

Science of Computational Logic

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Problem 6.1

Consider the language $\mathcal{L}(\mathcal{R}, \mathcal{F}, \mathcal{V})$, with $\mathcal{R} = \{p/0, q/0\}$.

Given the set of formulas $S = \{p \leftarrow \neg q, q \leftarrow \neg p\}$,

compute $C_C(S)$.

Solution

First we compute:

$$C_{S,p} = \neg q \leftarrow p$$

$$C_{S,q} = \neg p \leftarrow q$$

We observe that both p and q are defined predicate symbols.

Thus $\{(\forall X_{CWA}) \neg p(X_{CWA}) \mid p \in \mathcal{R} \setminus \mathcal{R}_D\} = \emptyset$.

Then

$$\begin{aligned} C_C(S) &= \{G \mid S \cup C_{S,p} \cup C_{S,q} \cup \mathcal{E}_C \models G\} \\ &= \{G \mid \{p \leftarrow \neg q, q \leftarrow \neg p, \neg q \leftarrow p, \neg p \leftarrow q\} \cup \mathcal{E}_C \models G\} \\ &= \{G \mid \{p \leftrightarrow \neg q\} \cup \mathcal{E}_C \models G\} \end{aligned}$$

(\mathcal{E}_C is the completion of \approx .)

(We observe that $C_C(S) \neq C_{CWA}(S) = \mathcal{L}(\mathcal{R}, \mathcal{F}, \mathcal{V})$.)