Exercise 2.1:
Use the Martelli-Montanari algorithm step by step to unify the following pairs of terms with variables $x$, $y$, and $z$. For each step indicate which rule you have used.

a) $f(g(x), g(c), y)$ and $f(g(g(y)), x, a)$

b) $f(b, x, x, y)$ and $f(b, g(y), g(g(z)), g(a))$

c) $f(x, g(z), g(z))$ and $f(h(y), y, g(h(x)))$

Give the corresponding most general unifier (mgu) or give the reason why the terms are not unifiable.

Exercise 2.2:
Consider the following program

$$
p(X) :- q(X), r(X).
\quad q(f(X)).
\quad r(f(a)).
$$

a) Give an SLD-derivation $\xi$ for the query $?- p(X)$ that uses the Prolog selection rule.

b) For each derivation step of $\xi$, give the resultant that is associated with this step (Sl. 3/18).

c) Give the resultants of every level $i$ of $\xi$ (Sl. 3/19).

Exercise 2.3:
Consider the query $?- \text{fact}(0, Y), \text{fact}(Y, s(0))$. together with the program

$$
\text{fact}(0, s(0)).
\quad \text{fact}(s(N), F) :- \text{fact}(N, G), \text{mul}(s(N), G, F).
$$

a) Give an SLD-derivation using the Prolog selection rule (you don’t have to show the multiplication in detail). Give the substitutions and the CAS.
b) Show that the Switching Lemma (Sl. 3/26) holds for the initial query (i.e., for \( n = 0 \)).

*Hint:* Give a second SLD-derivation selecting the second atom at the beginning and using the Prolog selection rule afterwards. Show the correspondence of both derivations.

**Exercise 2.4:**
Give the SLD-tree for the query \(?- p(X,Y).\) and the following program. Use Prolog’s selection and computation rule.

\[
\begin{align*}
p(X,Y) & :- q(X,Y), r(Y,X). \\
q(X,a) & :- s(X). \\
q(X,c) & :- s(X). \\
r(X,b) & :- t(X). \\
s(a). \\
s(b). \\
s(c). \\
t(a). \\
t(c).
\end{align*}
\]