



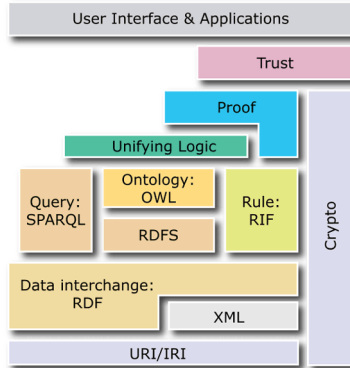
TECHNISCHE
UNIVERSITÄT
DRESDEN

FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

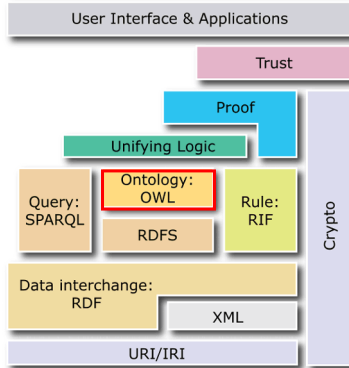
Hypertableau II

Sebastian Rudolph

Hypertableau II



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Agenda

- Recap
- The Hypertableau Rules
- Blocking in the Hypertableau Calculus
- Comparison Tableau and Hypertableau Calculus
- Summary

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Result of the Structural Transformation

by virtue of structural transformation and the known equivalences, we can rewrite an \mathcal{ALC} KB into an equisatisfiable one, which contains only axioms of the following shape (A and B being atomic):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$

$$A \sqsubseteq \exists r.B$$

$$A \sqsubseteq \forall r.B$$

$$\exists r.A \sqsubseteq B$$

$$\forall r.A \sqsubseteq B$$

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$
$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

in case $m = 0$, the rule head contains $\perp(x)$.

$A \sqsubseteq \exists r.B$	$A(x) \rightarrow (\exists r.B)(x)$
$A \sqsubseteq \forall r.B$	$A(x) \wedge r(x, y) \rightarrow B(y)$
$\exists r.A \sqsubseteq B$	$r(x, y) \wedge A(y) \rightarrow B(x)$
$\forall r.A \sqsubseteq B$	$\rightarrow (\exists r. \neg A)(x) \vee B(x)$

Simplification via Structural Transformation

- structural transformation introduces new concepts for complex concept subexpressions
- taking polarity into account, it suffices to introduce subsumption instead of equivalences

Polarity in Structural Transformation

We define the polarity of a concept C inside a formula as follows:

- C occurs in C positively,
- C occurs in $\neg D$ positively (negatively) if C occurs in D negatively (positively),
- C occurs in $D \sqcap E$ or $D \sqcup E$ positively (negatively), if C occurs positively (negatively) in D or E ,
- C occurs in $\exists r.D$ or $\forall r.D$ positively (negatively), if C occurs positively (negatively) in D ,
- C occurs in $D \sqsubseteq E$ positively (negatively), if C occurs positively (negatively) in E or negatively (positively) in D .

A concept occurs positively (negatively) in an (\mathcal{ALC}) TBox \mathcal{T} , if C occurs positively (negatively) in an axiom in \mathcal{T} .

\rightsquigarrow a concept may occur both positively and negatively in an axiom

Optimized Transformation with Polarity

Let \mathcal{T} be an \mathcal{ALC} TBox. For every concept (sub-)expression C in \mathcal{T} , we introduce a fresh atomic concept A_C and define the function $\text{st}(C)$ as follows:

$$\begin{array}{lll} \text{st}(A) = A & \text{st}(\neg C) = \neg A_C & \text{st}(\exists r.C) = \exists r.A_C \\ \text{st}(\top) = \top & \text{st}(C \sqcap D) = A_C \sqcap A_D & \text{st}(\forall r.C) = \forall r.A_C \\ \text{st}(\perp) = \perp & \text{st}(C \sqcup D) = A_C \sqcup A_D & \end{array}$$

The result of the structural transformation of a TBox \mathcal{T} is a TBox \mathcal{T}' with the following axioms:

- $A_C \sqsubseteq \text{st}(C)$ for every concept C occurring positively in \mathcal{T} ,
- $\text{st}(C) \sqsubseteq A_C$ for every concept C occurring negatively in \mathcal{T} ,
- $A_C \sqsubseteq A_D$ for every GCI $C \sqsubseteq D \in \mathcal{T}$.

