TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic ∴ Group

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Knowledge Representation and Reasoning

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Exercises 5

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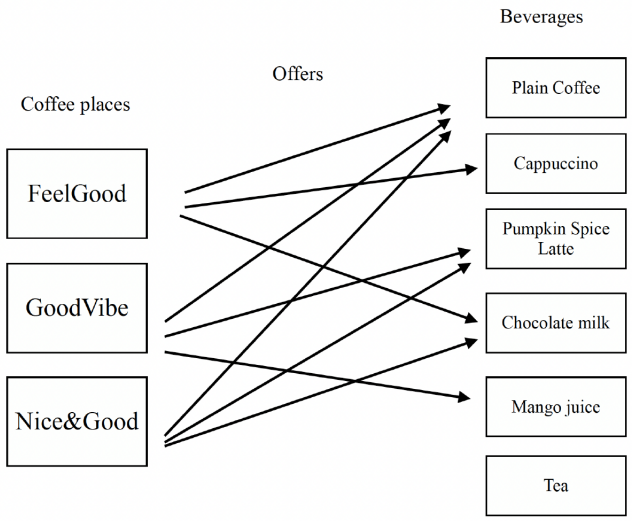
Problem 1.
We say that an atomic role $\mathcal{R}$ is satisfiable w.r.t. a TBox $\mathcal{T}$ if there exists a model $\mathcal{I}$ of $\mathcal{T}$ such that $\mathcal{R}^{\mathcal{I}} \neq \emptyset$. rite down the following:

1. Write a satisfiable $\mathcal{ALC} - TBox$ such that Role $\mathcal{R}$ is unsatisfiable w.r.t. $\mathcal{T}$. An unsatisfiable $\mathcal{ALC}$-knowledge base $\mathcal{K}$ whose TBox is satisfiable and whose ABox contains only role assertions.
2. Reduce the problem of checking satisfiability of an atomic role w.r.t. an $\mathcal{ALC} - T\text{-}Box$ such that role $\mathcal{R}$ is unsatisfiable w.r.t. $\mathcal{T}$

Problem 2.
Consider the following interpretation of three coffee places and some beverages they offer:



Do the following:

1. Formalize the queries 1-3 in the language of conjunctive queries.

Query1: All beverages offered by some coffee place.

Query2: All beverages offered by FeelGood and Nice & Good.

Query3: The coffee place that offers cappuccino.

2. Provide the answers to those queries given the interpretations above.
3. Construct two additional queries: one that yields $\{FeelGood, Nice\&Good\}$ as the only answers and one that yields $\{PlainCoffee, PumpkinSpiceLatte, MangoJuice\}$ as the only answers.

Problem 3.
Consider the following TBox $\mathcal{T}$:

$$\begin{aligned} \exists hasFather.\top &\sqsubseteq Person \\ \exists hasFather^{-}.\top &\sqsubseteq Person \\ Person &\sqsubseteq \exists hasFather \end{aligned}$$

Consider also the following ABox $\mathcal{A}$:

$$\mathcal{A} = \{Person(John),\ Person(Nick),\ Person(Toni),\ hasFather(John,\ Nick),\ hasFather(Nick,Toni)\}$$

Provide the certain answers to the following queries:

$q_1(x, y)$ $hasFather(x, y)$

$q_2(x)$ $\exists y.hasFather(x, y)$

$q_3(x)$ $\exists y_1, y_2, y_3.hasFather(x, y_1) \wedge hasFather(y_1, y_2) \wedge hasFather(y_2, y_3)$

$q_4(x, y_3)$ $\exists y_1, y_2.hasFather(x, y_1) \wedge hasFather(y_1, y_2) \wedge hasFather(y_2, y_3)$