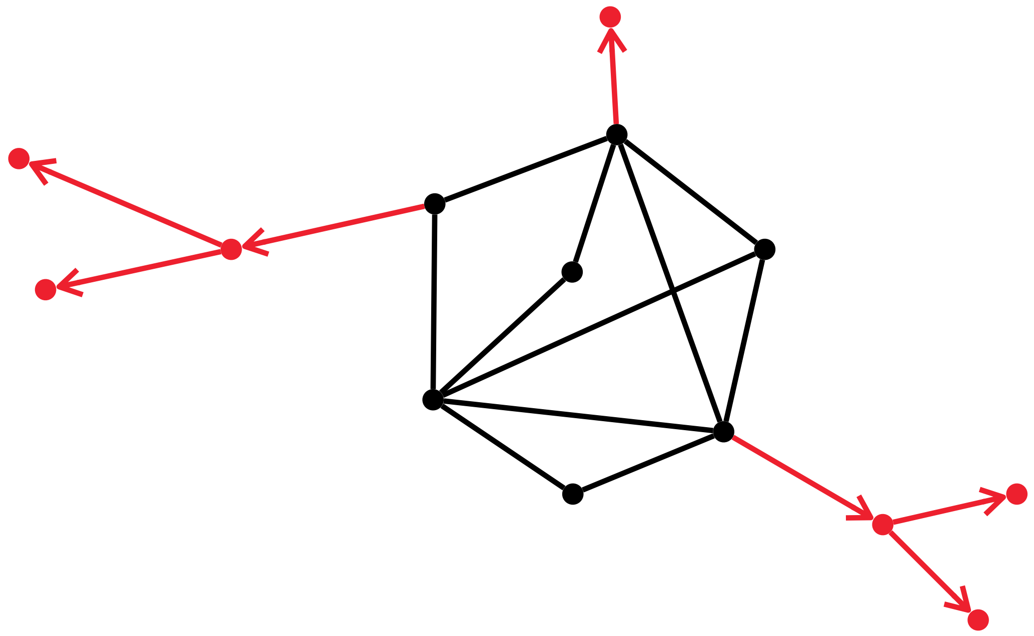

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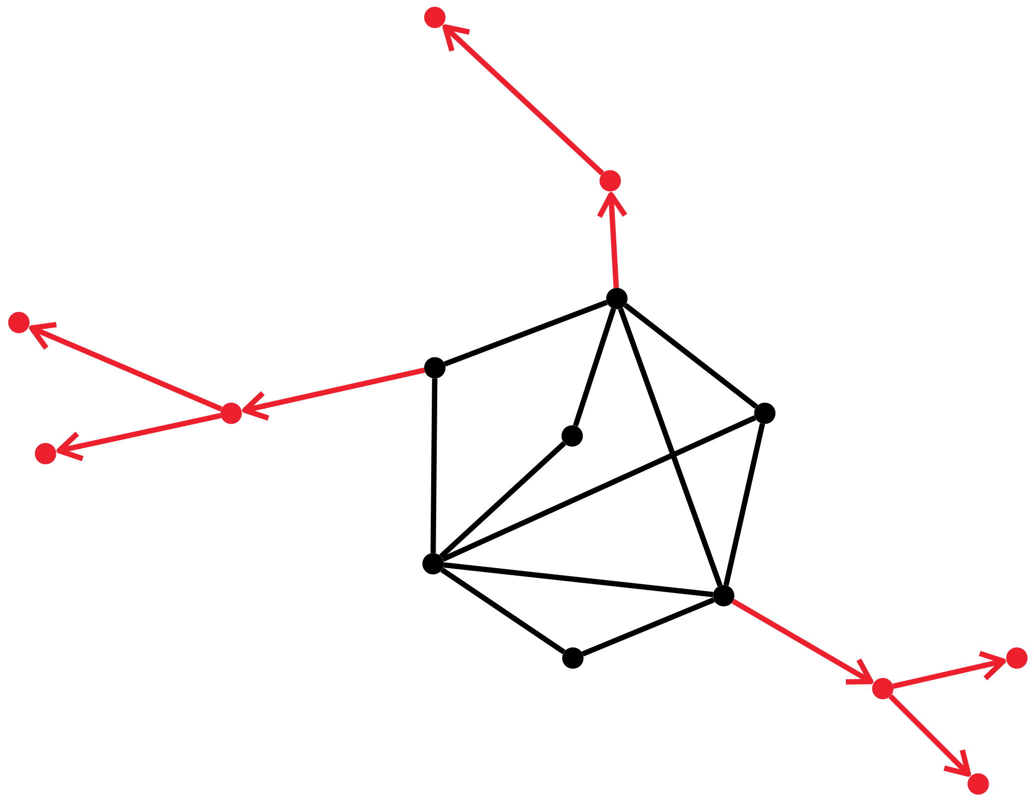

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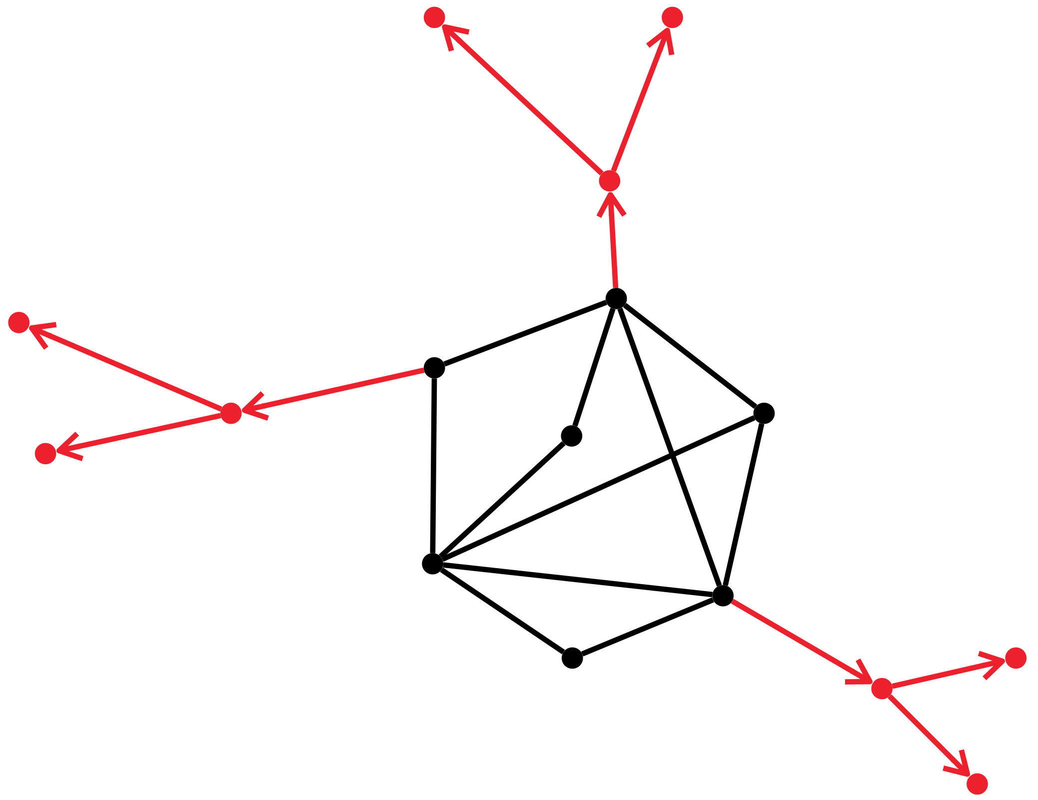

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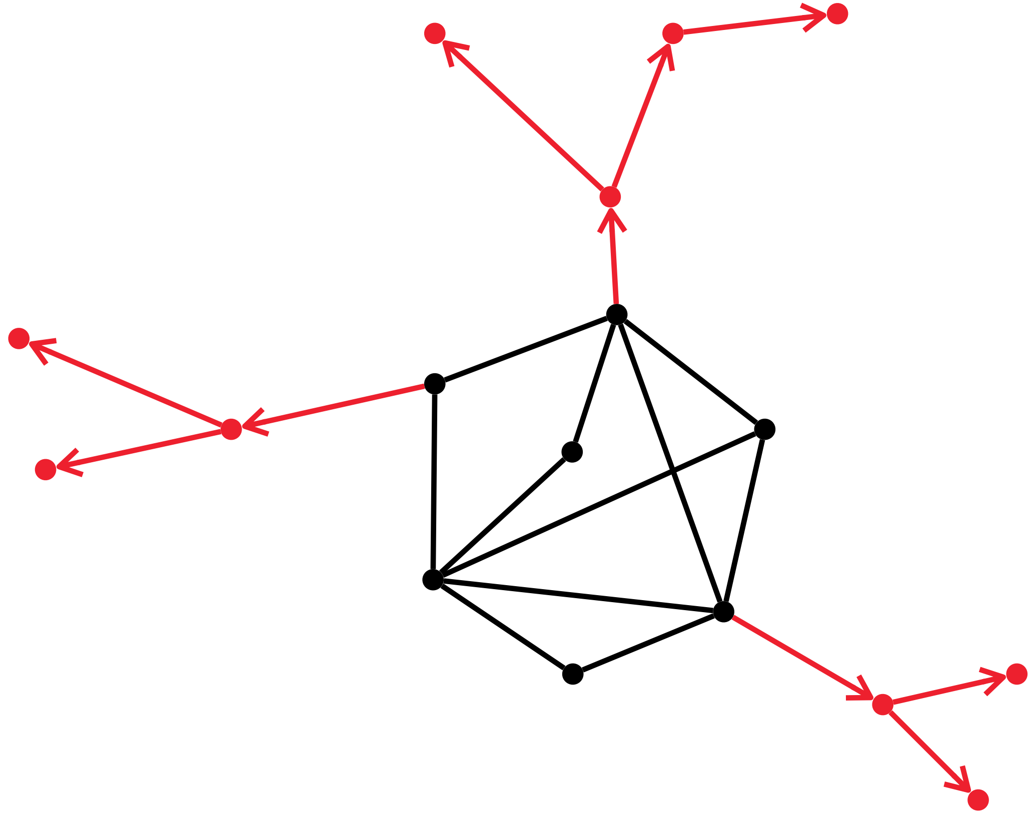

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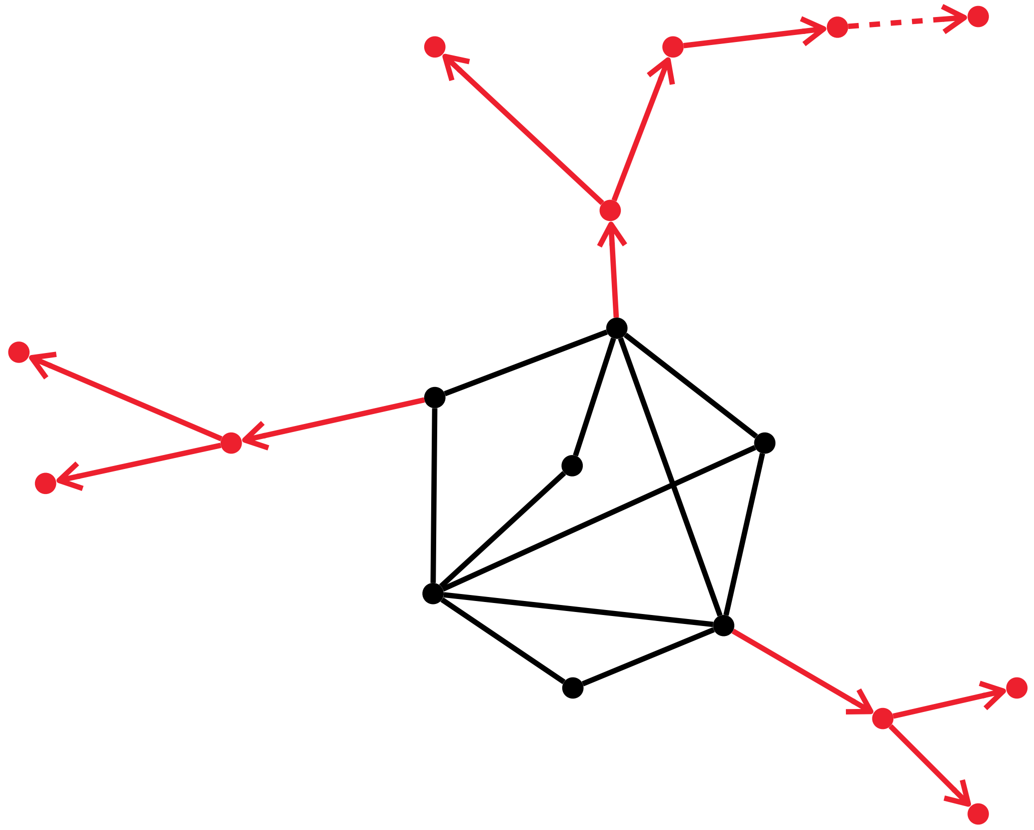

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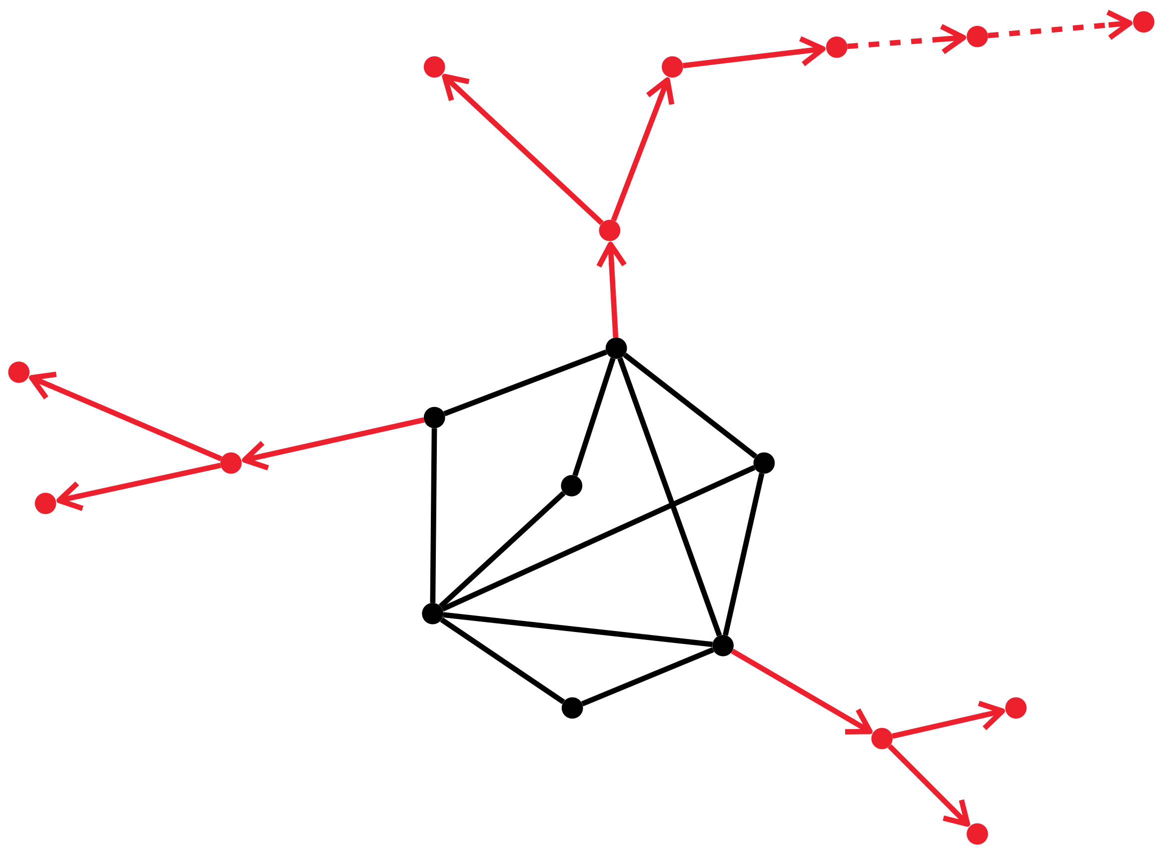

