# Syllogistic Reasoning under the Weak Completion Semantics

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# Syllogisms

All b are a
All b are c

What follows?

ALL B ARE A
ALL B ARE C

What follows?

| | | | | |
|---|---|---|---|---|
| All A are C | No A are C | Some A are C | Some A are not C | |
| All C are A | No C are A | Some C are A | Some C are not A | NVC |

# Reasoning Towards An Appropriate Logical Form

| Mood | NL | FOL | Short |
|---|---|---|---|
| Affirmative universal (A) | All *a* are *b*. | $\forall X(a(X) \rightarrow b(X))$ | A*ab* |
| Affirmative existential (I) | Some *a* are *b*. | $\exists X(a(X) \wedge b(X))$ | I*ab* |
| Negative universal (E) | No *a* are *b*. | $\forall X(a(X) \rightarrow \neg b(X))$ | E*ab* |
| Negative existential (O) | Some *a* are not *b*. | $\exists X(a(X) \wedge \neg b(X))$ | O*ab* |

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| | Figure 1 | Figure 2 | Figure 3 | Figure 4 |
|---|---|---|---|---|
| First Premise | a-b | b-a | a-b | b-a |
| Second Premise | b-c | c-b | c-b | b-c |

# Reasoning Towards An Appropriate Logical Form

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| | Figure 1 | Figure 2 | Figure 3 | Figure 4 |
|---|---|---|---|---|
| First Premise | a-b | b-a | a-b | b-a |
| Second Premise | b-c | c-b | c-b | b-c |

- There are 64 different pairs of premises and 512 different pairs of syllogisms.
- A problem can be completely specified by the quantifiers of the first and second premise and the figure. The example just discussed is denoted by AA4.

# Modeling Syllogisms

We model the Weak Completion Semantics to syllogisms and follow four principles:

1. Licenses for inferences
2. Existential Import and Gricean Implicature
3. Negation by Transformation
4. Unknown Generalization

# Licenses for Inferences

According to Stenning and van Lambalgen [2008], conditionals should be formalized by licenses for inferences:

$$p(X) \quad \leftarrow \quad q(X).$$

becomes

$$\begin{array}{lcl} p(X) & \leftarrow & q(X) \wedge \neg ab(X). \\ ab(X) & \leftarrow & \bot. \end{array}$$

# Existential Import/ Gricean Implicature

- Humans normally do not quantify over things that do not exist.
- Consequently, *for all* implies *there exists*.
- Humans require existential import for a conditional to be true.

# Negation by Transformation

- Logic programs do not allow negative literals as heads of clauses.
- Replace a negative conclusion $\neg p(X)$ by $p'(X)$ and add

$$\begin{array}{lll} p(X) & \leftarrow & \neg p'(X). \\ \mathrm{U} & \leftarrow & p(X) \wedge p'(X). \end{array}$$

where the second clause represents an integrity constraint.

# Negation by Transformation

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p(X) & \leftarrow & \neg p'(X). \\
\mathrm{U} & \leftarrow & p(X) \wedge p'(X).
\end{array}
$$

  where the second clause represents an integrity constraint.
- Combined with the principle of licenses for inferences, we obtain

$$
\begin{array}{lcl}
p(X) & \leftarrow & \neg p'(X) \wedge \neg ab(X). \\
ab(X) & \leftarrow & \bot. \\
\mathrm{U} & \leftarrow & p(X) \wedge p'(X).
\end{array}
$$

# Unknown Generalization

- Humans seem to distinguish between some a are b and some b are a.
- But in FOL, $\exists X(a(X) \wedge b(X)) \equiv \exists X(b(X) \wedge a(X))$.
- Humans seem to distinguish between some a are b and all a are b.
- If we learn that some a are b, then
  - there must be an object $o_1$ belonging to $a$ and $b$ (Gricean Implicature),
  - there must be another object $o_2$ belonging to $a$ and for which

    it is unknown whether it belongs to $b$ (Unknown Generalization).

