Master's Thesis Topic:
A Simplified Tableau Calculus for Standpoint LTL

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1 Summary

Linear temporal logic (LTL) is an important modal logic introduced by Pnueli [5] for reasoning about temporal concepts and dynamically evolving systems. It extends propositional logic by incorporating temporal operators that allow statements about the future of a given computation or process. The two primary operators are X ("next") and U ("until"), which enable the specification of properties over linear sequences of states. LTL has been extensively applied in fields such as program verification, model checking, and the formal specification of reactive systems.

Standpoint logic (SL) was recently introduced [4] as a relatively low-cost multi-agent logic with applications in the context of knowledge integration. Within the framework of standpoint logic, propositions may be 'wrapped' within modalities of the form $\Box_s$ and $\Diamond_s$ with s a standpoint, allowing for declarations of the form $\Box_s\varphi$ ('according to s, it is *unequivocal* that φ') and $\Diamond_s\varphi$ ('according to s, it is *conceivable* that φ'). Such modalities capture the semantic commitments of particular agents, yet do not require the nesting of semantic commitments within semantic commitments. This allows for SL to recover favorable computational properties; indeed, the satisfiability problem for SL is NP-complete [4].

In [3], *standpoint linear temporal logic* (SLTL) was introduced as a fusion of both LTL and SL. This allows for a logic which combines the multi-perspective capabilities of SL with the temporal features of LTL. This lets one model both the evolution of a system as well as changing standpoints over time.

As is the case for any logic, it is always important to develop a proof system for the logic that permits automated reasoning and an investigation of the logic's properties. For LTL, a relatively simple tableau calculus was developed by Reynolds that builds finite trees of sets of formulae and detects the satisfiability of formulae [6]. In [3], a tableau calculus was used to decide the satisfiability of formulae in SLTL. This tableau calculus builds tableaux with a rather complex structure, making the calculus unwieldy, difficult to use, and obfuscating proofs of soundness, completeness, and termination. It is conjectured that the tableau calculus for SLTL can be significantly refined and simplified so that tableaux have the structure of trees of sets of formulae, similar to Reynolds's tableau calculus for LTL. The goal of this thesis is to define a sound, complete, and terminating tableau calculus for SLTL in the style of Reynolds, which circumvents the unnecessary complexities of the tableau calculus in [3].

1.1 Goals

- Provide a simplified tableau calculus for SLTL that builds tableaux which are trees of sets of formulae.
- Prove soundness of the tableau calculus.
- Prove completeness of the tableau calculus.
- Prove termination of the tableau construction algorithm and analyze the computational complexity.

2 Thesis

Section 2.1 below provides a suggested structure of the Master's thesis, which covers the various important results and discussion that should be included in the thesis. The thesis should be written using a specific LaTeX template, explained in Section 2.2 below.

2.1 Structure of Thesis

The following is a suggested structure of the thesis. The student is free to organize the thesis as they see fit, but they should ensure that the presentation is clear, coherent, and supports the overall narrative of the research. The structure should adequately cover the background, motivation, technical developments, results, and conclusions of the work. Additionally, the student is encouraged to provide sufficient context and explanation to make the thesis accessible to a broader audience within the field.

1. **Introduction**: The first section of the report should discuss various logics: linear temporal logic LTL, standpoint logic SL, and standpoint linear temporal logic SLTL. It should cover how these logics came about historically, what their uses/applications are, and why they are practically/theoretically interesting. Moreover, the introduction should motivate the thesis, explaining issues with the current tableau calculus for SLTL, why the new tableau calculus is proposed, and what is accomplished in the thesis. Last, the introduction should provide an outline explaining what chapters/sections are included in the thesis and what is covered in each.

2. **Logical Preliminaries**: This section should define the language used for SLTL and the semantics of SLTL (i.e., what models are used and how formulae are interpreted). Also, it would be helpful to include examples of applying SLTL either here (or in the introduction) to show the kinds of scenarios that can be modelled with SLTL.