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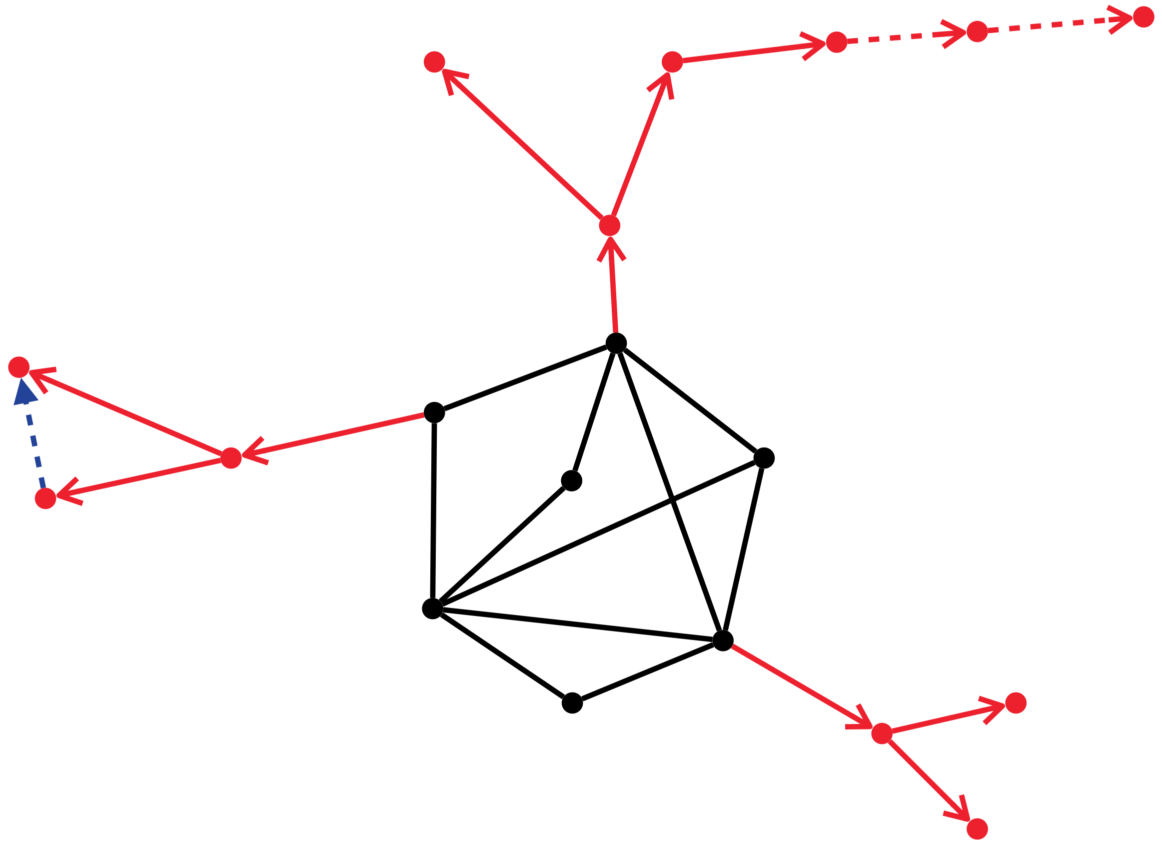

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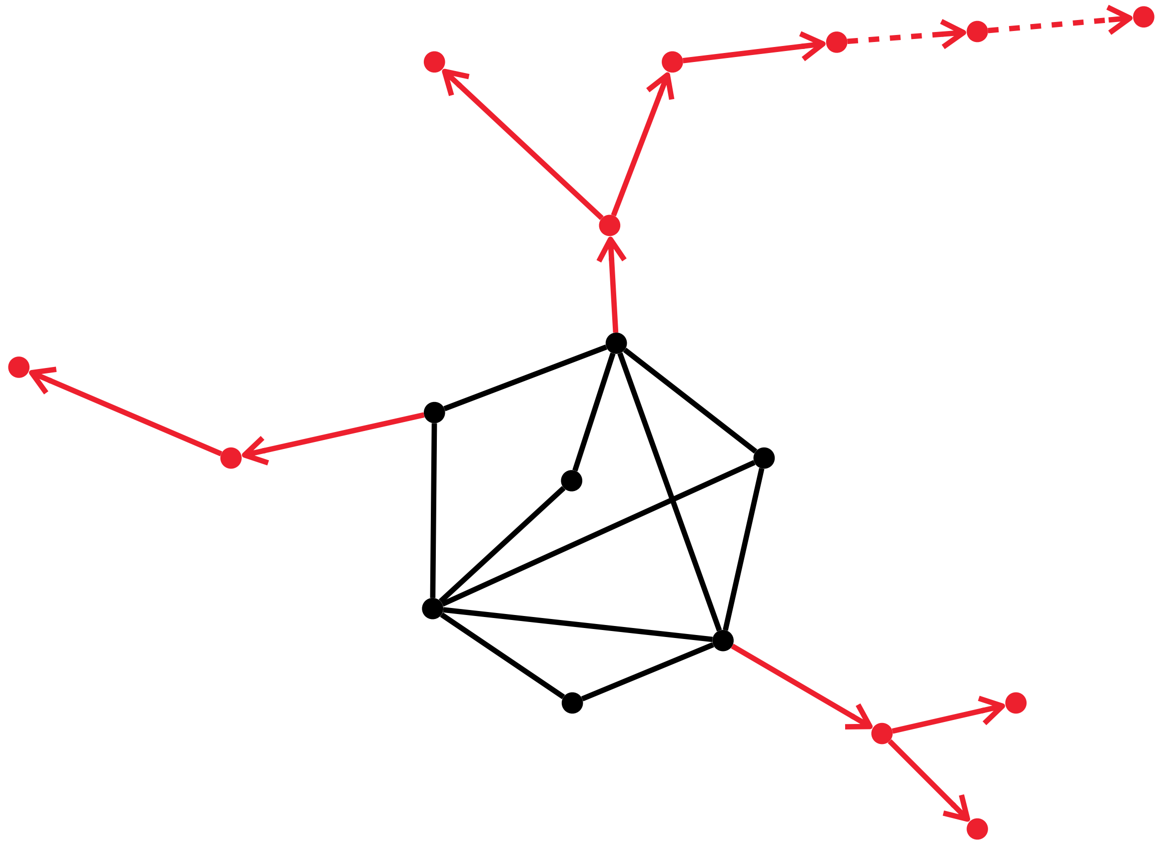

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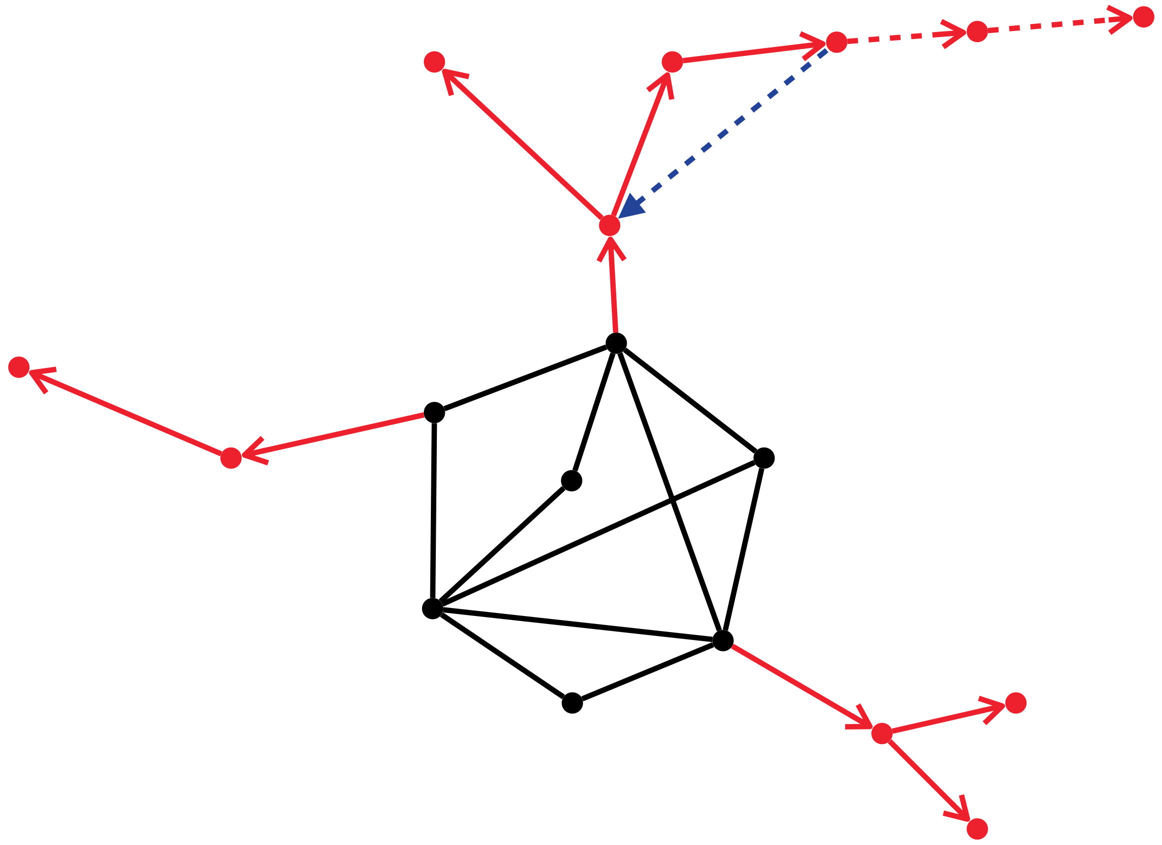

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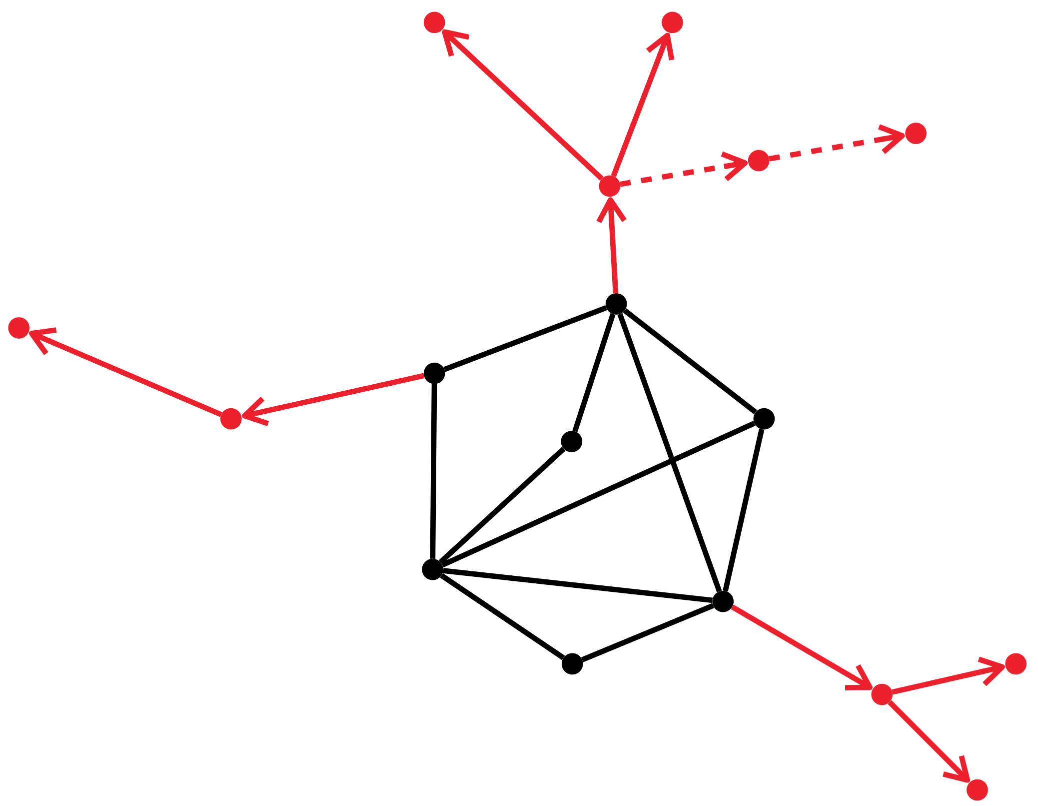

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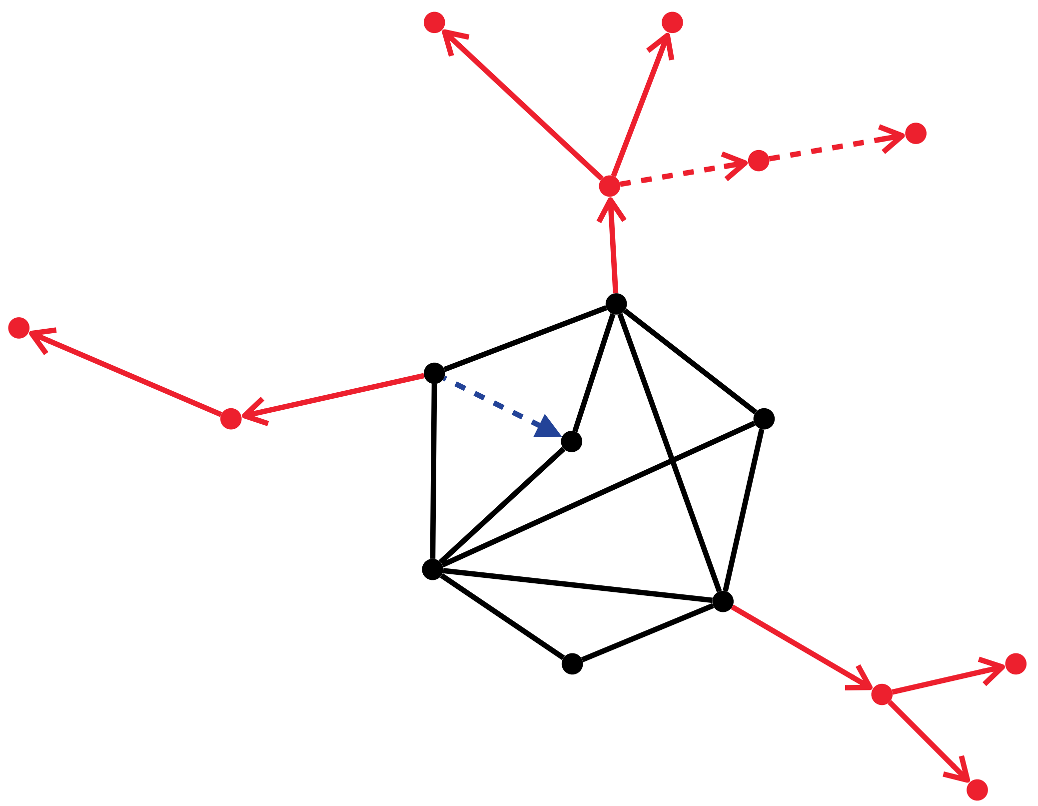

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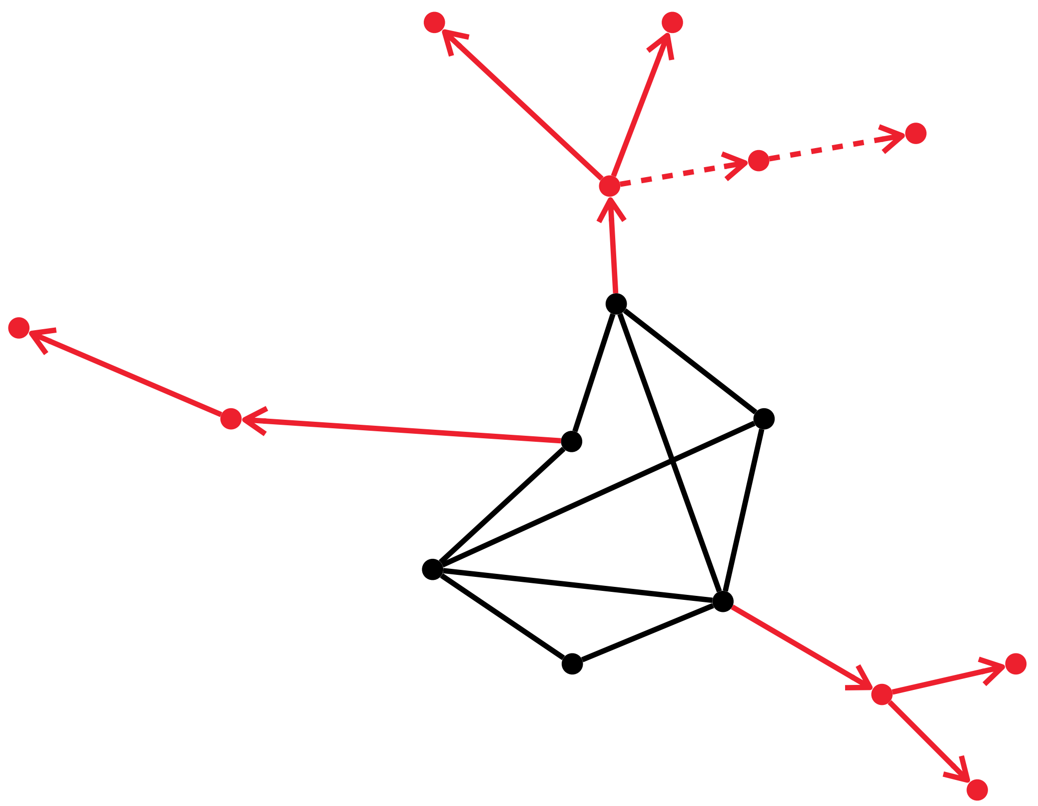