Simplification of Axioms

we can now use known equivalences to simplify the axioms further:

$$\{C \sqsubseteq D \sqcap E\} \equiv \{C \sqsubseteq D, C \sqsubseteq E\}$$

$$\{C \sqcup D \sqsubseteq E\} \equiv \{C \sqsubseteq E, D \sqsubseteq E\}$$

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Hypertableau Calculus: Basic Notions

- for a TBox \mathcal{T} , let $\text{cl}(\mathcal{T})$ be the corresponding set of clauses
- we assume that the ABox is non-empty
- we assume that the ABox contains only facts of the forms $A(a)$, $\neg A(a)$, $(\exists r.A)(a)$, $(\exists r.\neg A)(a)$, $r(a, b)$
- we write $\text{Vars}(\mathcal{T})$ to denote the set of variables in $\text{cl}(\mathcal{T})$
- we write $\text{Inds}(\mathcal{A})$ to denote the set of individual names in \mathcal{A}

The Hypertableau Calculus

HT-rule: for $A_1 \wedge \dots \wedge A_n \rightarrow B_1 \vee \dots \vee B_m \in \text{cl}(\mathcal{T})$
and mapping $\sigma: \text{Vars}(\mathcal{T}) \rightarrow \text{Inds}(\mathcal{A})$ with
 $\sigma(A_i) \in \mathcal{A}, \sigma(B_j) \notin \mathcal{A}$ for every $1 \leq i \leq n$,
pick a j with $1 \leq j \leq m$ and let $\mathcal{A} = \mathcal{A} \cup \{\sigma(B_j)\}$.

\exists -rule: for $(\exists r.C)(v) \in \mathcal{A}$ where v is not blocked and
there is no r successor v' of v with $C(v') \in \mathcal{A}$,
let $\mathcal{A} = \mathcal{A} \cup \{r(v, v'), C(v')\}$ where v' is a new node.

- the tableau is now represented as ABox
- the tableau is contradictory if the ABox contains $\perp(v)$ or $A(v)$ and $(\neg A)(v)$ for some individual v (and concept A)

Example Hypertableau

Let the TBox \mathcal{T} and ABox \mathcal{A} be given:

$$\mathcal{T} = \{\exists r.A \sqsubseteq A\} \quad \text{cl}(\mathcal{T}) = \{r(x,y) \wedge A(y) \rightarrow A(x)\}$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$\mathcal{A} = \{\neg A(a_0), r(a_0, b_1), r(b_1, a_1), \dots, r(a_{n-1}, b_n), r(b_n, a_n), A(a_n)\}$$

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the hypertableau algorithm does not exhibit non-determinism here

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Blocking in the Hypertableau

by now, blocking mechanism still undefined:

Definition (Blocking)

An individual $v \in \text{Inds}(\mathcal{A})$ **blocks** an individual $v' \in \text{Inds}(\mathcal{A})$ in an ABox \mathcal{A} **directly**, if:

- 1 v' is reachable from v ,
- 2 $\{A \mid A(v) \in \mathcal{A}\} = \{A \mid A(v') \in \mathcal{A}\}$; and
- 3 there is no directly blocked node v'' such that v' is reachable from v'' .

An individual $v' \in \mathcal{V}$ is **blocked** if either

- 1 v' is directly blocked or
- 2 there is a directly blocked node v such that v' is reachable from v .

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Blocking in the Hypertableau

- Like the original tableau, the ABox is seen as graph where every named individual is the root of a tree
- blocking depends only on atomic concepts
- can't we use subset blocking?

