

# RECENT ADVANCES IN REASONING WITH EXISTENTIAL RULES

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Knowledge-Based Systems

reporting joint work with

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### Existential Rules

#### Existential Rules are sentences of the form

$$\forall \vec{x}. \left( \varphi \rightarrow \exists \vec{v}. \psi \right)$$

where  $\varphi$  (body) and  $\psi$  (head) are conjunctions of atoms.

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#### Why?

- Rules are a powerful data query paradigm (Datalog!) applications in data management, program analysis, business analytics, social network analysis, . . .
- Existential rules are a powerful ontology language generalising Horn ontologies, lightweight OWL profiles, knowledge graph formalisms, . . .

Example:	Bicycle(a)	(1)
	$Bicycle(x) \to \exists v.hasPart(x,v) \land Wheel(v)$	(2)
	Wheel( $x$ ) $\rightarrow \exists w.hasPart(x, w) \land Spoke(w)$	(3)
	$Spoke(x) \to \exists u.partOf(x,u) \land Bicycle(u)$	(4)
	$hasPart(x, y) \rightarrow partOf(y, x)$	(5)
	$partOf(x, y) \land partOf(y, z) \rightarrow partOf(x, z)$	(6)
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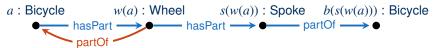
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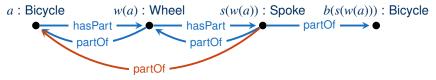
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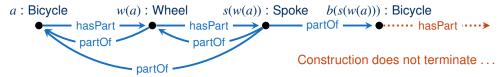
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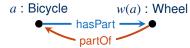
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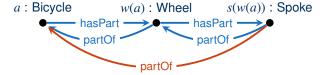
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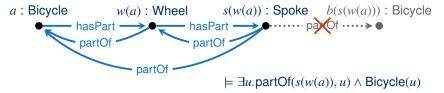
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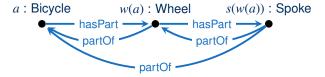
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#### Restricted chase computation:



→ restricted chase terminates, producing a finite model

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- Approach: skolemise ∃, perform chase, check if it stops;
   give up if a cyclic skolem term (with a repeated function symbol) appears
- Termination may depend on given facts, but: if the approach terminates on the critical instance (the set of all possible facts using a single constant "★") then it terminates on all sets of facts

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### **MFA check for the example:** we show only derivations from Bicycle(★)

★: Bicycle,W.,S.



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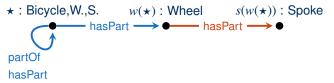


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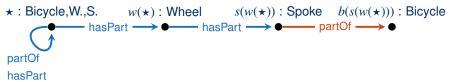


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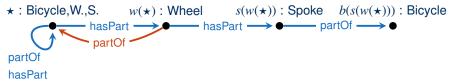


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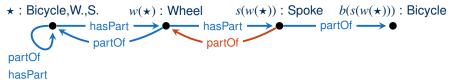


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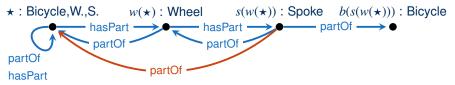


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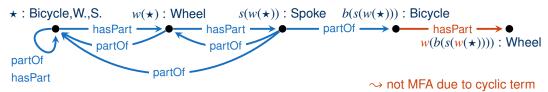


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# Restricted Acyclicity [IJCAI'17]

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### Restricted Model Faithful Acyclicity (RMFA):

- Perform a chase-like construction on the critical instance
- Only apply an  $\exists$ -rule with substitution  $\sigma$  if it is not blocked:
  - (1) find minimal amount of certain knowledge required for match  $\sigma$ ;
  - (2) check if this minimal knowledge already entails the rule head.
- Give up if procedure does not stop before a cyclic term occurs

### Theorem and Practice

**Theorem [IJCAl'17]:** Deciding if a set of rules is RMFA is 2ExpTime-complete even if the arity of predicates or the number of variables per rule is bounded.

- One can obtain slightly better bounds for DL ontologies (ExpTime)
- Criteria for making this tractable have been studied elsewhere, and seem to apply in many cases [ISWC'17]

**Practice:** We did not encounter major performance issues even for a prototype implementation. They arose mostly for rule sets that are artificially constructed to be "unreasonably" hard.