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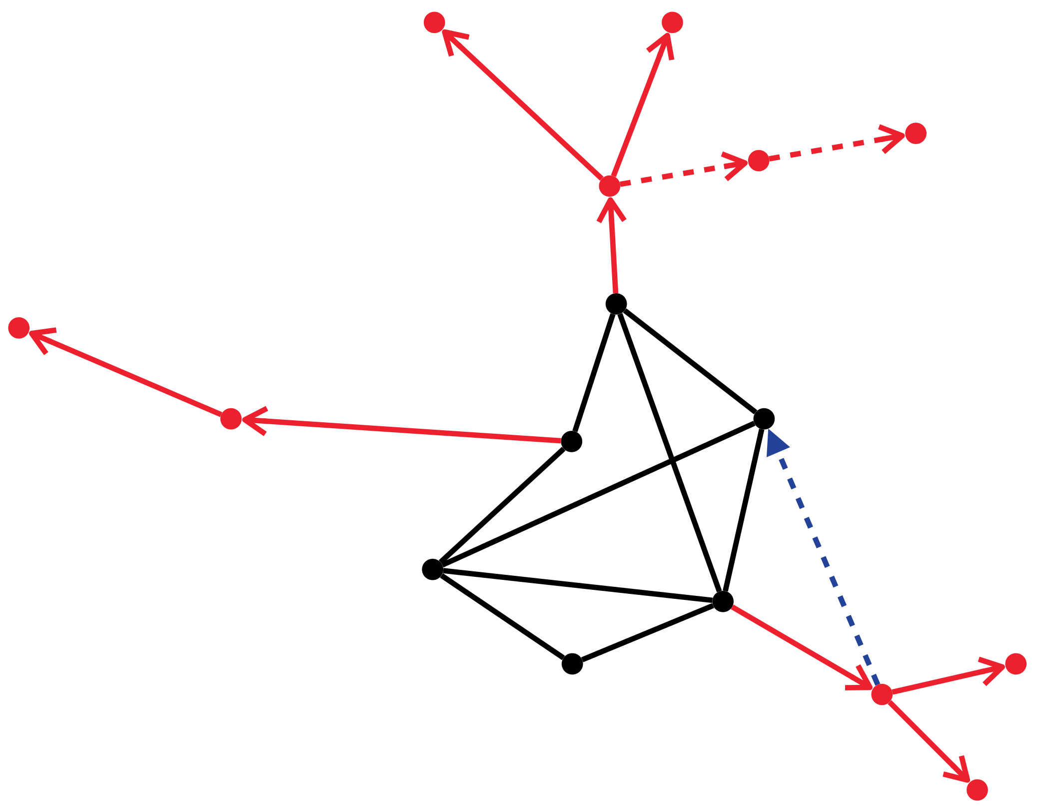

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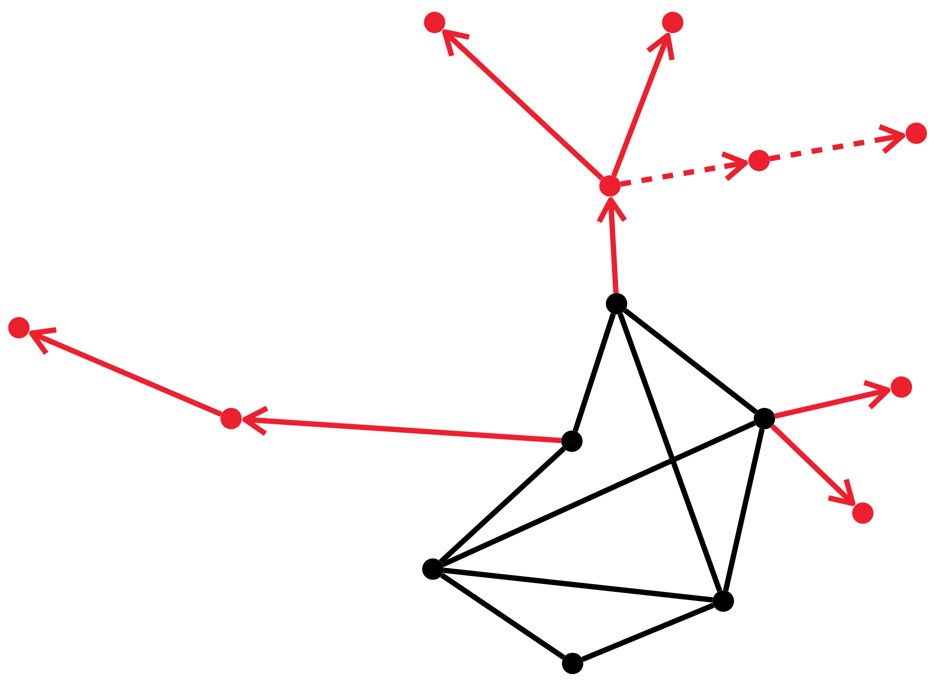

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“Unnamed-to-Named” Consequences

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Successor-to-predecessor

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Successor-to-predecessor

$C \sqsubseteq \exists R.D$

$D \sqsubseteq \exists S.E$

$\exists S.E \sqsubseteq F$

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$a : C$

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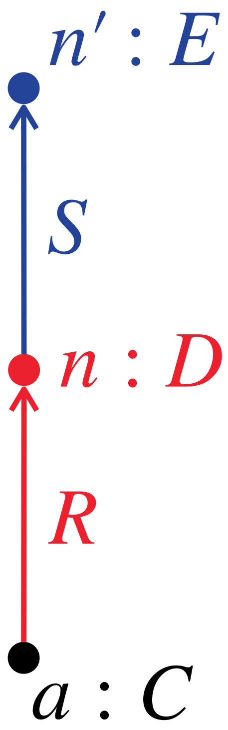

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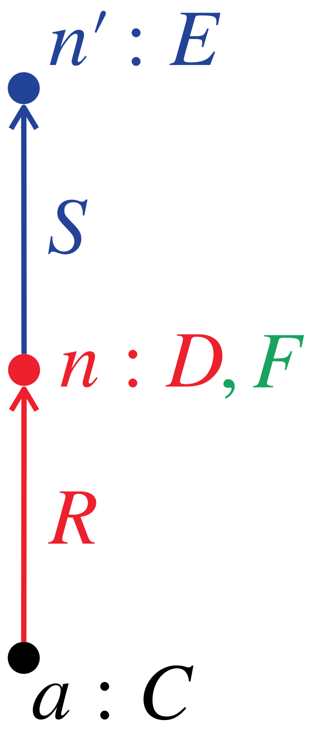

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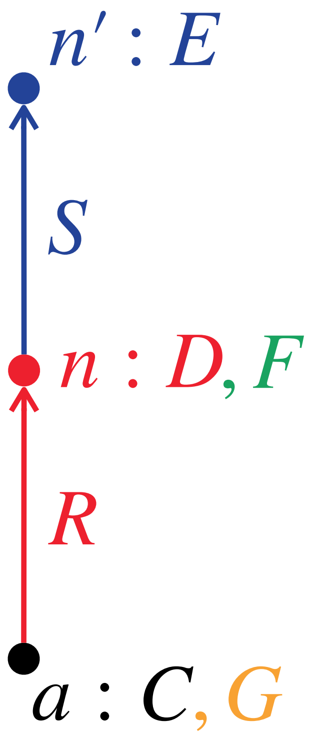

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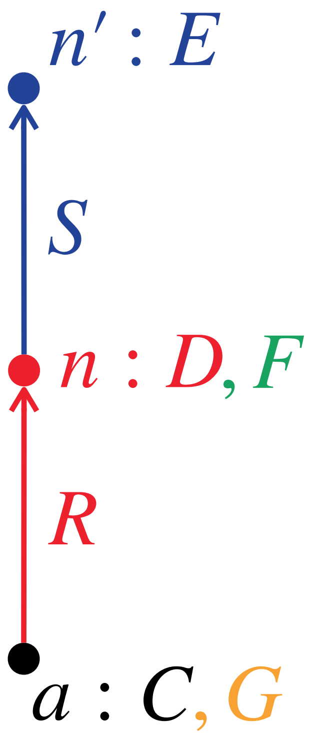

$C(x) \rightarrow G(x)$

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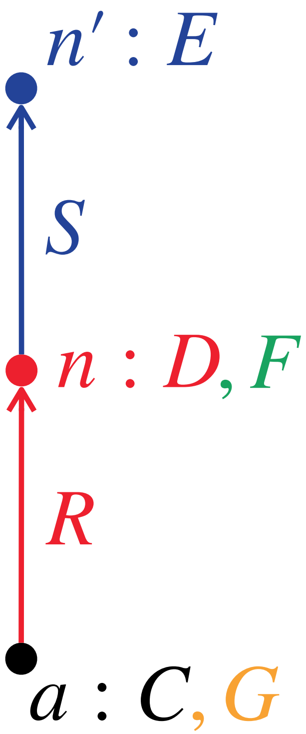

Folding

"Unnamed-to-Named" Consequences

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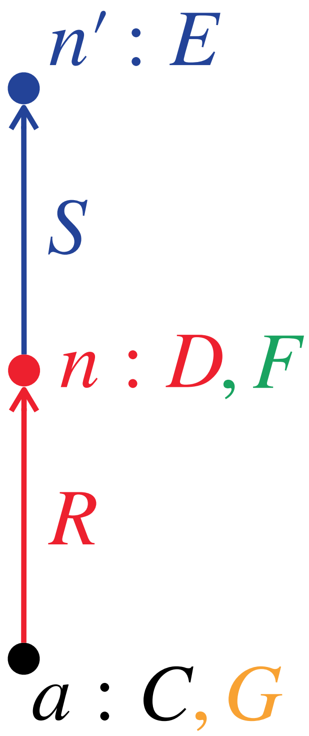

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$C \sqsubseteq \exists S \, . \, E$

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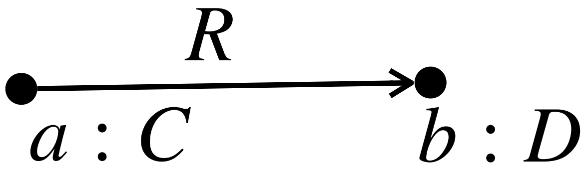

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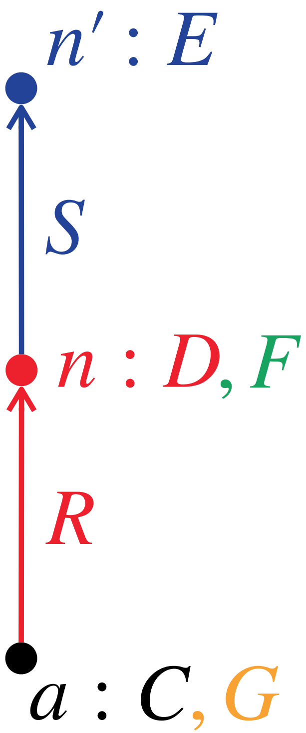

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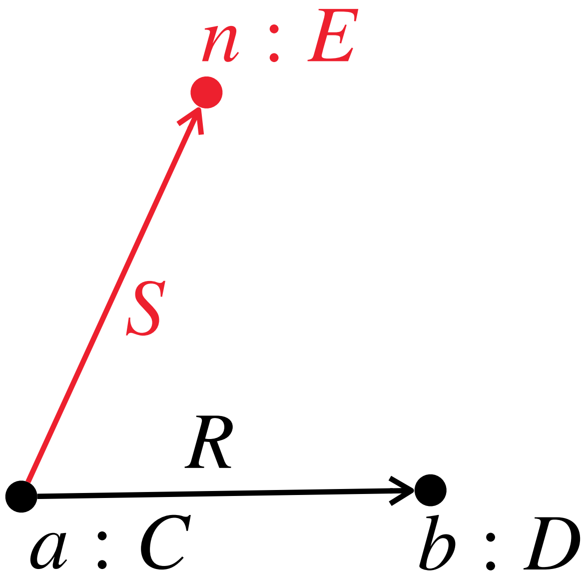

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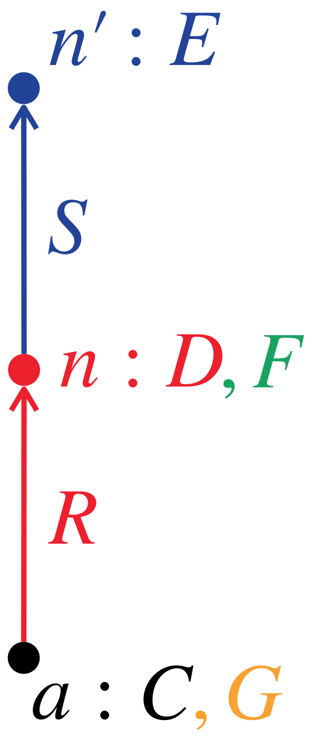
$C \sqsubseteq \exists R \,.\, D$

$D \sqsubseteq \exists S \,.\, E$

$\exists S \,.\, E \sqsubseteq F$

$\exists R \,.\, F \sqsubseteq G$

$C(x) \rightarrow G(x)$



Folding

$C \sqsubseteq \exists S \,.\, E$

$S \sqsubseteq R$

$E \sqsubseteq D$

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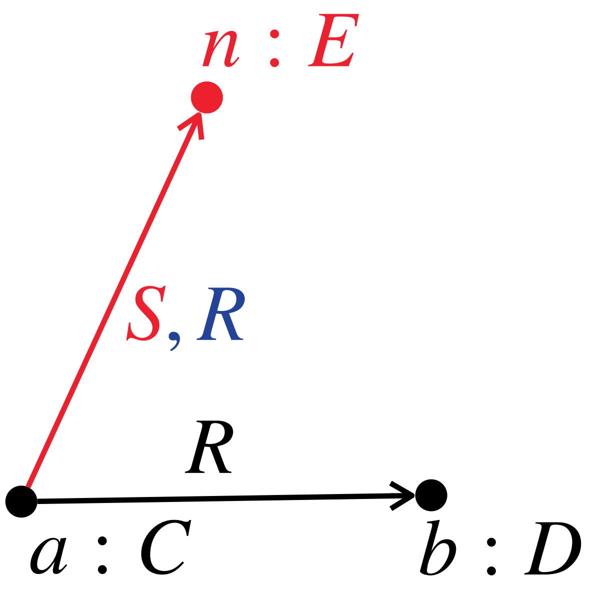

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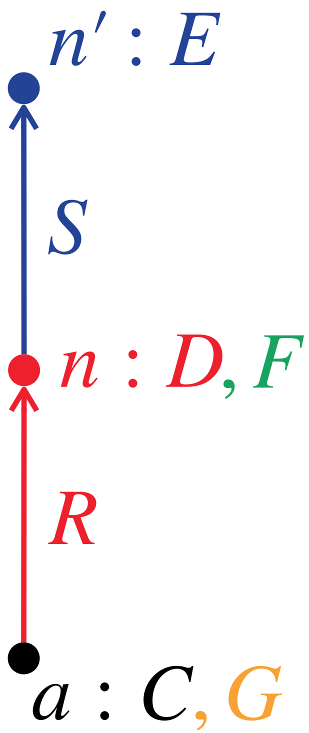

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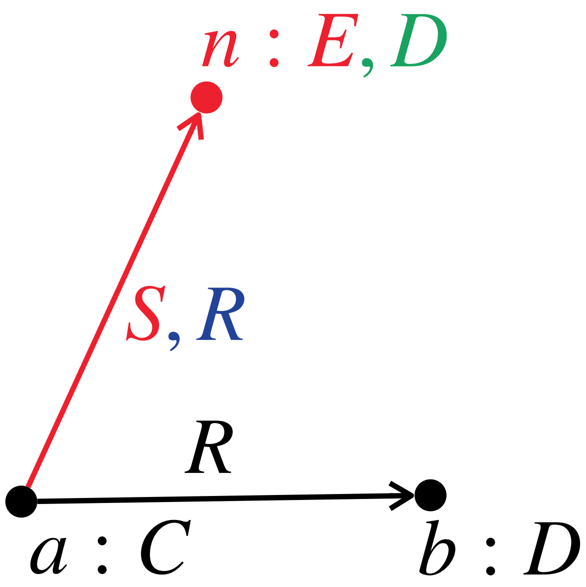

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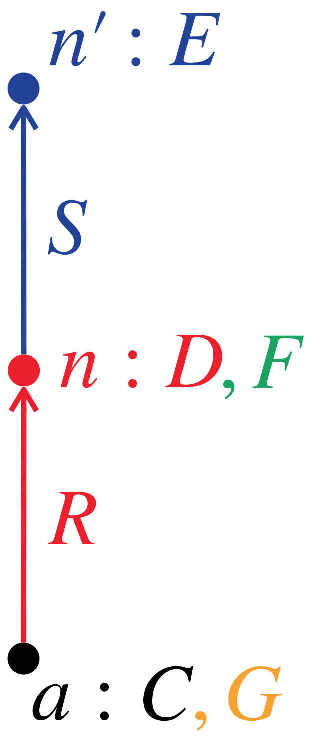

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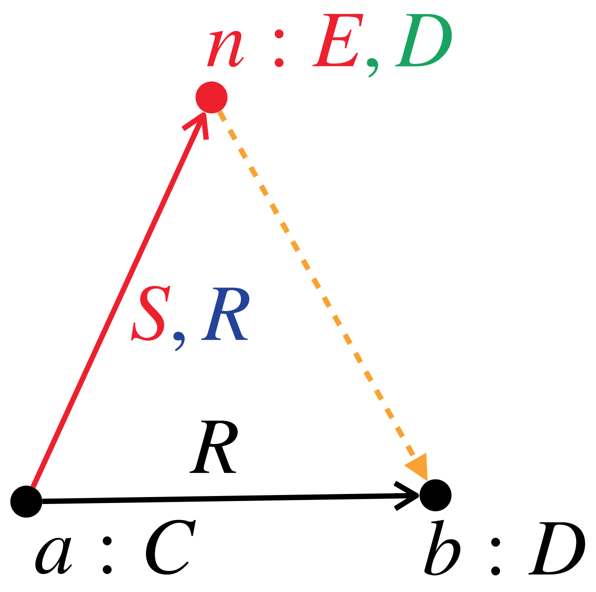

