Exercise 5.1. Let $A_{\text{LBA}}$ be the word problem of deterministic linear bounded automata. Show that $A_{\text{LBA}}$ is $\text{PSPACE}$-complete.

$$A_{\text{LBA}} = \{ \langle M, w \rangle \mid M \text{ is a (deterministic) LBA and } w \in \mathcal{L}(M) \}$$

Exercise 5.2. Consider the Japanese game go-moku that is played by two players $X$ and $O$ on a $19 \times 19$ board. Players alternately place markers on the board, and the first one to have five of its markers in a row, column, or diagonal wins.

Consider the generalised version of go-moku on an $n \times n$ board. Say that a position of go-moku is a placement of markers on such a board as it could occur during the game, together with a marker which player moves next. We define

$$\text{GM} = \{ \langle B \rangle \mid B \text{ is a position of go-moku where } X \text{ has a winning strategy} \}.$$ 

Show that $\text{GM}$ is in $\text{PSPACE}$.

Exercise 5.3. Show that the universality problem of nondeterministic finite automata $\text{ALL}_{\text{NFA}} = \{ \langle A \rangle \mid A \text{ an NFA accepting every valid input} \}$ is in $\text{PSPACE}$.

Hint: Prove that, if $\mathcal{L}(A) \neq \Sigma^*$ and $A$ has $n$ states, then there exists a word $w \in \Sigma^*$ of length at most $2^n$ such that $w \not\in \mathcal{L}(A)$. Then, use this fact to give a non-deterministic algorithm whose space consumption is polynomially bounded. Finally, apply Savitch’s Theorem.

Exercise 5.4. Show that the composition of logspace reductions again yields a logspace reduction.

Exercise 5.5. Show that the word problem $A_{\text{NFA}}$ of non-deterministic finite automata is $\text{NL}$-complete.

Exercise 5.6. Show that $\text{BIPARTITE} = \{ \langle G \rangle \mid G \text{ a finite bipartite graph} \}$ is in $\text{NL}$. For this show that $\text{BIPARTITE} \in \text{NL}$ and use $\text{NL} = \text{coNL}$. Hint: Show that a graph $G$ is bipartite if and only if it does not contain a cycle of odd length.