# All $y$ are $z$

'*All y are z*' is represented by the program $\mathcal{P}_{Ayz}$ which consists of the following clauses:

$$
\begin{array}{lcl}
z(X) & \leftarrow & y(X) \wedge \neg ab_{yz}(X). \\
ab_{yz}(X) & \leftarrow & \bot. \\
y(o) & \leftarrow & \top.
\end{array}
$$

# All $y$ are $z$

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- The first two clauses are obtained by the principle of licenses for inferences.

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- The first two clauses are obtained by the principle of licenses for inferences.
- The last clause follows by the principle of Gricean implicature.

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- The first two clauses are obtained by the principle of licenses for inferences.
- The last clause follows by the principle of Gricean implicature.

The least model of the weak completion of $\mathcal{P}_{Ayz}$ is

$$\langle \{y(o), z(o)\}, \{ab_{yz}(o)\} \rangle.$$

# No y are z (1)

'*No y are z*' in FOL can have different logical representations:

$$
\begin{array}{llll}
& \neg\exists X\,(y(X) \wedge z(X)) & & \\
\equiv & \forall X\,\neg(y(X) \wedge z(X)) & \text{by} & \neg\exists X \equiv \forall\neg X, \\
\equiv & \forall X\,(\neg y(X) \vee \neg z(X)) & \text{by} & \neg(A \wedge B) \equiv (\neg A \vee \neg B), \\
\equiv & \forall X\,(\neg z(X) \vee \neg y(X)) & \text{by} & (A \vee B) \equiv (B \vee A), \\
\equiv & \forall X\,(z(X) \rightarrow \neg y(X)) & \text{by} & (\neg A \vee B) \equiv (A \rightarrow B) \\
\equiv & \forall X\,(y(X) \rightarrow \neg z(X)) & \text{by} & A \rightarrow B \equiv \neg A \vee B \\
\equiv & \forall X\,(z(X) \rightarrow \neg y(X)) & \text{by} & A \rightarrow B \equiv \neg B \rightarrow \neg A \text{ and } \neg\neg A \equiv A
\end{array}
$$

# No y are z (2)

$\mathcal{P}_{Eyz}$ consists of the following clauses:

$$\begin{array}{lll} y'(X) & \leftarrow & z(X) \wedge \neg ab_{zny}(X). \\ ab_{zny}(X) & \leftarrow & \bot. \\ y(X) & \leftarrow & \neg y'(X) \wedge \neg ab_{nyy}(X). \\ z(o) & \leftarrow & \top. \\ ab_{nyy}(o) & \leftarrow & \bot. \end{array}$$

In addition we have the following integrity constraint:

$$\mathrm{U} \leftarrow y(X) \wedge y'(X).$$

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- The first two clauses in $\mathcal{P}_{Eyz}$ are obtained by licenses for inferences.

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z(o) & \leftarrow & \top. \\
ab_{nyy}(o) & \leftarrow & \bot.
\end{array}
$$

In addition we have the following integrity constraint:

$$\mathsf{U} \leftarrow y(X) \wedge y'(X).$$

- The first two clauses in $\mathcal{P}_{Eyz}$ are obtained by licenses for inferences.
- The third clause applying the principle of negation by transformation.

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In addition we have the following integrity constraint:

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- The first two clauses in $\mathcal{P}_{Eyz}$ are obtained by licenses for inferences.
- The third clause applying the principle of negation by transformation.
- In addition, this principle enforces the integrity constraint.

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y(X) & \leftarrow & \neg y'(X) \wedge \neg ab_{nyy}(X). \\
z(o) & \leftarrow & \top. \\
ab_{nyy}(o) & \leftarrow & \bot.
\end{array}
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In addition we have the following integrity constraint:

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- The first two clauses in $\mathcal{P}_{Eyz}$ are obtained by licenses for inferences.
- The third clause applying the principle of negation by transformation.
- In addition, this principle enforces the integrity constraint.
- The last two clauses of $\mathcal{P}_{Eyz}$ follows by the principle of Gricean implicature.