Example Hypertableau

$$\mathcal{T} = \{ \begin{array}{l} C \sqsubseteq \exists r.C \\ C \sqsubseteq \exists s.D \\ \exists s.D \sqsubseteq E \\ \top \sqsubseteq \forall r.(\neg E) \end{array} \}$$

$$\mathcal{A} = \{(\exists t.C)(a)\}$$

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 \text{cl}(\mathcal{T}) = \{
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$$\mathcal{A} = \{(\exists t.C)(a)\} \cup \{t(a, v_0), C(v_0)\}$$

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v_0 blocks v_1 (atomic concepts): $\mathcal{L}(v_1) = \{C\} \subseteq \mathcal{L}(v_1) = \{C\}$

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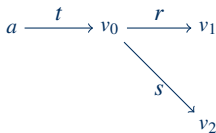
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 \quad C \sqsubseteq \exists r.C \\
 \quad C \sqsubseteq \exists s.D \\
 \quad \exists s.D \sqsubseteq E \\
 \quad \top \sqsubseteq \forall r.(\neg E)\}
 \end{array}
 \quad
 \text{cl}(\mathcal{T}) = \{
 \begin{array}{l}
 C(x) \rightarrow (\exists r.C)(x) \\
 C(x) \rightarrow (\exists s.D)(x) \\
 s(x, y) \wedge D(y) \rightarrow E(x) \\
 r(x, y) \wedge E(y) \rightarrow \perp(x)
 \end{array}
 \}$$

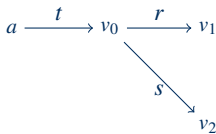


$$\begin{aligned}
 \mathcal{A} = & \{(\exists t.C)(a)\} \cup \{t(a, v_0), C(v_0)\} \cup \{(\exists r.C)(v_0)\} \cup \{(\exists s.D)(v_0)\} \\
 & \cup \{r(v_0, v_1), C(v_1)\} \cup \{\exists r.C(v_1)\} \cup \{\exists s.D(v_1)\} \\
 & \cup \{s(v_0, v_2), D(v_2)\}
 \end{aligned}$$

v_0 blocks v_1 (atomic concepts): $\mathcal{L}(v_0) = \{C\} \subseteq \mathcal{L}(v_1) = \{C\}$

Example Hypertableau

$$\begin{array}{l}
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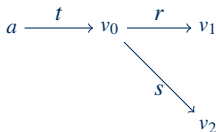


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 \}$$



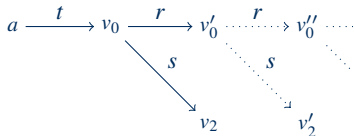
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 \end{aligned}$$

v_0 blocks v_1 (atomic concepts): $\mathcal{L}(v_1) = \{C\} \subseteq \mathcal{L}(v_0) = \{C\} \cup \{E\}$
 no further rule applicable

Example Hypertableau

When constructing the model, we run into troubles (recall: we replace the blocked individual by the blocking one, including the subtree rooted in the latter):

$$\text{cl}(\mathcal{T}) = \left\{ \begin{array}{ll} C(x) \rightarrow (\exists r.C)(x) & C(x) \rightarrow (\exists s.D)(x) \\ s(x, y) \wedge D(y) \rightarrow E(x) & r(x, y) \wedge E(y) \rightarrow \perp(x) \end{array} \right\}$$

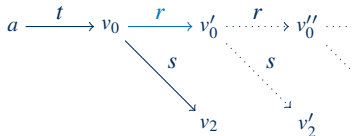


$$\mathcal{A} = \{ (\exists t.C)(a), t(a, v_0), C(v_0), E(v_0), (\exists r.C)(v_0), (\exists s.D)(v_0), \\ s(v_0, v_2), D(v_2), r(v_0, v'_0), C(v'_0), E(v'_0), \exists r.C(v'_0), \exists s.D(v'_0), \\ s(v'_0, v'_2), D(v'_2), r(v'_0, v''_0), C(v''_0), E(v''_0), \exists r.C(v''_0), \exists s.D(v''_0), \dots \}$$

Example Hypertableau

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$$\mathcal{A} = \{ (\exists t.C)(a), t(a, v_0), C(v_0), E(v_0), (\exists r.C)(v_0), (\exists s.D)(v_0), \\ s(v_0, v_2), D(v_2), r(v_0, v_0'), C(v_0'), E(v_0'), \exists r.C(v_0'), \exists s.D(v_0'), \\ s(v_0', v_2'), D(v_2'), r(v_0', v_0''), C(v_0''), E(v_0''), \exists r.C(v_0''), \exists s.D(v_0''), \dots \}$$