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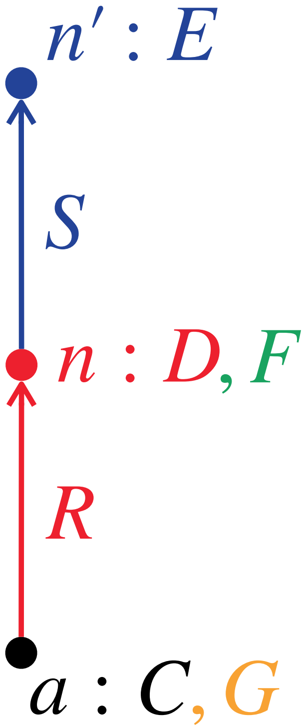

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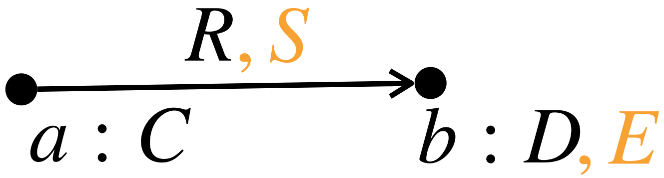

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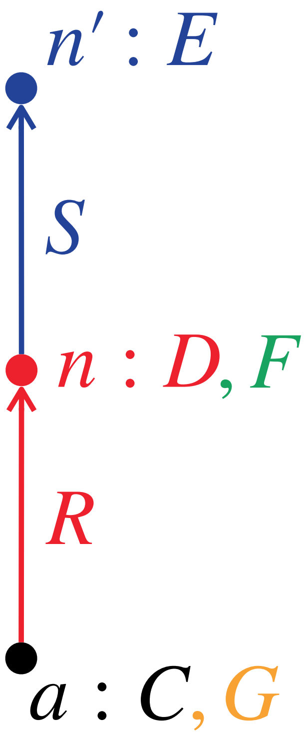

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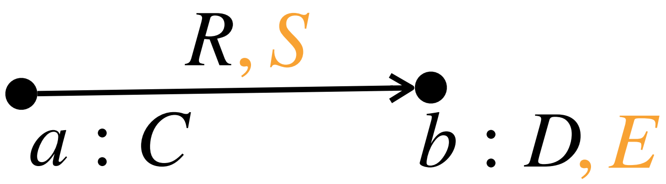

$$C(x) \wedge R(x, y) \wedge D(y) \rightarrow S(x, y)$$
$$C(x) \wedge R(x, y) \wedge D(y) \rightarrow E(y)$$

Computing IQ Rewritings for Horn-ALCHIQ

Consider some Horn-ALCHIQ TBox $\mathscr{T}$.
Then, the rule set $\mathscr{R}_{\mathscr{T}}$ defined as follows is an IQ-preserving rewriting for $\mathscr{T}$.

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For all $C_1 \sqcap \ldots \sqcap C_n \sqsubseteq D \in \Omega(\mathscr{T})$

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$\Omega(\mathcal{T})$ is the set of all axioms of one of the following forms entailed by $\mathcal{T}$.
$C_1 \sqcap \ldots \sqcap C_n \sqsubseteq D$
$C_1 \sqcap \ldots \sqcap C_n \sqsubseteq \exists (R_1 \sqcap \ldots \sqcap R_m) . (D_1 \sqcap \ldots \sqcap D_k)$

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Consequence-based Reasoning Calculi
[IJCAI 2017] Yevgeny
[AAAI 2012] Eiter et al.

$$\frac{M \sqsubseteq \exists S.(N \sqcap N') \quad N \sqsubseteq A}{M \sqsubseteq \exists S.(N \sqcap N' \sqcap A)} \ \mathbf{R}^{c}_{\sqsubseteq}$$

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$C_1(x) \wedge \ldots \wedge C_n(x) \rightarrow D(x) \in \mathcal{R}_{\mathcal{T}}$

For all $C \sqsubseteq \leq 1 R . D \in \mathcal{T}$,

$C(x) \wedge R(x, y) \wedge D(y) \wedge R(x, z) \wedge D(z) \rightarrow y \approx z \in \mathcal{R}_{\mathcal{T}}$,

$C(x) \wedge C_1(x) \wedge \ldots \wedge C_n(x) \wedge R(x, y) \wedge D(y) \rightarrow E(y) \in \mathcal{R}_{\mathcal{T}}$ if $C_1 \sqcap \ldots \sqcap C_n \sqsubseteq \exists R . (D \sqcap E) \in \Omega(\mathcal{T})$, and

$C(x) \wedge C_1(x) \wedge \ldots \wedge C_n(x) \wedge R(x, y) \wedge D(y) \rightarrow S(x, y) \in \mathcal{R}_{\mathcal{T}}$ if $C_1 \sqcap \ldots \sqcap C_n \sqsubseteq \exists (R \sqcap S) . D \in \Omega(\mathcal{T})$

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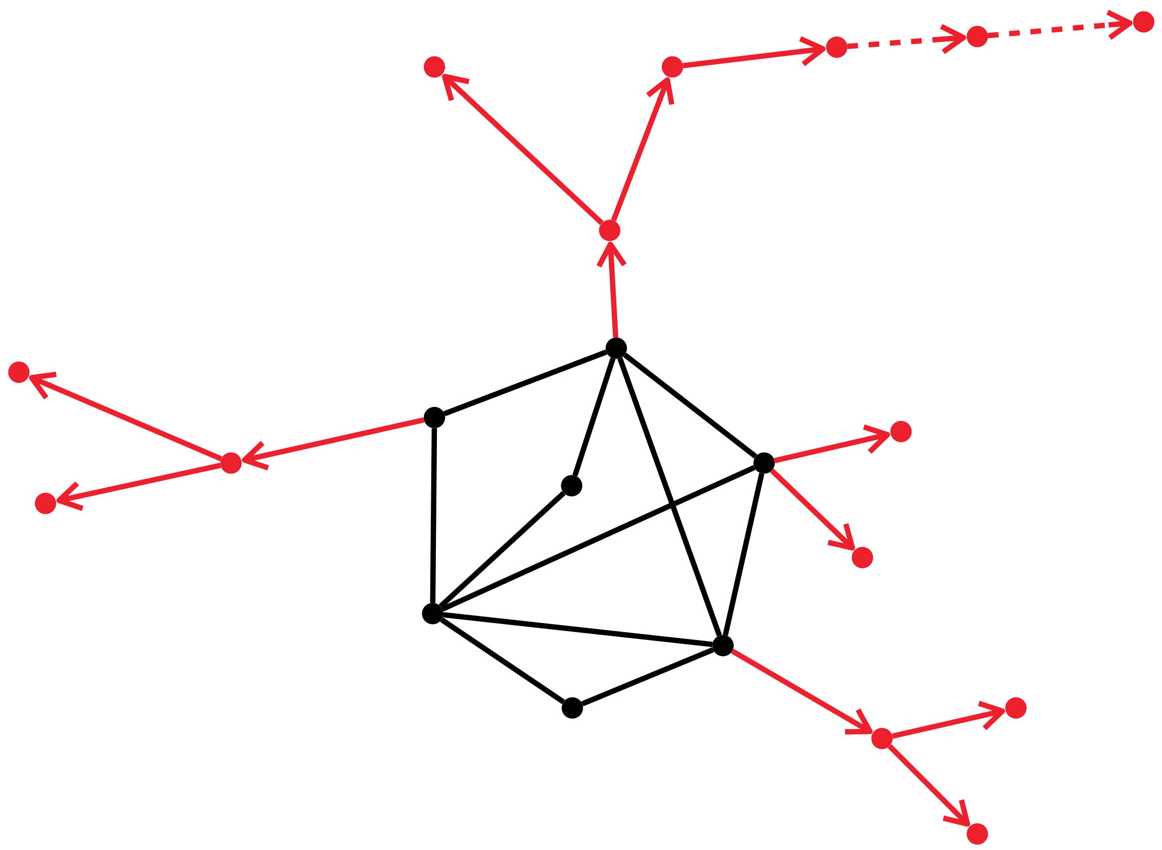
From Horn-SRIQ to Datalog

Tree Model Property



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$C \sqsubseteq \leq 1 R . D$

$R_1 \circ \ldots \circ R_n \sqsubseteq S$

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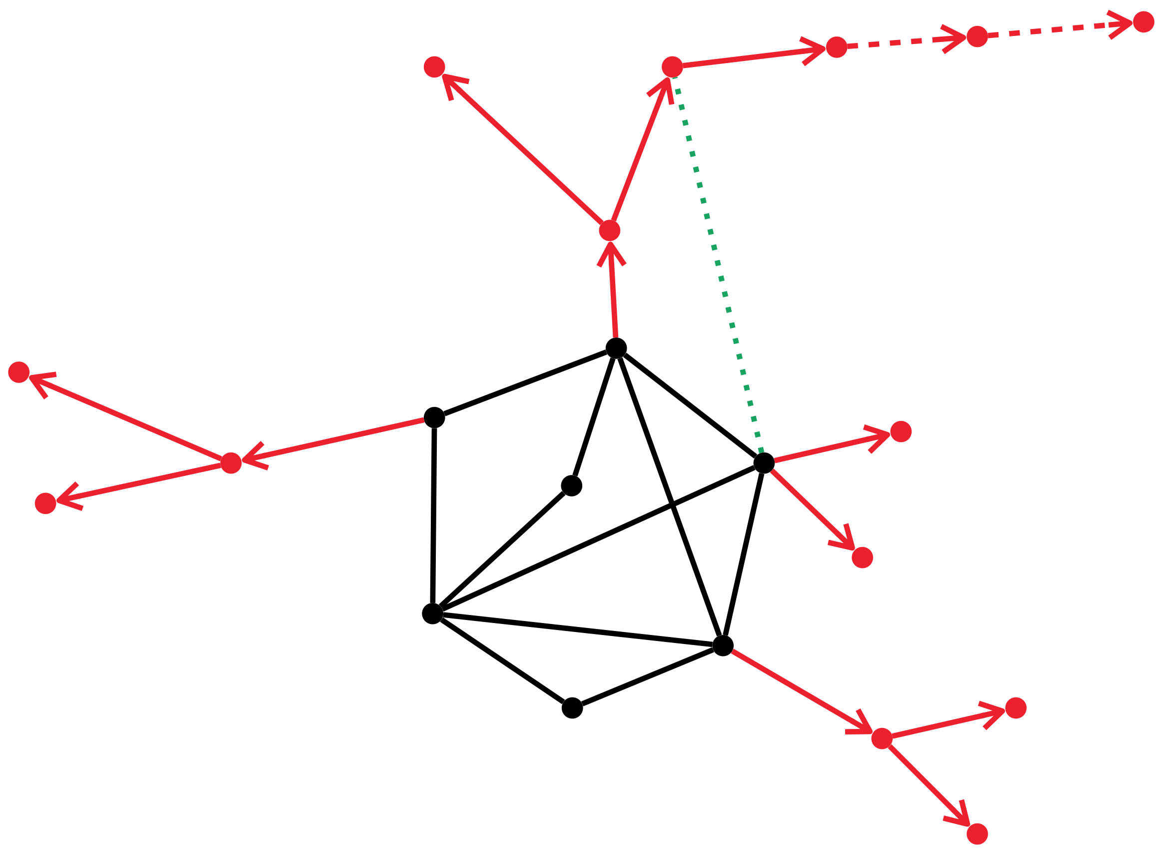

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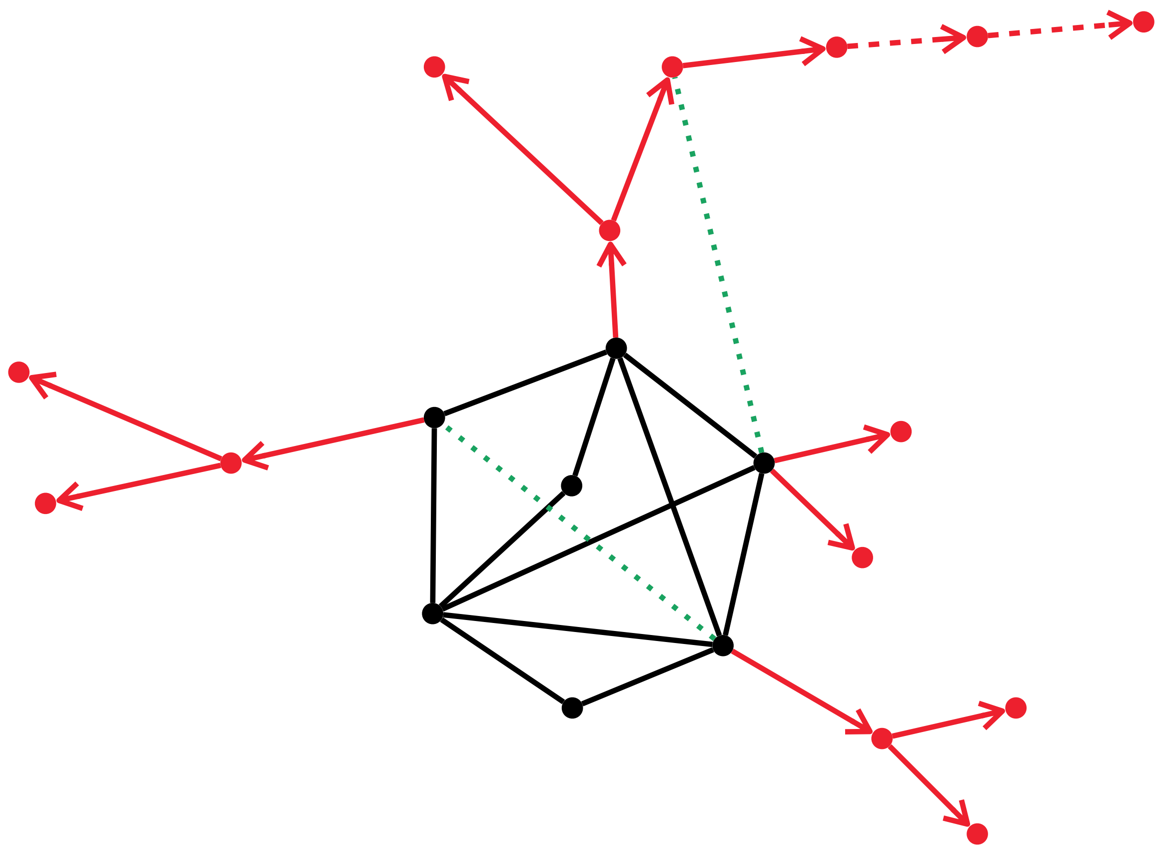

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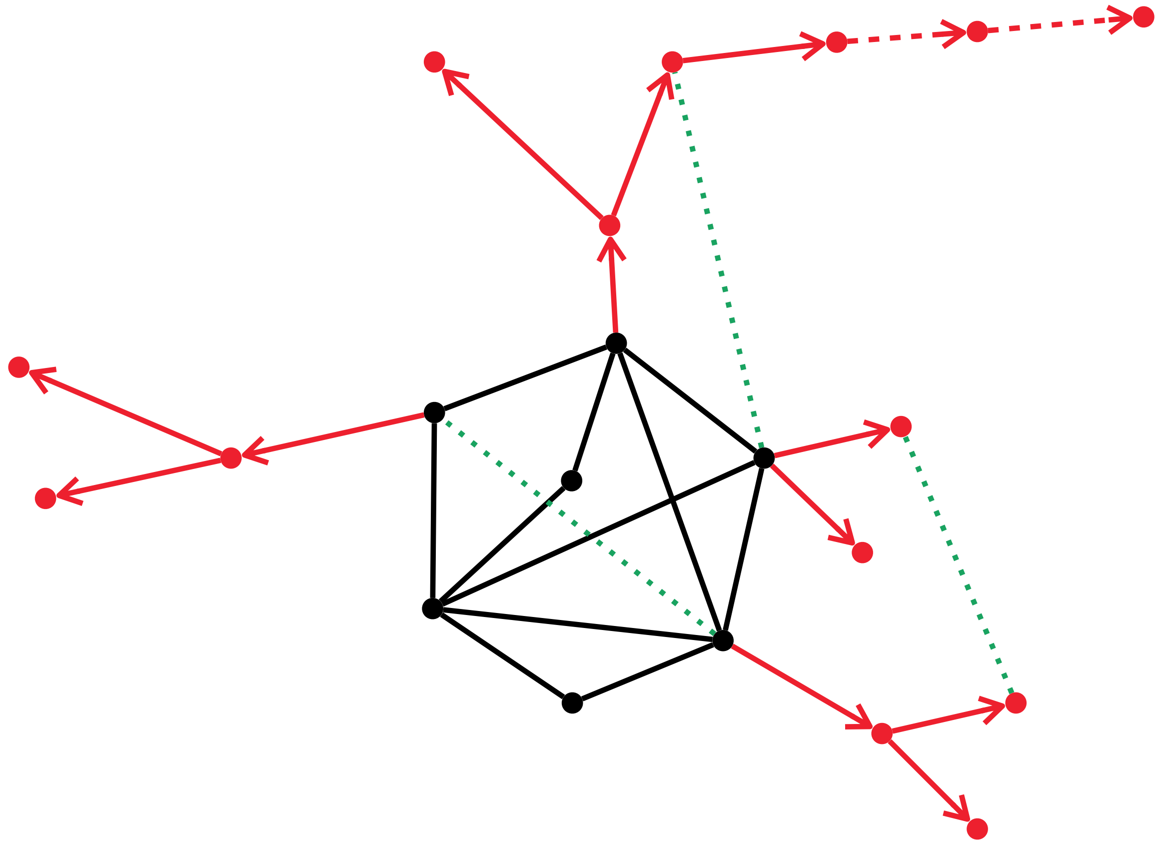