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y(X) & \leftarrow & \neg y'(X) \wedge \neg ab_{nyy}(X). \\
z(o) & \leftarrow & \top. \\
ab_{nyy}(o) & \leftarrow & \bot.
\end{array}
$$

In addition we have the following integrity constraint:

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- The first two clauses in $\mathcal{P}_{Eyz}$ are obtained by licenses for inferences.
- The third clause applying the principle of negation by transformation.
- In addition, this principle enforces the integrity constraint.
- The last two clauses of $\mathcal{P}_{Eyz}$ follows by the principle of Gricean implicature.

The least model of the weak completion of $\mathcal{P}_{Eyz}$ is

$$
\langle \{z(o), y'(o)\}, \{ab_{zny}(o), ab_{nyy}(o), y(o)\} \rangle.
$$

# Some y are z

'*Some y are z*' represented by the $\mathcal{P}_{Iyz}$, which consists of the following clauses:

$$
\begin{array}{lcl}
z(X) & \leftarrow & y(X) \wedge \neg ab_{yz}(X). \\
ab_{yz}(o_1) & \leftarrow & \bot. \\
y(o_1) & \leftarrow & \top. \\
y(o_2) & \leftarrow & \top.
\end{array}
$$

# Some y are z

'*Some y are z*' represented by the $\mathcal{P}_{Iyz}$, which consists of the following clauses:

$$\begin{array}{lcl} z(X) & \leftarrow & y(X) \wedge \neg ab_{yz}(X). \\ ab_{yz}(o_1) & \leftarrow & \bot. \\ y(o_1) & \leftarrow & \top. \\ y(o_2) & \leftarrow & \top. \end{array}$$

- The first two clauses are again obtained by the principle of using licenses for inferences.

# Some y are z

'*Some y are z*' represented by the $\mathcal{P}_{Iyz}$, which consists of the following clauses:

$$\begin{array}{lcl} z(X) & \leftarrow & y(X) \wedge \neg ab_{yz}(X). \\ ab_{yz}(o_1) & \leftarrow & \bot. \\ y(o_1) & \leftarrow & \top. \\ y(o_2) & \leftarrow & \top. \end{array}$$

- The first two clauses are again obtained by the principle of using licenses for inferences.
- The abnormality predicate is restricted to the object $o_1$, which is assumed to exist by the principle of Gricean implicature, represented by the third clause.

# Some y are z

'*Some y are z*' represented by the $\mathcal{P}_{Iyz}$, which consists of the following clauses:

$$\begin{array}{lcl} z(X) & \leftarrow & y(X) \wedge \neg ab_{yz}(X). \\ ab_{yz}(o_1) & \leftarrow & \bot. \\ y(o_1) & \leftarrow & \top. \\ y(o_2) & \leftarrow & \top. \end{array}$$

- The first two clauses are again obtained by the principle of using licenses for inferences.
- The abnormality predicate is restricted to the object $o_1$, which is assumed to exist by the principle of Gricean implicature, represented by the third clause.
- The fourth clause is obtained by the principle of unknown generalization.

# Some y are z

'*Some y are z*' represented by the $\mathcal{P}_{Iyz}$, which consists of the following clauses:

$$\begin{array}{lcl} z(X) & \leftarrow & y(X) \wedge \neg ab_{yz}(X). \\ ab_{yz}(o_1) & \leftarrow & \bot. \\ y(o_1) & \leftarrow & \top. \\ y(o_2) & \leftarrow & \top. \end{array}$$

- The first two clauses are again obtained by the principle of using licenses for inferences.
- The abnormality predicate is restricted to the object $o_1$, which is assumed to exist by the principle of Gricean implicature, represented by the third clause.
- The fourth clause is obtained by the principle of unknown generalization.

