What to expect

- The course has 12 lectures, 6 tutorials and a practical part
- Lecture is on Tuesday in DS 1, 7:30-9:00
- Tutorials and practical sessions are on Thursday in DS 1, 7:30-9:00
- Schedule and lecture material will be available at course web-page https://iccl.inf.tu-dresden.de/web/Problem_Solving_and_Search_in_Artificial_Intelligence_(WS2019)
- The **practical part** consists of solving (implementing) a problem and its presentation. Should be performed in groups of two, assignments will be ready at October 24th.
- Fixed Dates for practical part (see web-page)
  1) Analysis of the problem, group building
  2) Concept how to solve it
  3) Presentation of solution - last questions
- DEADLINE: 7th February, 2020
- EXAM: oral module examination

Literature

- plus additional articles
Agenda

1. Introduction
2. Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
3. Local Search, Stochastic Hill Climbing, Simulated Annealing
4. Tabu Search
5. Answer-set Programming (ASP)
6. Constraint Satisfaction Problems (CSP)
7. Evolutionary Algorithms/ Genetic Algorithms
8. Structural Decomposition Techniques (Tree/Hypertree Decompositions)
What are the Ages of my Three Sons?

Two men meet on the street. One gives the other a puzzle

A: "All three of my sons celebrate their birthday this very day! So, can you tell me how old each of them is?"

B: "Sure, but you’ll have to tell me something about them."

A: "The product of the ages of my sons is 36."

B: "That’s fine but I need more than just this."

A: "The sum of their ages is equal to the number of windows in that building."

B: "Still, I need an additional hint to solve your puzzle."

A: "My oldest son has blue eyes."

B: "Oh, this is sufficient!"
"The product of the ages of my sons is 36."

<table>
<thead>
<tr>
<th>son 1</th>
<th>son 2</th>
<th>son 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>1</td>
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<tr>
<td>9</td>
<td>2</td>
<td>2</td>
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<tr>
<td>6</td>
<td>6</td>
<td>1</td>
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<tr>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
What are the Ages of my Three Sons? ctd.

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<table>
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<tr>
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<th>son 2</th>
<th>son 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>1</td>
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<td>12</td>
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<tr>
<td>9</td>
<td>2</td>
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<td>1</td>
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<tr>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
What are the Ages of my Three Sons? ctd.

"The sum of their ages is equal to the number of windows in that building."

\[
\begin{align*}
36 & + 1 & + 1 & = & 38 \\
18 & + 2 & + 1 & = & 21 \\
12 & + 3 & + 1 & = & 16 \\
9 & + 4 & + 1 & = & 14 \\
9 & + 2 & + 2 & = & 13 \\
6 & + 6 & + 1 & = & 13 \\
6 & + 3 & + 2 & = & 11 \\
4 & + 3 & + 3 & = & 10
\end{align*}
\]
What are the Ages of my Three Sons? ctd.

"The sum of their ages is equal to the number of windows in that building."

<table>
<thead>
<tr>
<th>Age 1</th>
<th>Age 2</th>
<th>Age 3</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>1</td>
<td>1</td>
<td>38</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>2</td>
<td>11</td>
</tr>
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<td>4</td>
<td>3</td>
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\[
\begin{array}{ccc}
36 & + & 1 & + & 1 & = & 38 \\
18 & + & 2 & + & 1 & = & 21 \\
12 & + & 3 & + & 1 & = & 16 \\
 9 & + & 4 & + & 1 & = & 15 \\
\hline
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What was difficult on this problem?
Problem Solving

- Where to begin?
- You have to create the plan for generating a solution.
- Always consider all of the available data.
- Can you make connections between the goal and what is given?
Why are Some Problems Difficult to Solve?

- The number of possible solutions in the **search space** is too large for an exhaustive search.
- The problem is too complicated, and simplified models of the problem are useless.
- The **evaluation function** of the quality of a solution is noisy or varies with time, which requires an entire series of solutions.
- There are so many constraints that finding even one feasible answer is difficult, let alone searching for an optimal solution.
- The person solving the problem is inadequately prepared.
The Size of the Search Space

Boolean Satisfiability Problem (SAT)

Make a compound statement of Boolean variables evaluate to TRUE.

• For example, consider the following problem of 100 variables given in conjunctive normal form (CNF):

\[ F(x) = (x_{17} \lor \neg x_{37} \lor x_{73}) \land (\neg x_{11} \lor \neg x_{56}) \land \cdots \land (x_{2} \lor x_{43} \lor \neg x_{77} \lor \neg x_{89} \lor \neg x_{97}). \]

• **Challenge:** find the truth assignment for each variable \( x_i \), for all \( i = 1, \ldots, 100 \) s.t. \( F(x) = TRUE \).
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**Space of possible solutions.**

- Any binary string of length 100 is a possible solution.

- Two choices for each variable, and taken over 100 variables, generates \(2^{100}\) possibilities.
The Size of the Search Space ctd.

- Size of the search space $S$ is
  \[ |S| = 2^{100} \approx 10^{30} = 1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000. \]

- The number of bacterial cells on Earth is estimated at around $5 \times 10^{30}$.

- If we had a computer that could test 1000 strings per second and could have started at the beginning of time itself, 15 billion years ago (Big Bang!) we would have examined fewer than 1% of all the possibilities by now!