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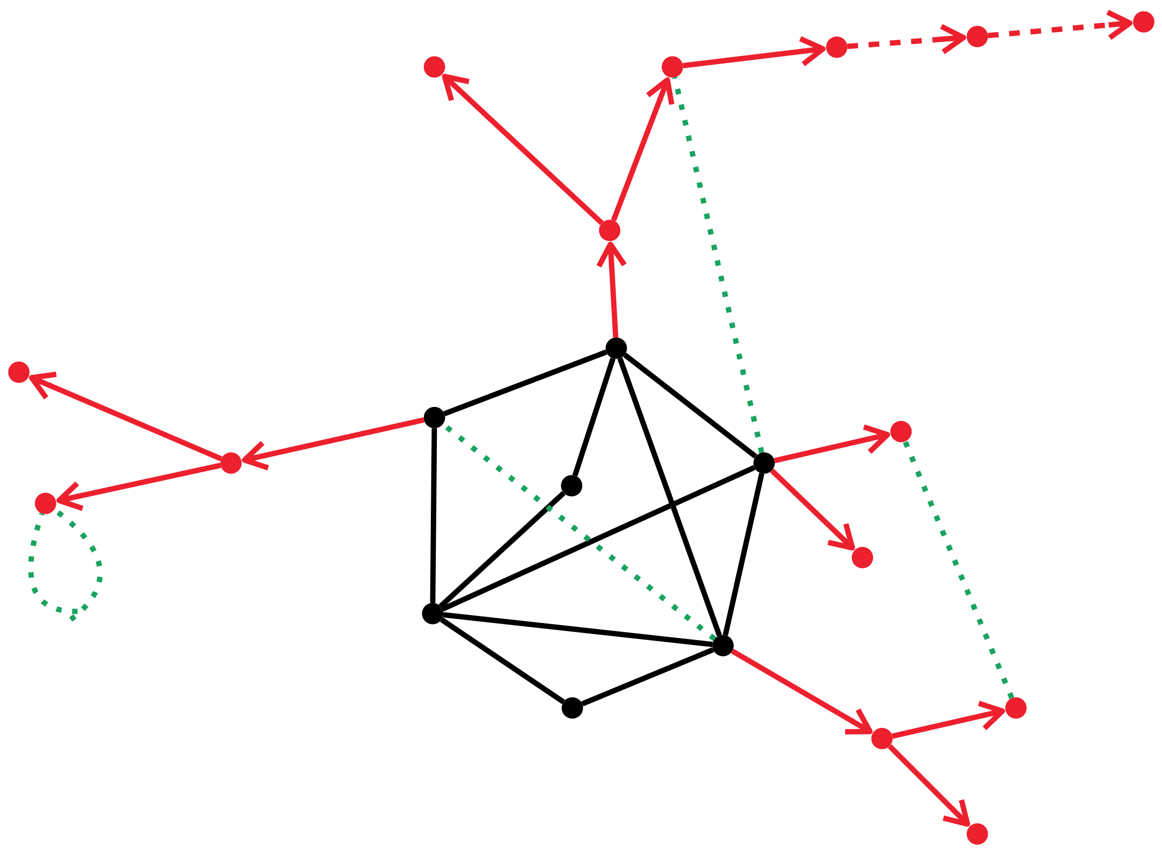

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Complex Roles and NFA

$$\mathcal{T} = \{R \circ S \circ T \sqsubseteq R, \quad V \circ X \circ Y \sqsubseteq R, \quad W \sqsubseteq X, \quad R \circ R \sqsubseteq R\}$$

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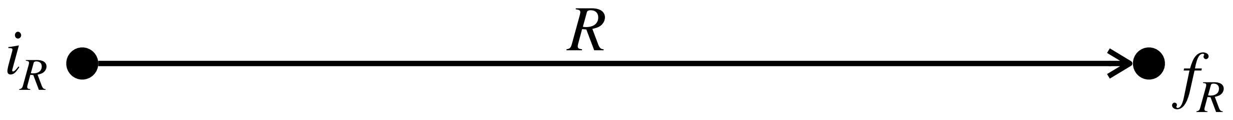

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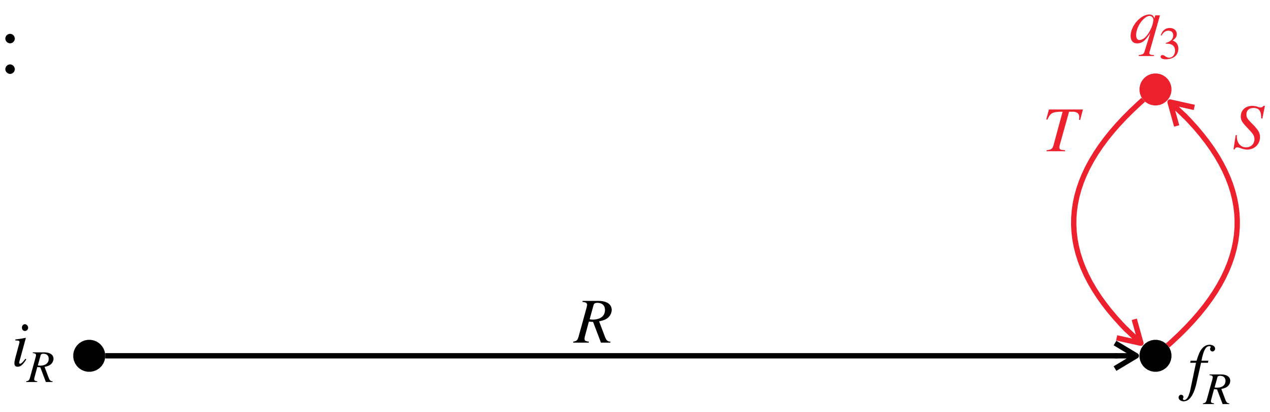

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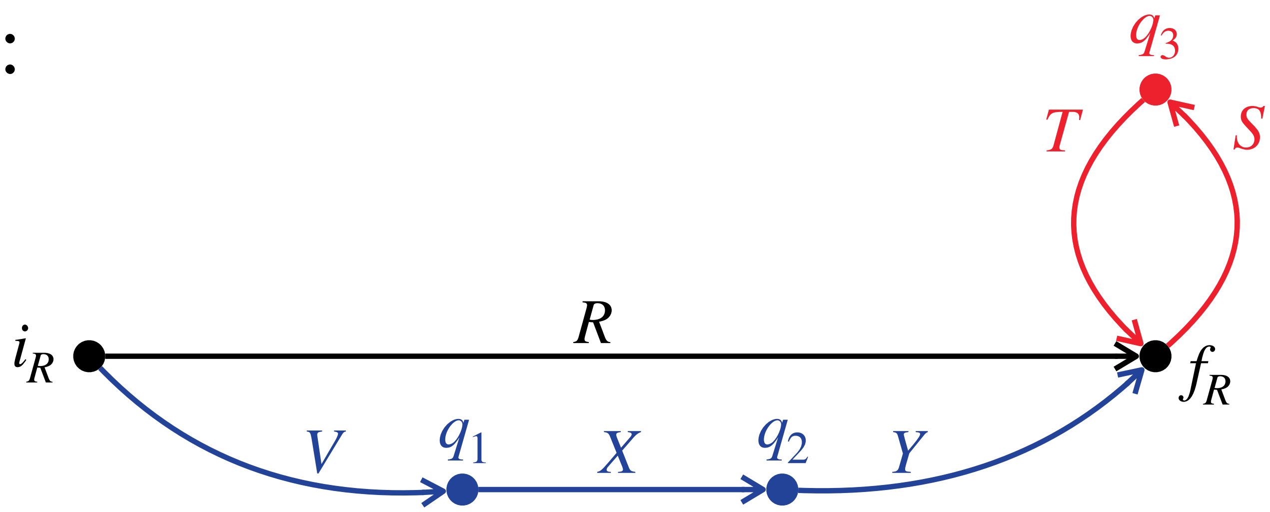

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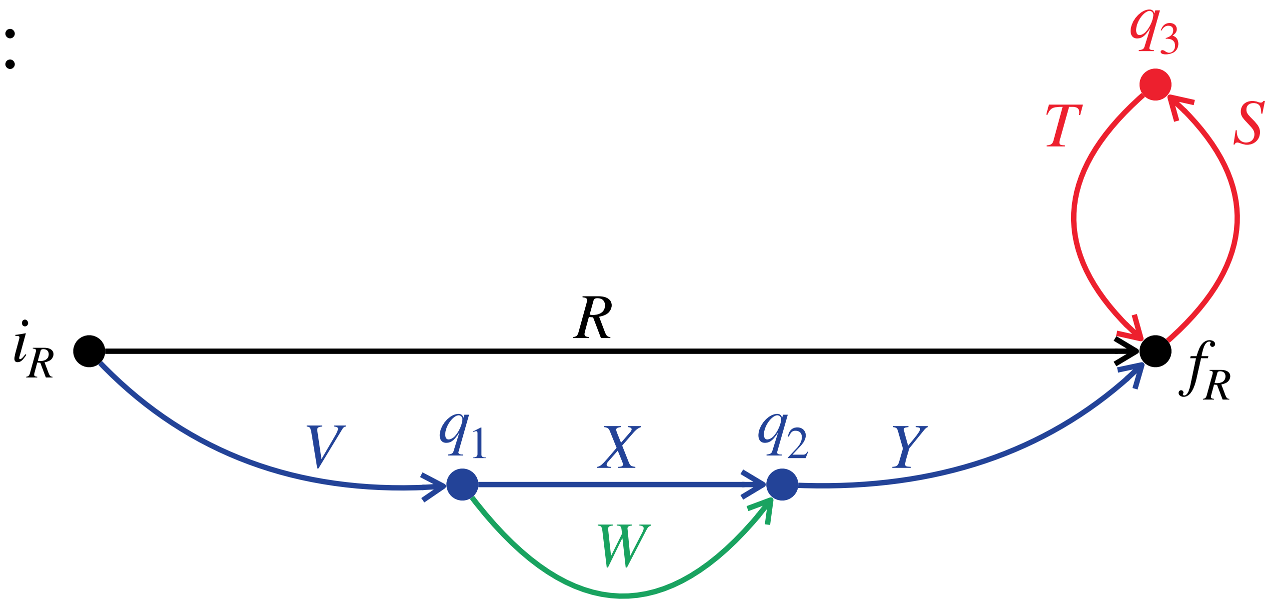

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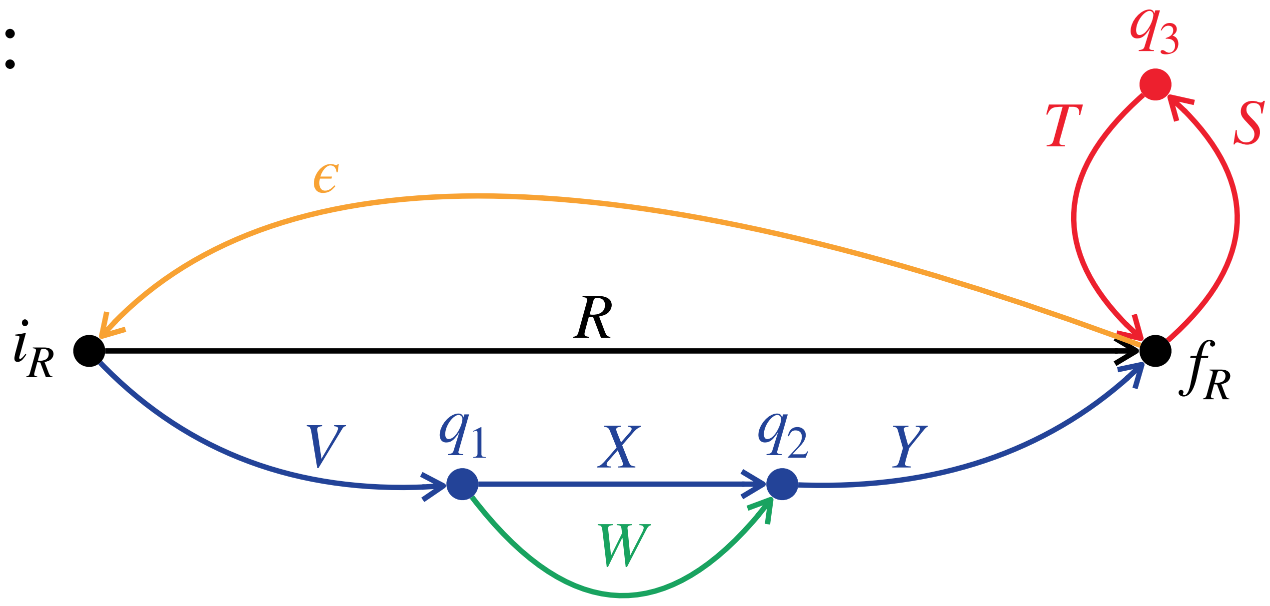
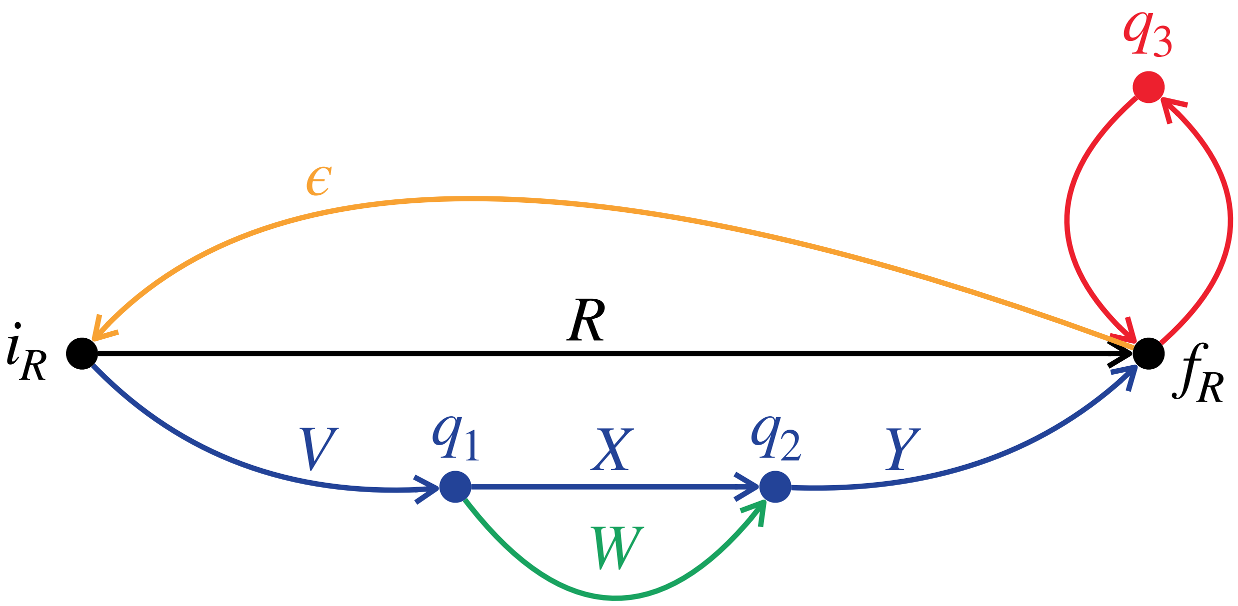


Box Pushing

$$A \sqsubseteq \forall R \,.\, B \in \mathcal{T}$$



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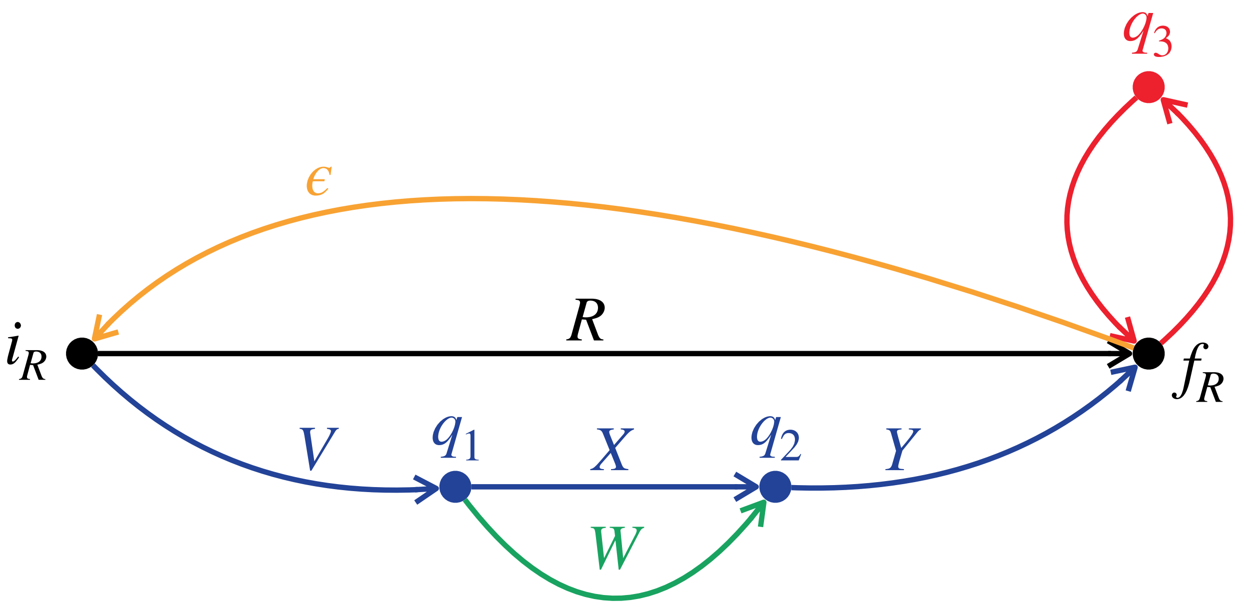