### Real-World Coverage

RMFA succeeds in detecting that our example has a finite restricted chase.

#### How about other practical rule sets?

- OWL ontologies can often be transformed into existential rules
- We studied 1220 ontologies obtained from two sources (MOWLcorp and Oxford ontology corpus)
- We also applied a new sufficient criterion RMFC that shows non-termination

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MFA (skolem chase termination) 884 (72.5%)

RMFA (restricted chase termination) 936 (76.7%) MFA + 52

RMFC (restricted chase non-termination) 239 (19.6%)

Termination not decided by our methods 45 (3.6%)
```

# VLog: A Column-Based Rule Engine [AAAI'16]

### VLog is an efficient implementation for large-scale rule reasoning

- Free and open source (C++)
- Command-line client and web interface
- Fully in-memory or using database back-end for input facts
- Supports existential quantifiers and arbitrary predicate arities

https://github.com/karmaresearch/vlog

#### Main reasoning algorithm: Bottom-up materialisation (chase)

- Semi-naive evaluation: only apply rules to matches that involve newly derived facts
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**Performance example:** We extracted a Datalog rule set of 9,396 rules from DB-pedia, and applied it to a set of 112M facts from the same source. On a laptop, VLog computes 33M derived facts in 20sec, using 585MiB of RAM.

### Restricted Chase in VLog

#### Since February 2018, VLog supports existential rule reasoning [unpublished]:

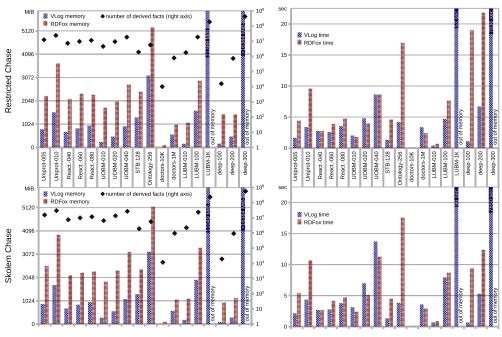
- Two chase variants: skolem chase and (1-parallel) resrticted chase
- Restricted chase gives priority to the execution of ∃-free rules
- → supports all rule sets that satisfy RMFA

#### We performed an extensive evaluation:

- using 18 challenging existential rule ontologies (many from a recent benchmark),
- producing several hundreds of billions of derived facts,
- on a laptop (2.2GHz Intel Core i7 CPU [4 cores], 16GB RAM 1600MHz DDR3).

We compare against RDFox, a leading rule engine

# Results: Memory and Time



### Conclusion

Existential rules are a powerful ontology and data analysis language

The chase is a versatile reasoning procedure, but it may not terminate

### Summary of results:

- RMFA: the first criterion for restricted chase termination (TTBOOK)
- RMFC: the first criterion for non-termination of any chase (TTBOOK)
- VLog: a very memory-efficient and surprisingly fast existential rule reasoner

#### What's next? (potentially including student projects)

- Optimisations (we only do vanilla restricted chase so far)
- Applications, e.g., existential rule reasoning for automated deduction?
- Existential rules for enriched knowledge graphs/attributed logics?
- Adding numeric reasoning (linear programming, CSPs, ...)
- Coping with (some types of) infinite models

### References

- [IJCAl'17] David Carral, Irina Dragoste, Markus Krötzsch: Restricted chase (non)termination for existential rules with disjunctions. In Carles Sierra, ed., Proceedings of the 26th International Joint Conference on Artificial Intelligence (IJCAl'17), 922-928, 2017.
- [ISWC'17] David Carral, Irina Dragoste, Markus Krötzsch: Tractable Query Answering for Expressive Ontologies and Existential Rules. In Claudia d'Amato et al., eds., Proceedings of the 16th International Semantic Web Conference (ISWC'17), volume 10587 of LNCS. Springer 2017.
- [AAAI'16] Jacopo Urbani, Ceriel Jacobs, Markus Krötzsch: Column-Oriented Datalog Materialization for Large Knowledge Graphs. In Dale Schuurmans, Michael P. Wellman, eds., Proceedings of the 30th AAAI Conference on Artificial Intelligence, 258-264. AAAI Press 2016.