The least model of the weak completion of $\mathcal{P}_{Iyz}$ is

$$\langle \{y(o_1), y(o_2), z(o_1)\}, \{ab_{yz}(o_1)\}\rangle.$$

## Some y are not z

'*Some y are not z*' represented by $\mathcal{P}_{Oyz}$ consists of the following clauses:

$$
\begin{array}{lcl}
z'(X) & \leftarrow & y(X) \wedge \neg ab_{ynz}(X). \\
ab_{ynz}(o_1) & \leftarrow & \bot. \\
z(X) & \leftarrow & \neg z'(X) \wedge \neg ab_{nzz}(X). \\
y(o_1) & \leftarrow & \top. \\
y(o_2) & \leftarrow & \top. \\
ab_{nzz}(o_1) & \leftarrow & \bot. \\
ab_{nzz}(o_2) & \leftarrow & \bot.
\end{array}
$$

In addition, we need the integrity constraint

$$
\mathsf{U} \leftarrow z(X) \wedge z'(X).
$$

# Some y are not z

'*Some y are not z*' represented by $\mathcal{P}_{Oyz}$ consists of the following clauses:

$$
\begin{array}{lcl}
z'(X) & \leftarrow & y(X) \wedge \neg ab_{ynz}(X). \\
ab_{ynz}(o_1) & \leftarrow & \bot. \\
z(X) & \leftarrow & \neg z'(X) \wedge \neg ab_{nzz}(X). \\
y(o_1) & \leftarrow & \top. \\
y(o_2) & \leftarrow & \top. \\
ab_{nzz}(o_1) & \leftarrow & \bot. \\
ab_{nzz}(o_2) & \leftarrow & \bot.
\end{array}
$$

In addition, we need the integrity constraint

$$\mathsf{U} \leftarrow z(X) \wedge z'(X).$$

- The first four clausses and the integrity constraints are derived as in $\mathcal{P}_{Eyz}$.

# Some y are not z

'*Some y are not z*' represented by $\mathcal{P}_{Oyz}$ consists of the following clauses:

$$
\begin{array}{lcl}
z'(X) & \leftarrow & y(X) \wedge \neg ab_{ynz}(X). \\
ab_{ynz}(o_1) & \leftarrow & \bot. \\
z(X) & \leftarrow & \neg z'(X) \wedge \neg ab_{nzz}(X). \\
y(o_1) & \leftarrow & \top. \\
y(o_2) & \leftarrow & \top. \\
ab_{nzz}(o_1) & \leftarrow & \bot. \\
ab_{nzz}(o_2) & \leftarrow & \bot.
\end{array}
$$

In addition, we need the integrity constraint

$$\mathrm{U} \leftarrow z(X) \wedge z'(X).$$

- The first four clausses and the integrity constraints are derived as in $\mathcal{P}_{Eyz}$.
- The fifth clause of $\mathcal{P}_{Oyz}$ is obtained by the principle of unknown generalization.

# Some y are not z

'*Some y are not z*' represented by $\mathcal{P}_{Oyz}$ consists of the following clauses:

$$\begin{array}{lcl} z'(X) & \leftarrow & y(X) \wedge \neg ab_{ynz}(X). \\ ab_{ynz}(o_1) & \leftarrow & \bot. \\ z(X) & \leftarrow & \neg z'(X) \wedge \neg ab_{nzz}(X). \\ y(o_1) & \leftarrow & \top. \\ y(o_2) & \leftarrow & \top. \\ ab_{nzz}(o_1) & \leftarrow & \bot. \\ ab_{nzz}(o_2) & \leftarrow & \bot. \end{array}$$

In addition, we need the integrity constraint

$$\mathsf{U} \leftarrow z(X) \wedge z'(X).$$

- The first four clausses and the integrity constraints are derived as in $\mathcal{P}_{Eyz}$.
- The fifth clause of $\mathcal{P}_{Oyz}$ is obtained by the principle of unknown generalization.

The least model of the weak completion of $\mathcal{P}_{Oyz}$ is

$$\langle \{y(o_1), y(o_2), z'(o_1)\}, \{ab_{ynz}(o_1), ab_{nzz}(o_1), ab_{nzz}(o_2), z(o_1)\} \rangle.$$

# Three Examples

## Syllogism AA4

ALL B ARE A
ALL B ARE C

What follows?