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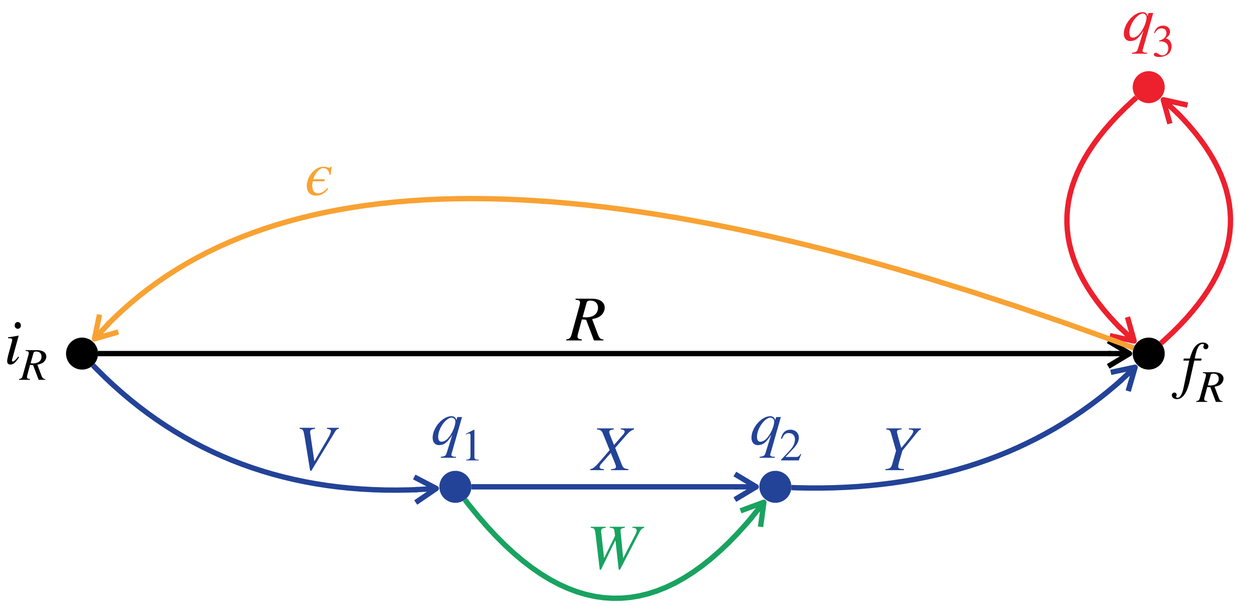

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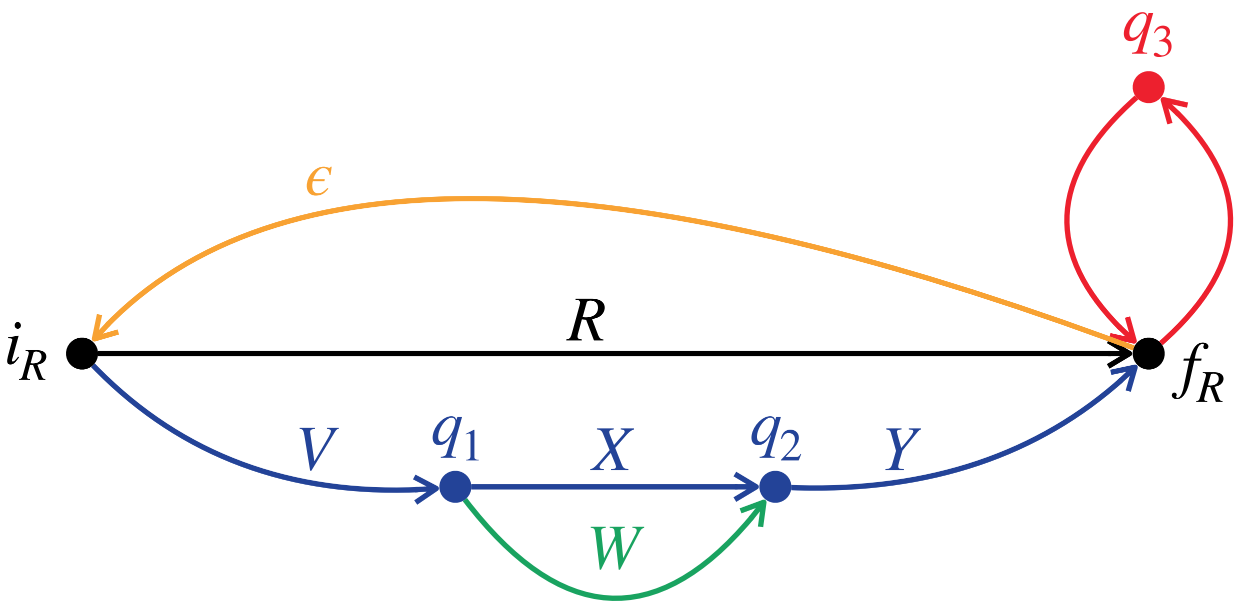

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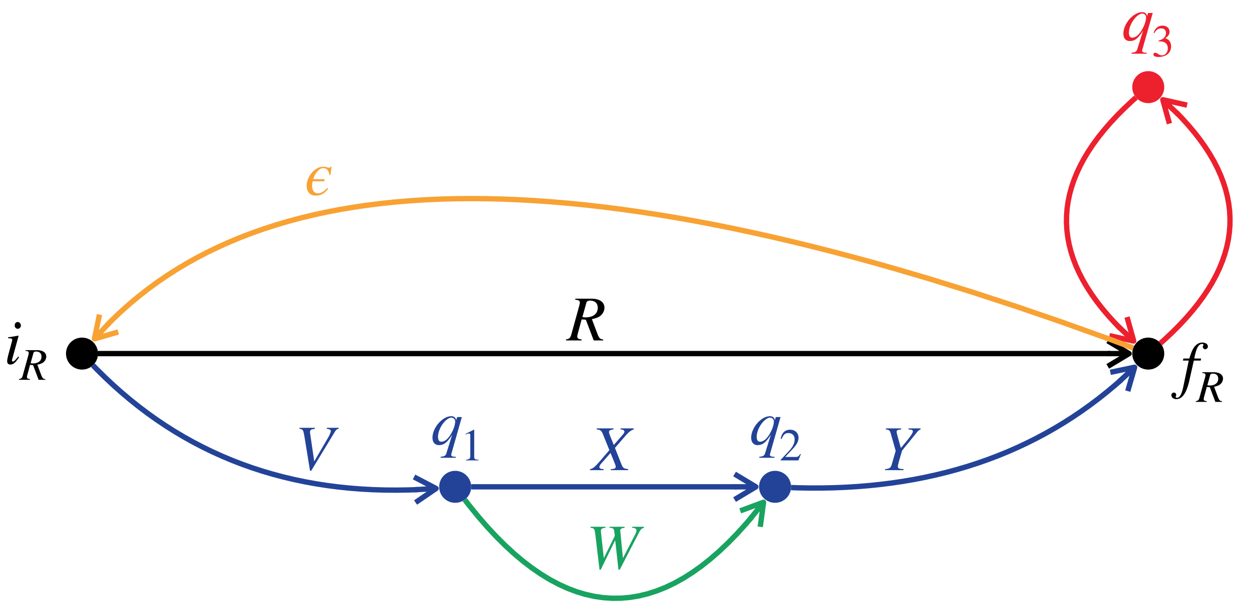

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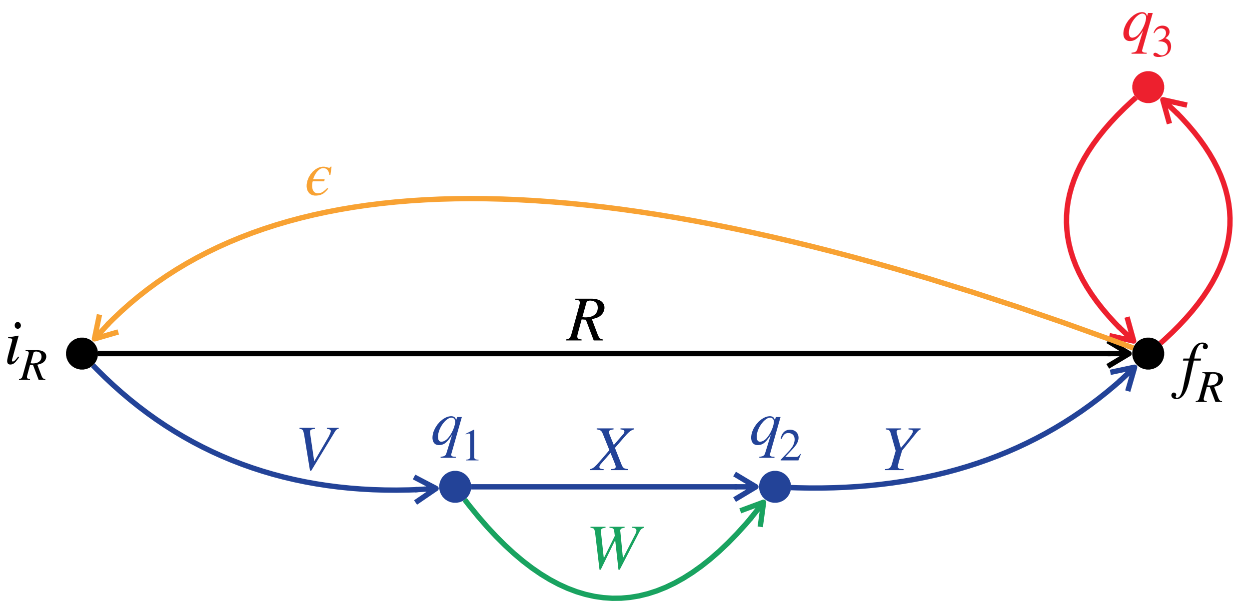

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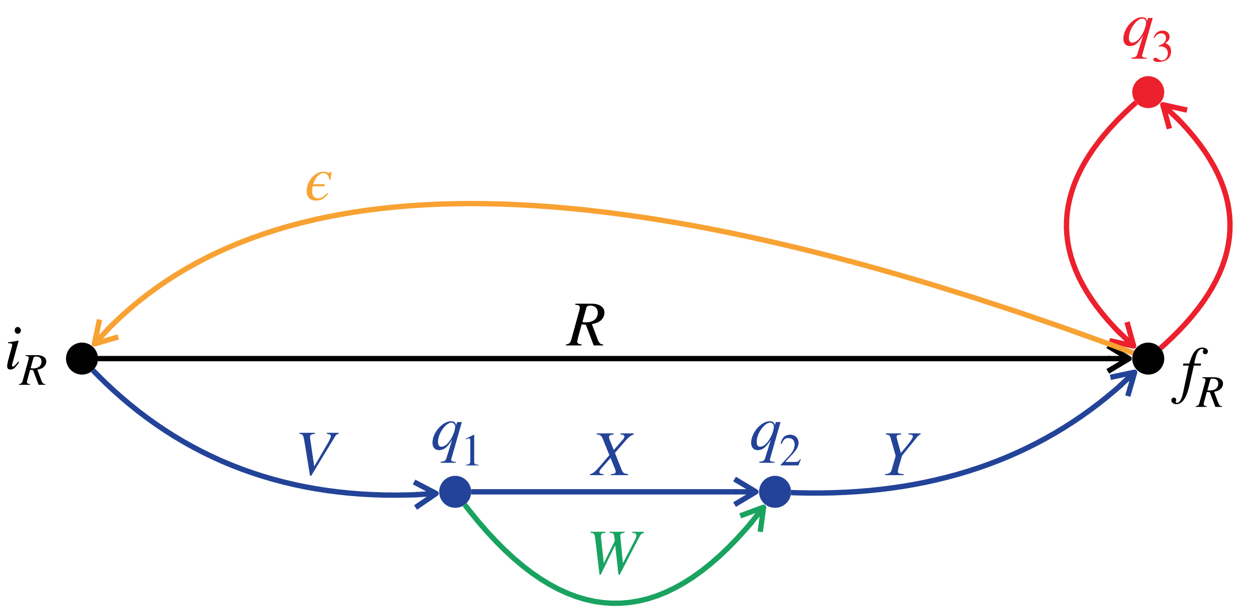

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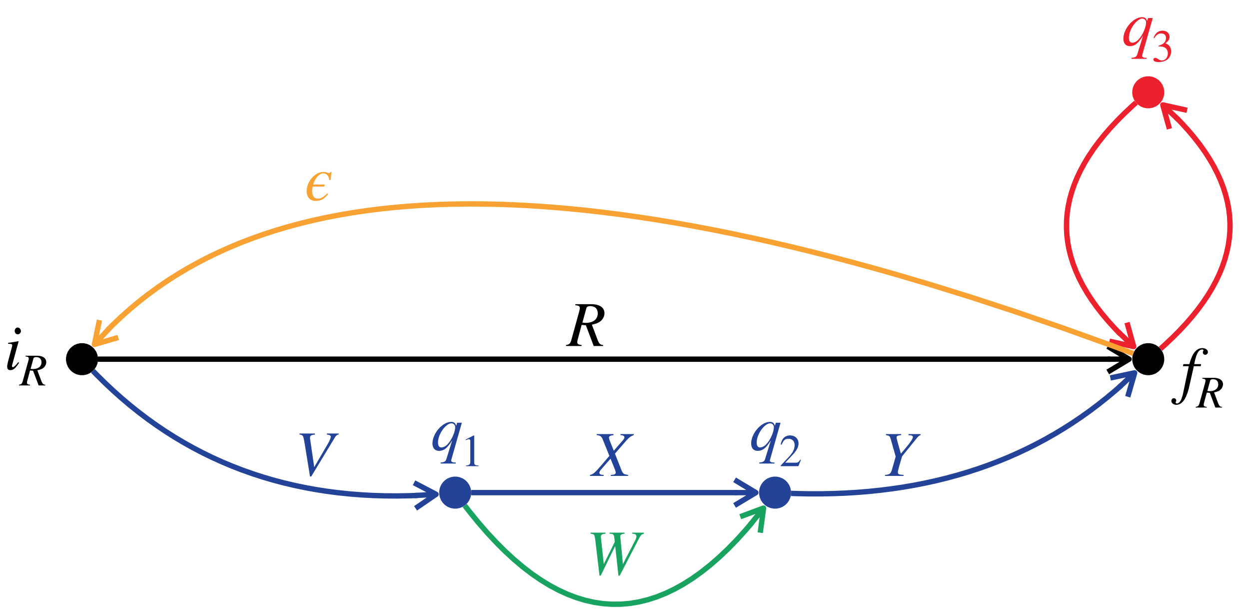

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$$\mathsf{ResearchGroup} \sqsubseteq \forall \mathsf{HasMember}.\mathsf{Researcher}$$
$$\mathsf{Researcher} \sqsubseteq \exists \mathsf{HasMember}^{-}.\mathsf{ResearchGroup}$$
$$\mathsf{Collaborates} \circ \mathsf{HasMember}^{-} \circ \mathsf{HasMember} \sqsubseteq \mathsf{HasConflict}$$
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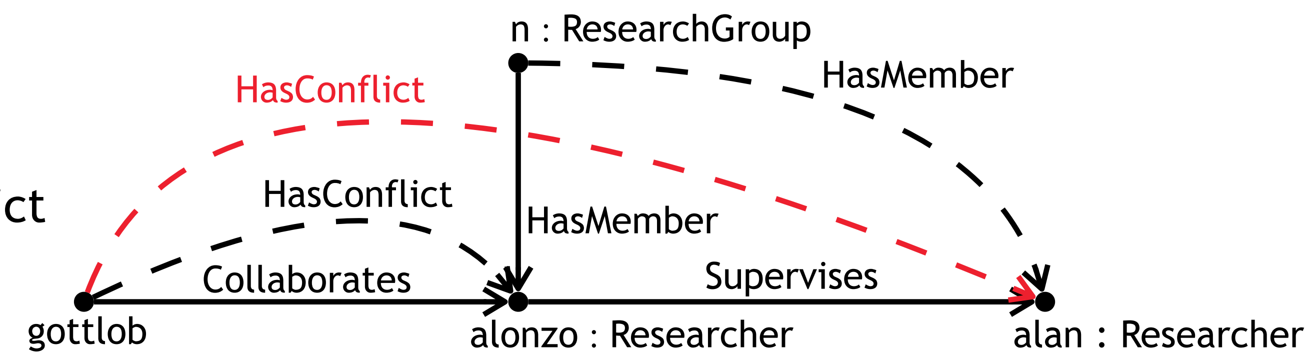

