Exercise 4.1
Recall the definition of a graph and define the graph coloring problem.

Exercise 4.2
Given a graph $G$ and a set of colors $C$, give a CNF encoding for the graph coloring problem such that

1. the encoded propositional logic formula is satisfiable iff there exists a proper coloring for $G$ with $C$ and
2. each model of this formula encodes such a proper coloring.

Exercise 4.3
Implement a program that can parse DIMACS graph coloring instances in the format below and encodes the input graph coloring problem for a given number of colors into CNF.

- comment lines start with ‘c’
- problem description line: p edge #nodes #edges
- edge lines start with ‘e’ followed by two nodes and a new line

Below an example instance:

```
c this is a test instance
p edge 4 5
e 1 2
e 1 3
e 2 4
e 4 1
```

After encoding and solving the input problem, the program should either answer with “unsatisfiable” or with a proper coloring.

Exercise 4.4
Improve your program with

- additional clauses that constraint each vertex to at most one color and
- an encoding that generates a MaxSAT problem that encodes the minimum number of colors that yield a proper coloring.