Comparison Blocking in Tableau vs. Hypertableau Calculus

- subset blocking does not work in hypertableau
- including non-atomic concepts doesn't help
- axiom $\exists s.D \sqsubseteq E$ equivalent with $D \sqsubseteq \forall s^-.E$

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$$\begin{array}{ll} \exists s.D \sqsubseteq E & D \sqsubseteq \forall s^-.E \\ \forall s.(\neg D) \sqcup E & \neg D \sqcup \forall s^-.E \end{array}$$

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$$\begin{array}{ll} \exists s.D \sqsubseteq E & D \sqsubseteq \forall s^-.E \\ \forall s.(\neg D) \sqcup E & \neg D \sqcup \forall s^-.E \\ s(x, y) \wedge D(y) \rightarrow E(x) & D(x) \wedge s^-(x, y) \rightarrow E(y) \end{array}$$

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$\exists s.D \sqsubseteq E$	$D \sqsubseteq \forall s^-.E$
$\forall s.(\neg D) \sqcup E$	$\neg D \sqcup \forall s^-.E$
$s(x, y) \wedge D(y) \rightarrow E(x)$	$D(x) \wedge s^-(x, y) \rightarrow E(y)$
	$D(x) \wedge s(y, x) \rightarrow E(y)$

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	$D(x) \wedge s(y, x) \rightarrow E(y)$
	$D(y) \wedge s(x, y) \rightarrow E(x)$

- for inverse roles, even the tableau calculus needs equality blocking
- for the axiom $\exists s.D \sqsubseteq E$ the tableau calculus requires a choice (GCI \rightsquigarrow disjunction)

Agenda

- Recap
- The Hypertableau Rules
- Blocking in the Hypertableau Calculus
- Comparison Tableau and Hypertableau Calculus
- Summary

Remarks Hypertablau

- hypertableau requires equality blocking
- if number restrictions or functionality are involved, pairwise blocking is needed
- inverse roles disappear inside the rules (variable positions are swapped)
- translation of the rules is more complicated in practice (further heuristics are used to avoid disjunctions)
- for evaluating the rules, semi-naïve evaluation can be used
- dependency directed backtracking can be used just like in the tableau
- (in)equality for functionality / number restrictions:
 - $\text{Func}(f)$ corresponds to the rule $f(x, y_1) \wedge f(x, y_2) \rightarrow y_1 \approx y_2$
 - special rule \approx -rule for merging and pruning

Comparison Tableau vs. Hypertablau

- hypertableau has a more costly preprocessing
- nondeterminism can be often avoided
- blocking requires equality
- ABox is extended via rules analog to expanding the tableau
- good mechanism for evaluating rules
- implementation as ABox instead of graph structures makes blocking checks more difficult (\rightsquigarrow optimization via hashing)
- both tableau and hypertableau can be extended to OWL 2 (but treatment of nominals difficult)
- hypertableau implemented in HermiT, tableau in FaCT++ and Pellet

Agenda

- Recap
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Summary

- we have treated the basic calculi that are targeted at model construction
- reasoning for OWL profiles can be implemented with more efficient methods (\rightsquigarrow consequence-based procedures)
- tableau and hypertableau are correct and terminating
 - corresponding proofs (particularly for more expressive logics) rather complicated
- in practice diverse optimizations used (other kinds of blocking, caching of model parts, heuristics, etc.)
- normally applicable for medium size knowledge bases, but depends a lot from the axioms' complexity
- worst-case for \mathcal{ALC} KBs: ExpTime, for OWL 1 DL: NExpTime and for OWL 2 DL: 2-NExpTime

What We Haven't Talked About (in Detail)

- number restrictions (except functionality in the tableau)
- datatypes (specific algorithms)
- nominals and particularly the interplay with inverses and number restrictions
- optimized rule translation for hypertableau
- further optimizations: disjunction learning, told subsumers, etc.
- precise complexity considerations