# Syllogism AA4

ALL B ARE A
ALL B ARE C

What follows?

| | | | | |
|---|---|---|---|---|
| All A are C | No A are C | Some A are C | Some A are not C | |
| All C are A | No C are A | Some C are A | Some C are not A | NVC |

# Syllogism AA4

ALL B ARE A
ALL B ARE C

What follows?

| | | | | |
|---|---|---|---|---|
| All A are C | No A are C | Some A are C | Some A are not C | |
| All C are A | No C are A | Some C are A | Some C are not A | NVC |

Valid Conclusions

# Syllogism AA4

ALL B ARE A
ALL B ARE C

What follows?

| | | | | |
|---|---|---|---|---|
| All A are C | No A are C | Some A are C | Some A are not C | |
| All C are A | No C are A | Some C are A | Some C are not A | NVC |

Majority's Conclusions

# Syllogism AA4

$\mathcal{P}_{\text{AA4}}$ consists of the following clauses:

$$
\begin{array}{lcl}
a(X) & \leftarrow & b(X) \wedge \neg ab_{ba}(X) \\
b(o_1) & \leftarrow & \top \\
ab_{ba}(X) & \leftarrow & \bot \\
c(X) & \leftarrow & b(X) \wedge \neg ab_{bc}(X) \\
ab_{bc}(X) & \leftarrow & \bot \\
b(o_2) & \leftarrow & \top
\end{array}
$$

The least model of the weak completion of $\mathcal{P}_{\text{AA4}}$ is

$$
\langle \{b(o_1),\ b(o_2),\ a(o_1),\ a(o_2), c(o_1),\ c(o_2)\}, \\
\{ab_{ba}(o_1),\ ab_{ba}(o_2),\ ab_{bc}(o_1),\ ab_{bc}(o_2)\}\rangle.
$$

- This model entails both '*all a are c*' and '*all c are a*'.
- Analogously this also holds for '*all c are a*'.
- This prediction matches partially with the answers from participants who concluded Aac and NVC.

# Syllogism OA4

Some B are not A
All B are C

What follows?

# Syllogism OA4

Some B are not A
All B are C

What follows?

| | | | | |
|---|---|---|---|---|
| All A are C | No A are C | Some A are C | Some A are not C | |
| All C are A | No C are A | Some C are A | Some C are not A | NVC |

# Syllogism OA4

Some B are not A
All B are C

What follows?

| | | | | |
|---|---|---|---|---|
| All A are C | No A are C | Some A are C | Some A are not C | |
| All C are A | No C are A | Some C are A | Some C are not A | NVC |

Valid Conclusion

# Syllogism OA4

Some B are not A
All B are C

What follows?

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|---|---|---|---|---|
| All A are C | No A are C | Some A are C | Some A are not C | |
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Majority's Conclusions

# Syllogism EA2

No B are A
All C are B

What follows?

# Syllogism EA2

No B are A
All C are B

What follows?

| | | | | |
|---|---|---|---|---|
| All A are C | No A are C | Some A are C | Some A are not C | |
| All C are A | No C are A | Some C are A | Some C are not A | NVC |

# Syllogism EA2

NO B ARE A
ALL C ARE B

What follows?

| | | | | |
|---|---|---|---|---|
| All A are C | No A are C | Some A are C | Some A are not C | |
| All C are A | No C are A | Some C are A | Some C are not A | NVC |

Valid Conclusions

# Syllogism EA2

No B are A
All C are B

What follows?

| | | | | |
|---|---|---|---|---|
| All A are C | No A are C | Some A are C | Some A are not C | |
| All C are A | No C are A | Some C are A | Some C are not A | NVC |

Majority's Conclusions

# References

K. Stenning and M. van Lambalgen. Human Reasoning and Cognitive Science. A Bradford Book. MIT Press, Cambridge, MA, 2008. ISBN 9780262195836.