- Trying out all alternatives is out of the question.

- Choice of which evaluation function to use.

- Solutions closer to the right answer should yield better evaluations than those who are far away.

- If we try a string $x$ and $F(x)$ returns TRUE, we are done. But what if $F(x)$ returns FALSE?

- How to find a function which gives more than just "right" or "wrong"?
The Size of the Search Space ctd.

Traveling Salesperson Problem (TSP)

- Given $n$ cities and the distances between each pair of cities;
- Traveling salesperson must visit every city exactly once and return home covering the shortest distance.
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Search Space

- Set of permutations of $n$ cities.
- $2n$ different ways (for symmetrical TSP) to represent one tour.
- There are $n!$ ways to permute $n$ numbers.
- $|S| = n!/(2n) = (n - 1)!/2$
The Size of the Search Space ctd.

- \(|S| = \frac{n!}{(2n)} = \frac{(n - 1)!}{2}\)
- For any \(n > 6\), number of possible solutions to the TSP with \(n\) cities is larger than the number of possible solutions to the SAT problem with \(n\) variables.
- For \(n = 6\): \(5!/2 = 60\) solutions to the TSP and \(2^6 = 64\) solutions to a SAT.
- For \(n = 7\): 360 solutions to the TSP and 128 to the SAT.
- Search space increases very quickly with increasing \(n\).
- A 50-city TSP has more solutions than existing liters of water on the planet.
- However, the evaluation function for the TSP is more straightforward than for SAT.
- Table with distances between each pair of cities.
- After \(n\) addition operations we could calculate the distance of any candidate tour and use this to evaluate its merit.
- \(cost = dist(15, 3) + dist(3, 11) + \cdots + dist(6, 15)\)
Modeling the problem

- We only find the solution to a **model** of the problem.
- All models are simplifications of the real world.
- **Problem → Model → Solution**
  1. Use an approximate model of a problem and find the precise solution: **Problem → Model\(_a\) → Solution\(_p(\text{Model}\(_a\))\)**
  2. Use a precise model of the problem and find an approximate solution: **Problem → Model\(_p\) → Solution\(_a(\text{Model}\(_p\))\)**
- Which one is better?
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  1. Use an approximate model of a problem and find the precise solution: Problem $\rightarrow$ Model$_a$ $\rightarrow$ Solution$_p$(Model$_a$)
  2. Use a precise model of the problem and find an approximate solution: Problem $\rightarrow$ Model$_p$ $\rightarrow$ Solution$_a$(Model$_p$)

- Which one is better?
- Solution$_a$(Model$_p$) is better than Solution$_p$(Model$_a$).
Change over time

Problems may change
- before you model them,
- while you derive a solution, and
- after you execute the solution.

TSP - Travel time between two cities depends on many factors:
- traffic lights
- slow-moving trucks
- flat tire
- weather
- many more...

TU Dresden, 15th October 2019

PSSAI
Constraints

- Almost all practical problems pose constraints
- Two types of constraints:
  - Hard constraints, and
  - Soft constraints.
- Constraints make the search space smaller, but
  - It is hard to create operators that will act on feasible solution and generate in turn new feasible solutions that are an improvement of previous solution.
  - The geometry of search space gets tricky.
Constraints ctd.

Timetable of the classes at a college in one semester

We are given

- list of courses that are offered;
- list of students assigned to each class;
- professors assigned to each class;
- list of available classrooms, and information for size and other facilities that each offer.
Constraints ctd.

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Construct timetables that fulfill hard constraints:
- Each class must be assigned to an available room that has enough seats and requisite facilities.
- Students who are enrolled in more than one class can not have their classes held at the same time on the same day.
- Professors can not be assigned to teach courses that overlap in time.
Timetable - Soft Constraints:

- Courses that meet *twice a week* should preferably be assigned to Mondays and Wednesdays or Tuesdays and Thursdays.
- Courses that meet *three times per week* should preferably be assigned to Mondays, Wednesdays, and Fridays.
- Course time should be assigned so that students do *not* have to take final exams for multiple courses without any break in between.
- If more than one room satisfies the requirements for a course and is available at the designated time, the course should be assigned to the room with the *capacity that is closest to the class size*. 
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- Any timetable that meets the hard constraints is feasible.
- The timetable has to be optimized in the light of soft constraints.
- Each soft constraint has to be quantified.
- We can evaluate two candidate assignments and decide that one is better than other.
Solve the Problem!

- Mr. Smith and his wife invited four other couples for a party.
- When everyone arrived, some of the people in the room shook hands with some of the others.
- Nobody shook hands with their spouse and nobody shook hands with the same person twice.
- After that, Mr. Smith asked everyone how many times they shook someone’s hand.
- He received different answers from everybody.
- How many times did Mrs. Smith shake someone’s hand?
Problem solving is difficult for several reasons:

- Complex problems often pose an enormous number of possible solutions.
- To get any sort of solution at all, we often have to introduce simplifications that make the problem tractable. As a result, the solutions that we generate may not be very valuable.
- The conditions of the problem change over time and might even involve other people who want to fail you.
- Real-world problems often have constraints that require special operations to generate feasible solutions.