3. **Tableau Calculus**: This section should define the new tableau calculus for SLTL. This means that the various inference rules need to be defined, how a tableau is constructed should be defined, and examples of tableaux recognizing the (un)satisfiability of interesting formulae should be given to showcase how the system works. Also, open and closed tableau should be defined.

4. **Soundness**: This section should prove the soundness of the tableau calculus, i.e., that if a formula has an open tableau, then the formula is satisfiable.

5. **Completeness**: This section should prove the completeness of the tableau calculus, i.e., that if a formula is satisfiable, then it has an open tableau.

6. **Tableau Algorithm and Complexity**: This section should provide an algorithm that takes a formula as input and constructs either an open or closed tableau for the formula in a finite period of time. The computational complexity of this algorithm should be analyzed and proven.

7. **Conclusion**: Summarize what was accomplished in the thesis. One should also discuss any other important insights gained from this work and discuss potential avenues for future work.

2.2 Thesis Template

The thesis is expected to be written in LaTeX. Please email the supervisor (details below) to request the template.

3 Reading

The student is responsible for conducting the research for the project and must demonstrate a thorough understanding of the scientific context related to the thesis. The following *required reading* provides a useful starting point for their research, while the *additional reading* offers further valuable insights for the project. For any questions regarding additional reading or other sources, the student is encouraged to contact the supervisor. The student is also welcome to incorporate any other scientifically reviewed and relevant sources or findings into their thesis.

3.1 Required Reading

The following papers are essential for the thesis project and should be read and understood:

1. Pneuli 1977 [5] - This is the seminal paper introducing linear temporal logic LTL.
2. Gómez Álvarez and Rudolph 2021 [4] - This is one of the foundational papers on standpoint logic.
3. Reynolds 2017 [6]- This paper introduces one-pass tableau for LTL and will serve as a basis for the new tableau calculus defined in this project.
4. Gigante et al. 2022 [3] - This paper introduces the standpoint linear temporal logic SLTL and its first proposed tableau calculus.

3.2 Additional Reading

The following reading is recommended but not required:

1. Geatti et al. 2024 [2] - This paper demonstrates how efficient SAT-solving can be performed using Reynolds one-pass tableau for LTL.
2. Demri and Wałęga 2024 [1] - This paper clarifies the computational complexity of SLTL and proves the logic EXPSPACE-complete.

4 Supervision

The contact details of the supervisor, Dr. Tim Lyon, are as follows:

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References

1. Demri, S., Wałęga, P.A.: Computational complexity of standpoint LTL (2024), `https://arxiv.org/abs/2408.08557`
2. Geatti, L., Gigante, N., Montanari, A., Venturato, G.: Sat meets tableaux for linear temporal logic satisfiability. Journal of Automated Reasoning **68**(2), 6 (2024). `https://doi.org/10.1007/s10817-023-09691-1`
3. Gigante, N., Gómez Álvarez, L., Lyon, T.S.: Standpoint Linear Temporal Logic. In: Proceedings of the 20th International Conference on Principles of Knowledge Representation and Reasoning. pp. 311–321 (8 2023). `https://doi.org/10.24963/kr.2023/31`
4. Gómez Álvarez, L., Rudolph, S.: Standpoint logic: Multi-perspective knowledge representation. In: Neuhaus, F., Brodaric, B. (eds.) Proceedings of the 12th International Conference on Formal Ontology in Information Systems. FAIA, vol. 344, pp. 3–17. IOS Press (2021)
5. Pnueli, A.: The temporal logic of programs. In: 18th Annual Symposium on Foundations of Computer Science. pp. 46–57 (1977). `https://doi.org/10.1109/SFCS.1977.32`
6. Reynolds, M.: A new rule for LTL tableaux. In: Cantone, D., Delzanno, G. (eds.) 7th International Symposium on Games, Automata, Logics and Formal Verification. EPTCS, vol. 226, pp. 287–301 (2016). `https://doi.org/10.4204/EPTCS.226.20`