Technische Universität Dresden

Computational Logic ∴ Group

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Knowledge Representation and Reasoning

Winter 2026 Term

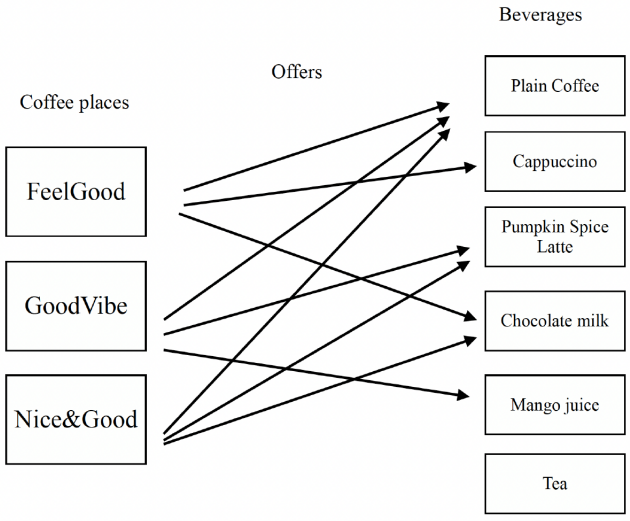
Exercises 5

23.11.2026

Problem 1.
We say that an atomic role R is satisfiable w.r.t. a TBox $\mathcal{T}$ if there exists a model $\mathcal{I}$ of $\mathcal{T}$ such that $R^{\mathcal{I}} \neq \emptyset$.

1. Write a satisfiable $\mathcal{ALC}$ TBox such that Role R is unsatisfiable w.r.t. $\mathcal{T}$.
2. Reduce the problem of checking satisfiability of an atomic role w.r.t. an $\mathcal{ALC}$ TBox such that role R is unsatisfiable w.r.t. $\mathcal{T}$
3. Write an unsatisfiable $\mathcal{ALC}$ knowledge base $\mathcal{K}$ whose ABox contains only role assertions and whose TBox is satisfiable and does not feature unsatisfiable roles.

Problem 2.
Consider the following interpretation of three coffee places and some beverages they offer:



Do the following:

1. Formalize the queries 1-3 in the language of conjunctive queries.

Query1: All beverages offered by some coffee place.

Query2: All beverages offered by both FeelGood and Nice & Good.

Query3: All coffee places that offer cappuccino.

2. Provide the answers to those queries given the interpretations above.
3. Construct two additional queries: one that yields $\{FeelGood, Nice\&Good\}$ as the only answers and one that yields $\{PlainCoffee, PumpkinSpiceLatte, MangoJuice\}$ as the only answers.

Problem 3.
Consider the following TBox $\mathcal{T}$:

$$\exists hasFather.\top \sqsubseteq Person$$
$$\exists hasFather^{-}.\top \sqsubseteq Person$$
$$Person \sqsubseteq \exists hasFather$$

Consider also the following ABox $\mathcal{A}$:

$$\mathcal{A} = \{Person(John),\ Person(Nick),\ Person(Toni),\ hasFather(John,\ Nick),\ hasFather(Nick,Toni)\}$$

Provide the certain answers to the following queries:

$q_1(x, y)$ $hasFather(x, y)$

$q_2(x)$ $\exists y.hasFather(x, y)$

$q_3(x)$ $\exists y_1, y_2, y_3.hasFather(x, y_1) \wedge hasFather(y_1, y_2) \wedge hasFather(y_2, y_3)$

$q_4(x, y_3)$ $\exists y_1, y_2.hasFather(x, y_1) \wedge hasFather(y_1, y_2) \wedge hasFather(y_2, y_3)$