Technische Universität Dresden

Prof. Dr. Sebastian Rudolph

Introduction to Formal Concept Analysis

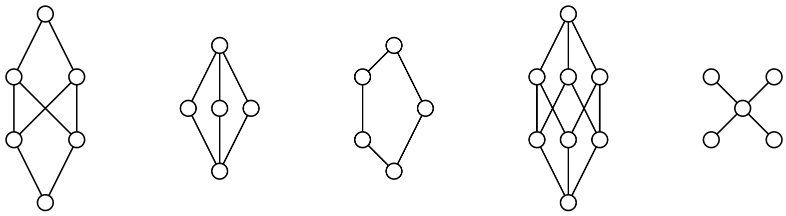
Exercise Sheet 2, Winter Semester 2017/18

Exercise 1 (line diagram)

a) Define: What is a lattice?

b) Find a preferably small lattice and draw its line diagram.

c) Which of the following line diagrams does not represent a lattice? Why?



(i) *(ii)* *(iii)* *(iv)* *(v)*

Exercise 2 (complete lattice)

a) Define: What is a complete lattice?

b) Can you find a *complete* lattice among the lattices of Exercise 1c?

c) Let $P := (M, \leq)$ be an ordered set such that for every subset X of M the infimum $\bigwedge X$ exists. Show that P is a complete lattice.

Exercise 3

Prove the following theorem:

Let $(L, \leqslant)$ be a lattice with supremum and infimum defined as usual. For any elements $x, y, z \in L$ holds:

(i) $x \wedge y = y \wedge x$	(ii) $x \vee y = y \vee x$
(iii) $x \wedge (y \wedge z) = (x \wedge y) \wedge z$	(iv) $x \vee (y \vee z) = (x \vee y) \vee z$
(v) $x \wedge (x \vee y) = x$	(vi) $x \vee (x \wedge y) = x$
(vii) $x \wedge x = x$	(viii) $x \vee x = x$

Exercise 4 (the basic theorem of formal concept analysis)

1. Show that $\underline{L} := (L, |)$ with $L := \{2^i 3^j 5^k \in \mathbb{N} \mid 0 \leq i, j \leq 2; 0 \leq k \leq 3\}$ is a complete lattice ($a|b$ is shorthand for the relation "a divides b").
2. Draw the line diagram for $\underline{L}$.
3. Which are the supremum-irreducible elements?
4. Which are the infimum-irreducible elements?
5. Give a formal context (G, M, I) such that its concept lattice is isomorphic to $\underline{L}$. Give the isomorphism explicitly.
6. How could the fact that $\underline{L}$ and $\underline{\mathfrak{B}}(G, M, I)$ are isomorphic be shown using the Basic Theorem of Formal Concept Analysis?