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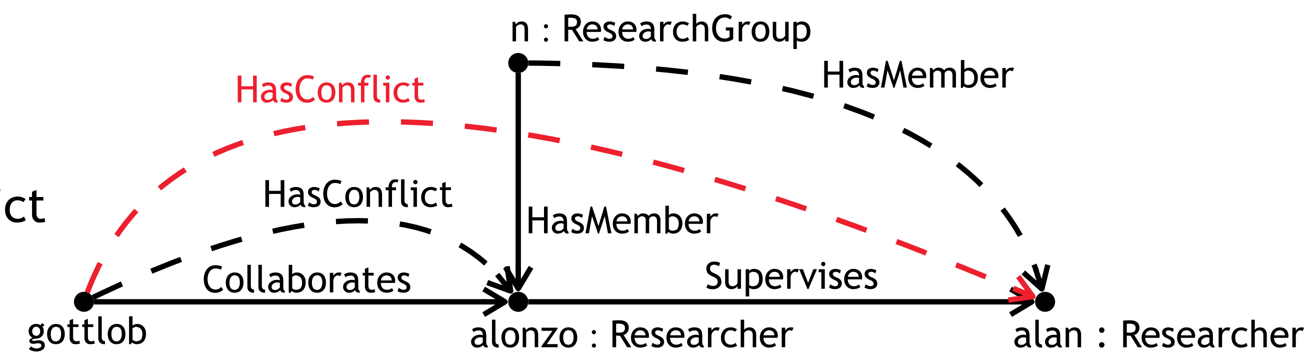
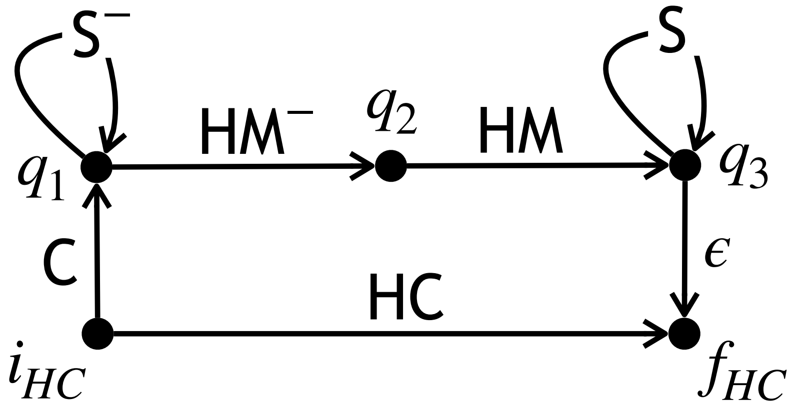
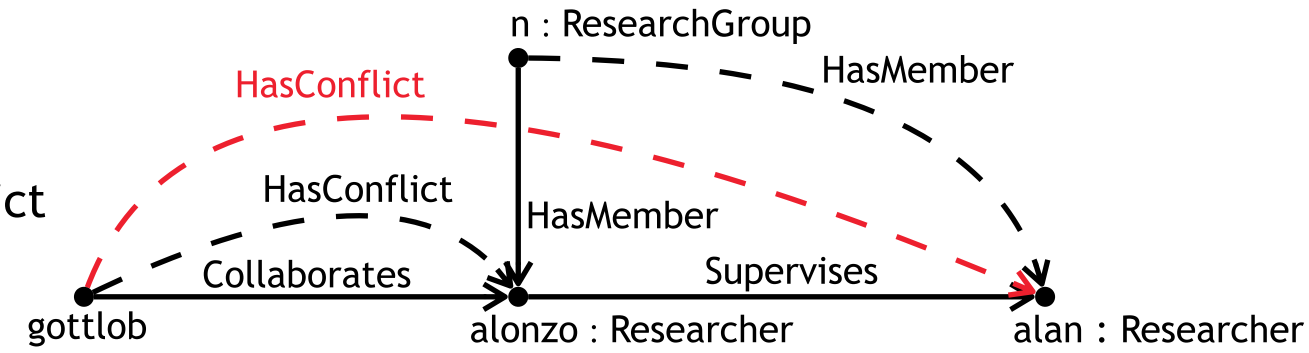
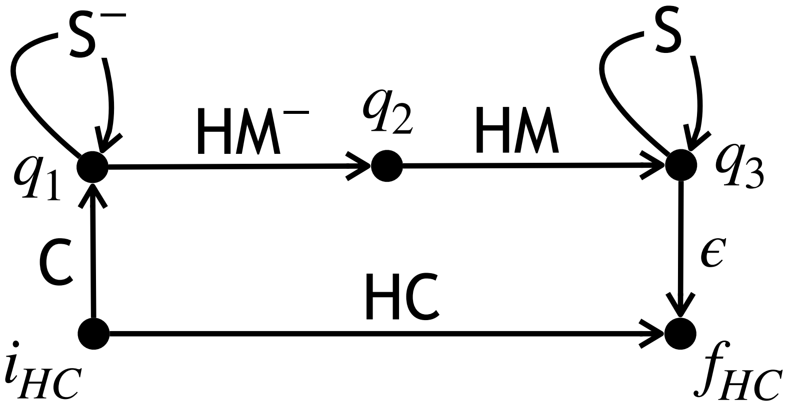




ResearchGroup ⊑ ∀HasMember . Researcher
Researcher ⊑ ∃HasMember⁻ . ResearchGroup
Collaborates ∘ HasMember⁻ ∘ HasMember ⊑ HasConflict
HasMember ∘ Supervises ⊑ HasMember





$$\text{Researcher}(x) \rightarrow \text{HasConflict}_{q_1,q_3}(x)$$
$$\text{Collaborates}(x, y) \rightarrow \text{HasConflict}_{q_1}(x, y)$$
$$\text{HasConflict}_{q_1}(x, y) \wedge \text{HasConflict}_{q_1,q_3}(y) \rightarrow \text{HasConflict}_{q_3}(x, y)$$
$$\text{HasConflict}_{q_3}(x, y) \wedge \text{Supervises}(y, z) \rightarrow \text{HasConflict}_{f_{HC}}(x, y)$$
$$\text{HasConflict}_{f_R}(x, y) \rightarrow \text{HasConflict}(x, y)$$

ResearchGroup $\sqsubseteq \forall$HasMember . Researcher
Researcher $\sqsubseteq \exists$HasMember$^-$. ResearchGroup
Collaborates $\circ$ HasMember$^-$ $\circ$ HasMember $\sqsubseteq$ HasConflict
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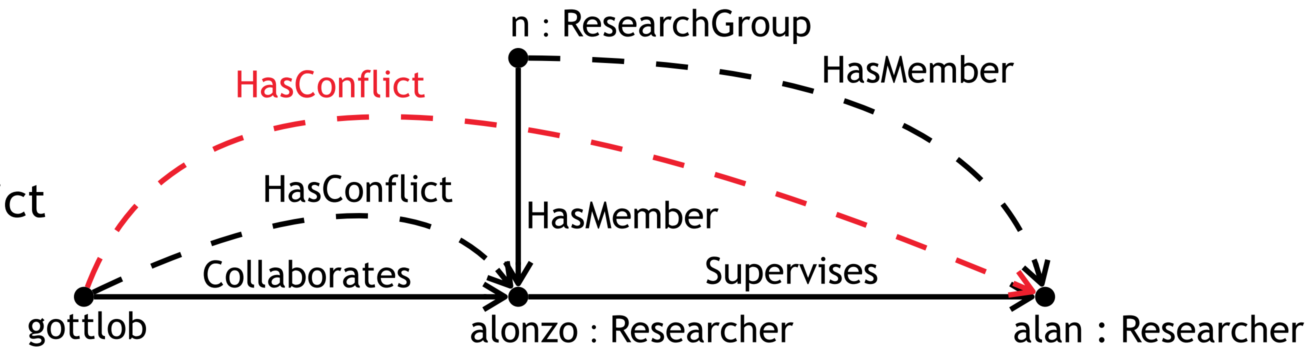

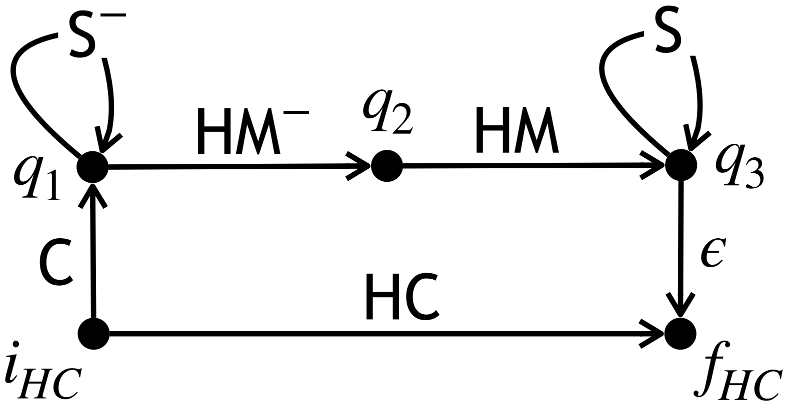

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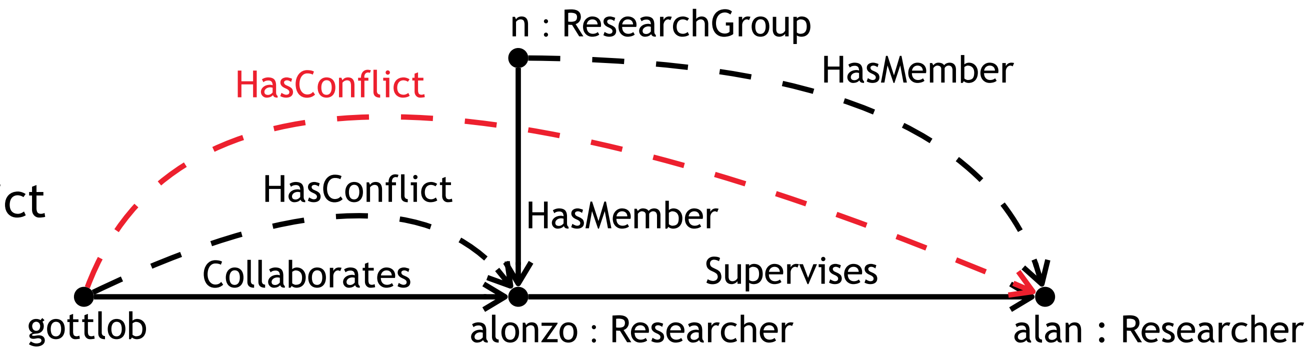

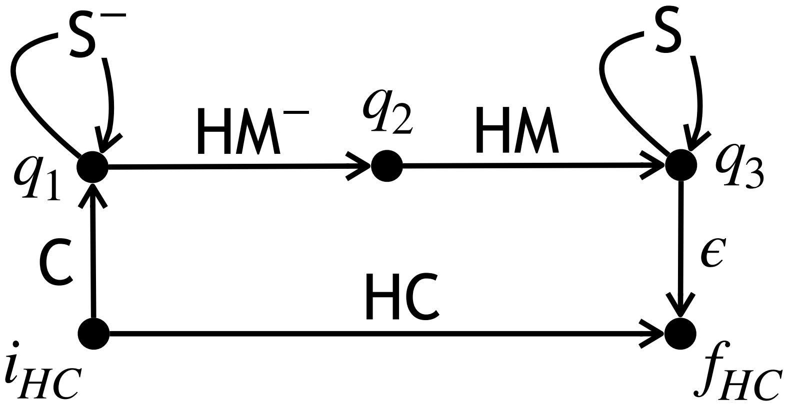

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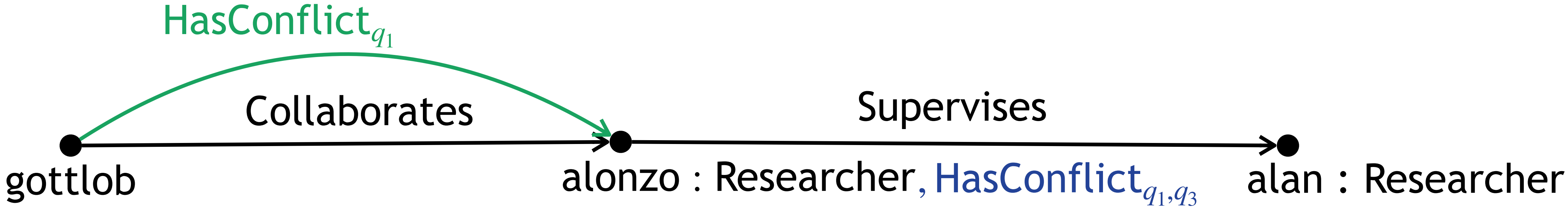

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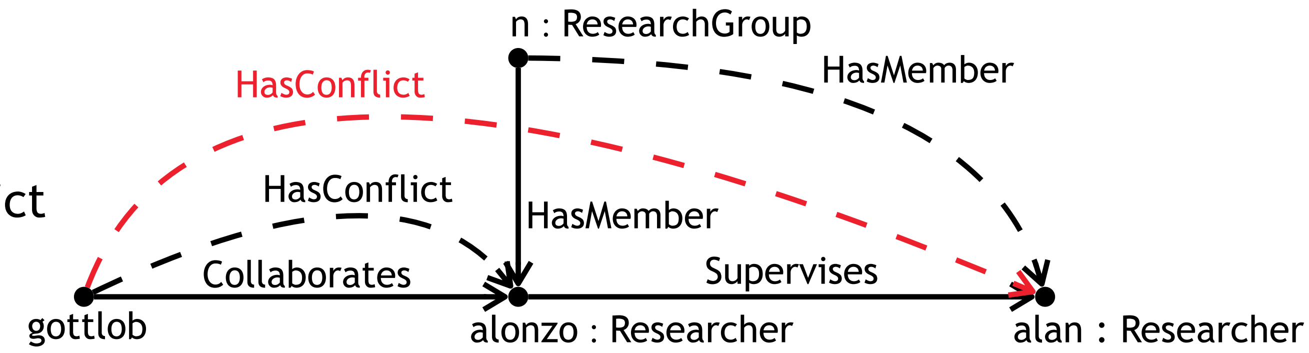

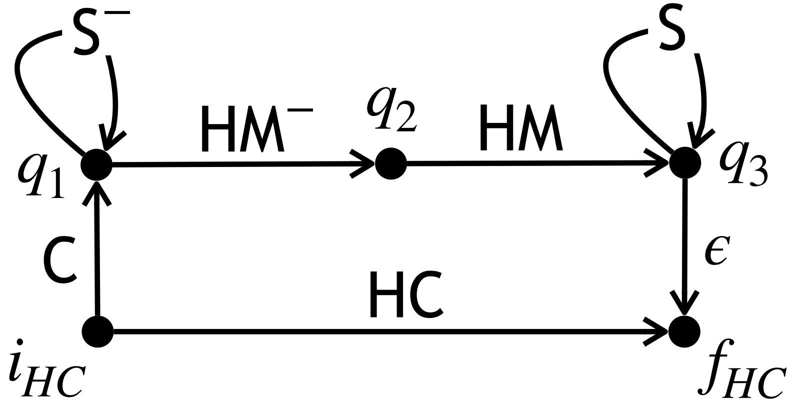

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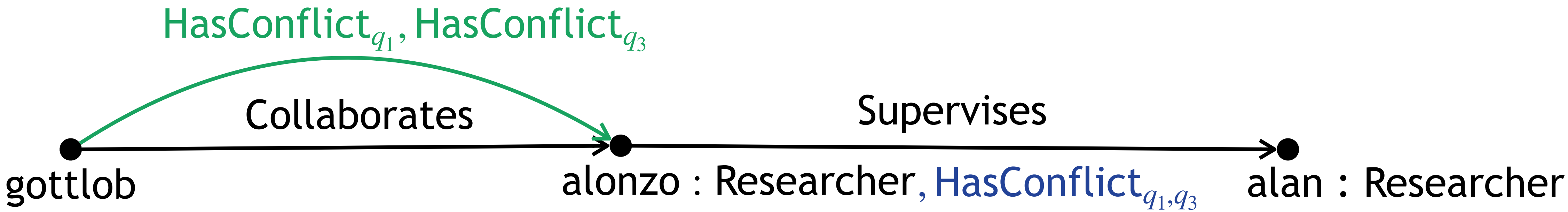

$\mathsf{ResearchGroup} \sqsubseteq \forall \mathsf{HasMember} . \mathsf{Researcher}$
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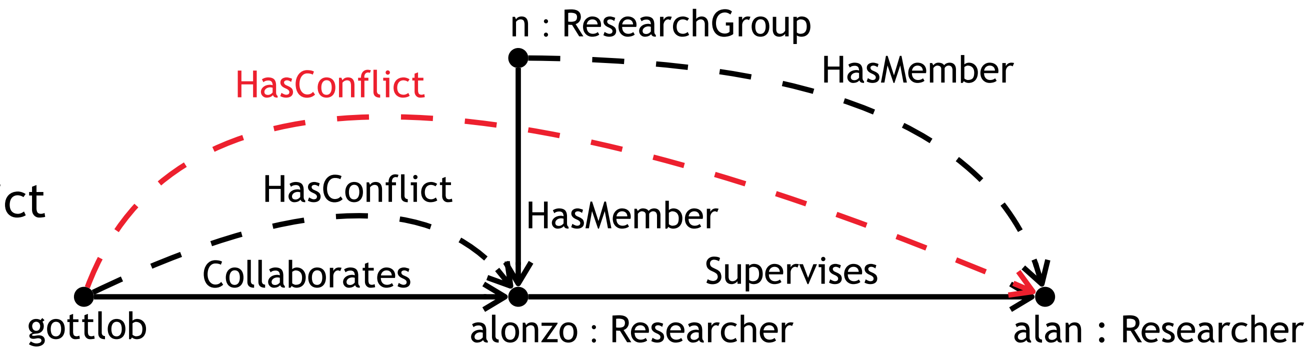

