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Complexity Theory

Exercise 5: Space Complexity

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Exercise 5.1. Consider the Japanese game *go-moku* that is played by two players X and O on a 19x19 board. Players alternately place markers on the board, and the first one to have five of their markers consecutively in a row, column, or diagonal wins.

Consider the generalized version of go-moku on an $n \times n$ board. Say that a *position* of go-moku is a placement of markers on such a board as it could occur during the game, together with a marker which player moves next. We define

$$\mathbf{GM} = \{\langle B \rangle \mid B \text{ is a position of go-moku where X has a winning strategy}\}.$$

Show that **GM** is in PSPACE.

Exercise 5.2. Show that the universality problem of nondeterministic finite automata

$$\mathbf{ALL}_{\text{NFA}} = \{\langle \mathcal{A} \rangle \mid \mathcal{A} \text{ an NFA accepting every valid input}\}$$

is in PSPACE.

Hint:
Proof that, if $\mathbf{L}(\mathcal{A}) \neq \Sigma^*$ and $\mathcal{A}$ has n states, then there exists a word $w \in \Sigma^*$ of length at most 2^n such that $w \notin \mathbf{L}(\mathcal{A})$. Then, use this fact to give a non-deterministic algorithm whose space consumption is polynomially bounded. Finally, apply Savitch's Theorem.

Exercise 5.3. Consider the following problem: given a string of parentheses, decide if they are properly matched and nested, (e.g., (()())() satisfy the requirement, but ()) or)(do not). Show that this problem is in L.

Exercise 5.4. Is it true that **CLIQUE** $\leq_L$ **INDEPENDENTSET**?

Exercise 5.5. The class NL consists of all problems solvable by nondeterministic log-space Turing machines. Can you give an equivalent verifier-based characterization, analogous to the standard verifier-based definition of NP?

Exercise 5.6. Let $A, B \subseteq \{0,1\}^*$ and $\varnothing \neq B \neq \{0,1\}^*$. Prove that, if $A \in \text{NL}$ and $B \leq_L A$, then $B \leq_p$ **PALINDROMES**.

PALINDROMES $= \{w \mid w \in \{0,1\}^*$ is a palindrome (reads the same from left to right and from right to left)$\}$

Exercise 5.7. Show that the composition of log-space reductions yields a log-space reduction.

Exercise 5.8. Show that the word problem $\mathsf{A}_{\mathsf{NFA}}$ of non-deterministic finite automata is NL-complete.

Exercise 5.9. Show that

$$\mathsf{BIPARTITE} = \{ \langle G \rangle \mid G \text{ a finite bipartite graph} \}$$

is in NL. For this show that $\overline{\mathsf{BIPARTITE}} \in \mathsf{NL}$ and use $\mathrm{NL} = \mathrm{coNL}$.

Hint:
Show that a graph G is bipartite if and only if it does not contain a cycle of odd length.