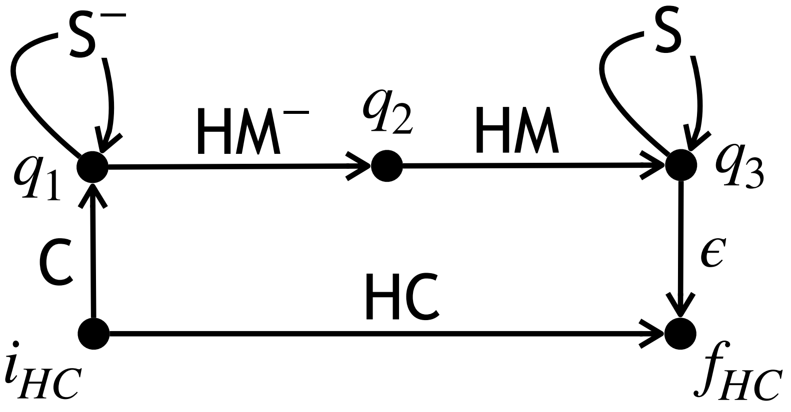

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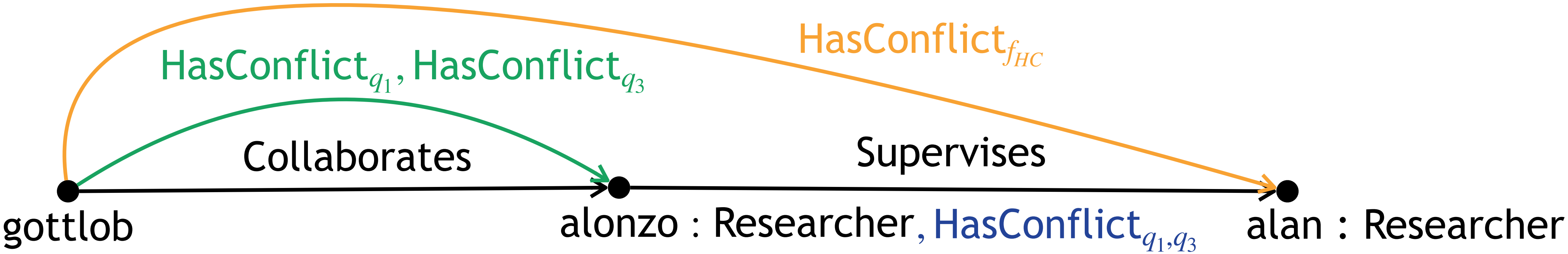

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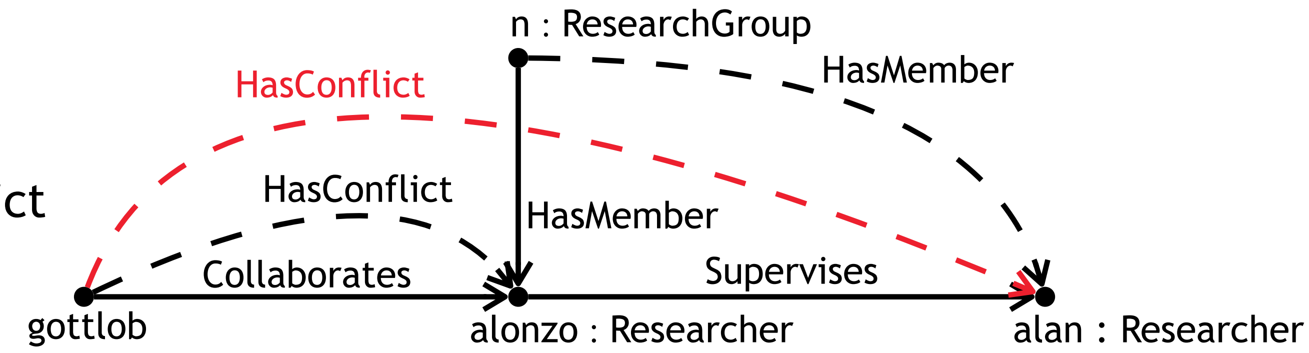

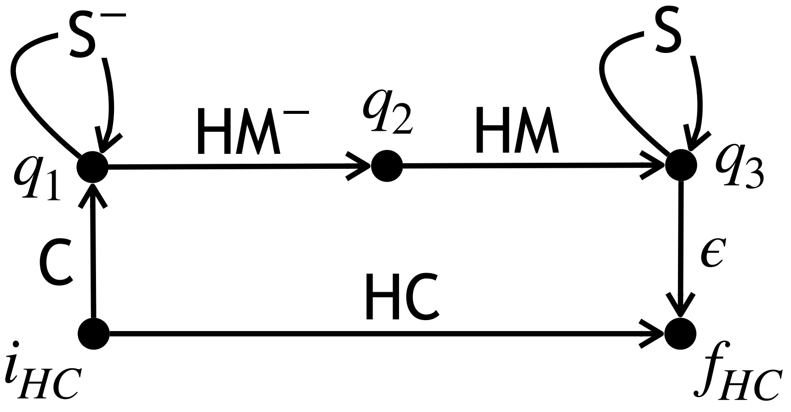

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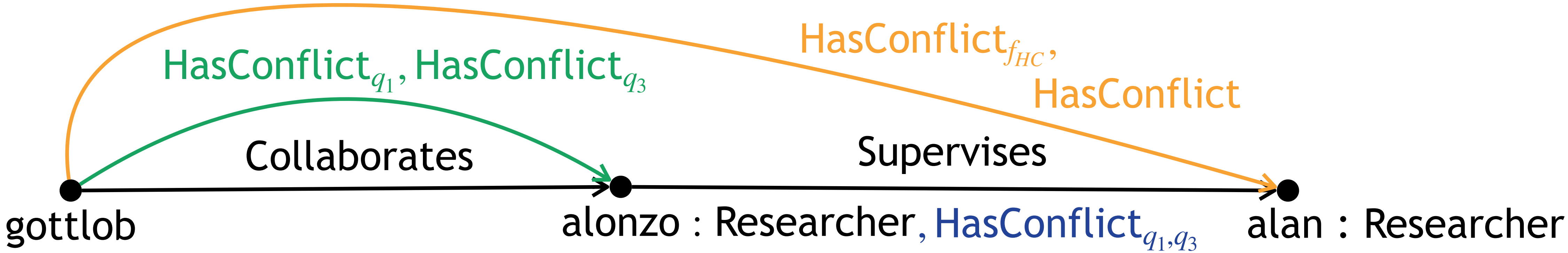
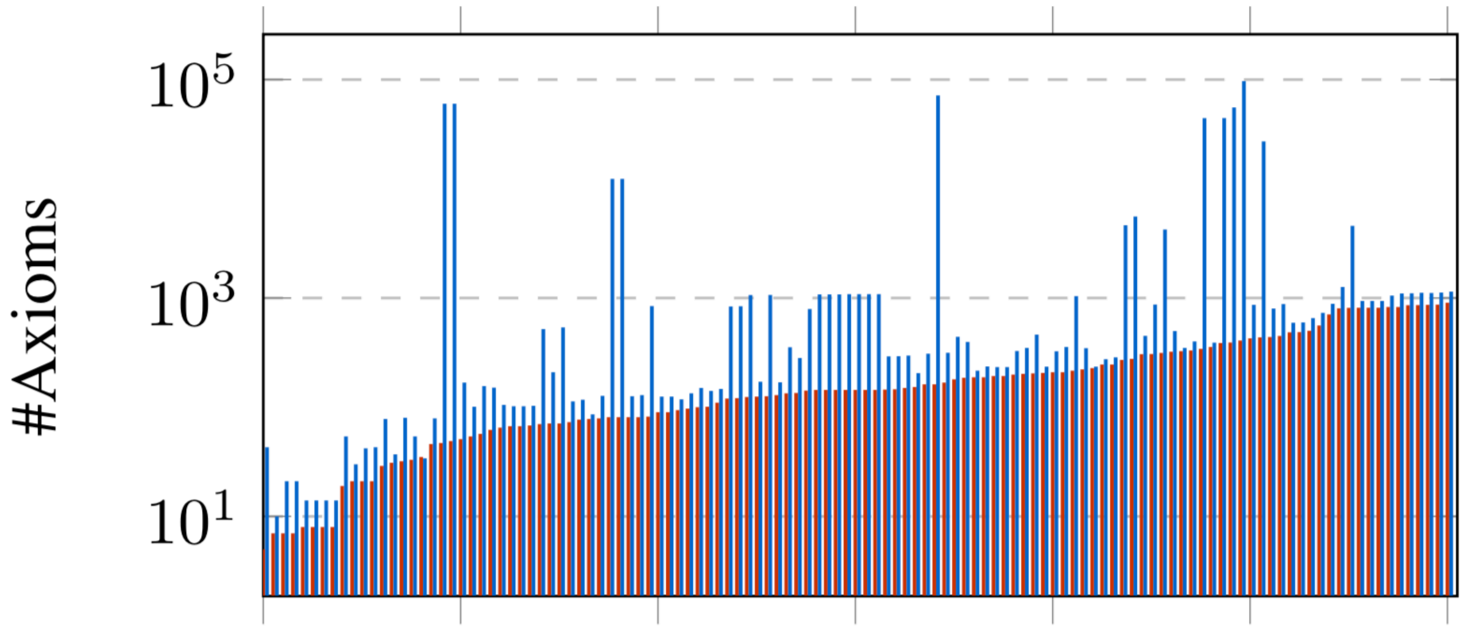


Evaluation

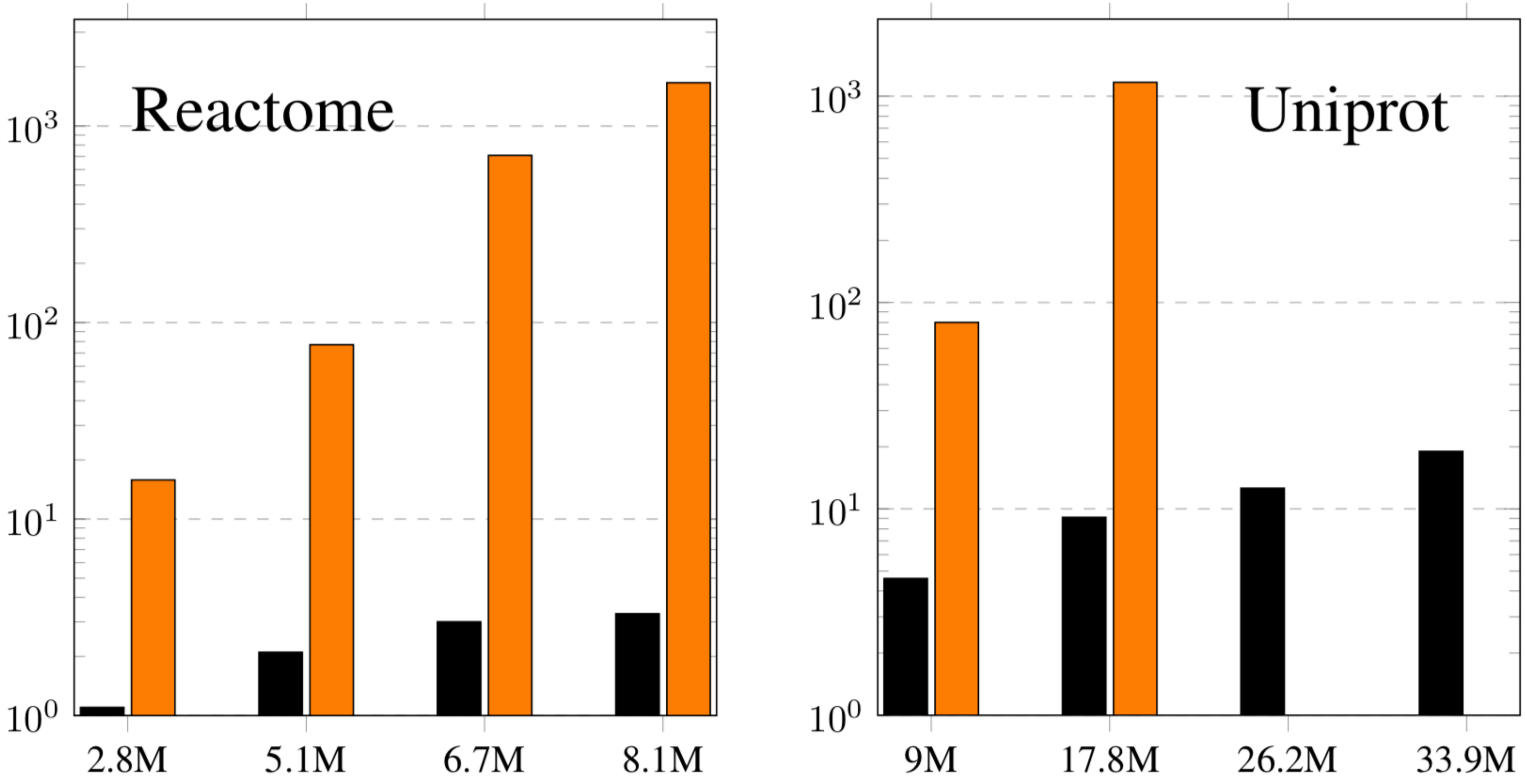
Size of Rewritings

- 187 ontologies from the MOWL Corpus

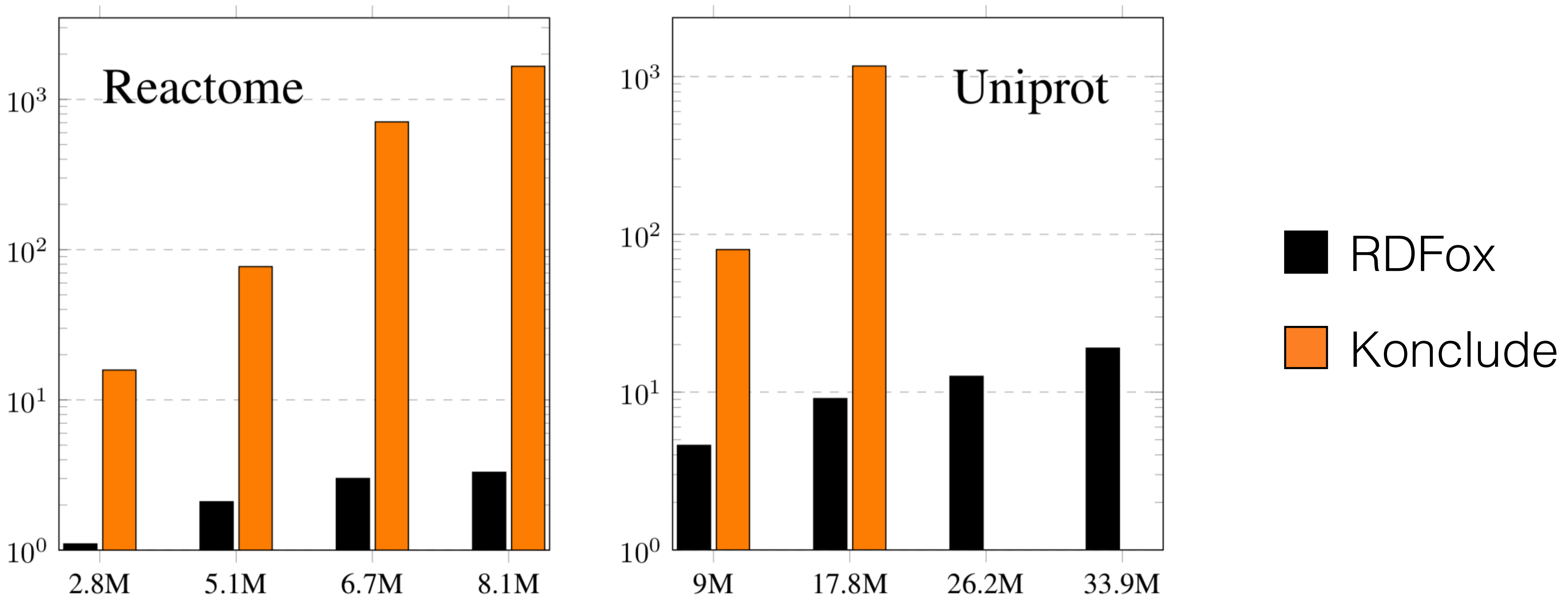
- 121 computed rewritings



Rewritings Performance



Rewritings Performance



From Horn-$\mathcal{SRIQ}$ to Datalog: A Data-Independent Transformation that Preserves Assertion Entailment

David Carral, Larry González, Patrick Koopmann

TECHNISCHE UNIVERSITÄT DRESDEN