Syllogistic Reasoning under the Weak Completion Semantics

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Reasoning Towards An Appropriate Logical Form

Mood	NL	FOL	Short
Affirmative universal (A)	All *a* are *b*.	$\forall X(a(X) \rightarrow b(X))$	A*ab*
Affirmative existential (I)	Some *a* are *b*.	$\exists X(a(X) \wedge b(X))$	I*ab*
Negative universal (E)	No *a* are *b*.	$\forall X(a(X) \rightarrow \neg b(X))$	E*ab*
Negative existential (O)	Some *a* are not *b*.	$\exists X(a(X) \wedge \neg b(X))$	O*ab*

- We assume that humans understand quantifiers with existential import, i.e.,

  For all implies *there exists.*

- The second and the third mood, I and E, each implies two facts about something, e.g. about some constant o. I.e. the example for I can be represented as

$$\mathcal{P}_{\mathrm{I}} = \{a(o) \leftarrow \top, b(o) \leftarrow \top\}$$

- The consequence in the third mood, E, is the negation of $b(X)$. The program representing the example for E together with the existential import, is

$$\mathcal{P}_{\mathrm{E}} = \{b'(X) \leftarrow a(X), b(X) \leftarrow \neg b(X), a(o) \leftarrow \top\}$$

Three Examples‡

‡ Experimental Results from (Khemlani and Johnson-Laird, 2012)

Syllogism IE1

Some A are B
No B are C

What follows?

All A are C	No A are C	Some A are C	Some A are not C	
All C are A	No C are A	Some C are A	Some C are not A	NVC

Syllogism: IE1

$$\mathcal{P}_{IE1} \quad = \quad \{a(o_1) \leftarrow \top, b(o_1) \leftarrow \top, c'(X) \leftarrow b(X), c(X) \leftarrow \neg c'(X), b(o_2) \leftarrow \top\}$$

Its weak completion is

$$\begin{array}{rcl} a(o_1) & \leftrightarrow & \top \\ b(o_1) & \leftrightarrow & \top \\ c'(o_1) & \leftrightarrow & b(o_1) \\ c(o_1) & \leftrightarrow & \neg c'(o_1) \end{array} \qquad \begin{array}{rcl} & & \\ b(o_2) & \leftrightarrow & \top \\ c'(o_2) & \leftrightarrow & b(o_2) \\ c(o_2) & \leftrightarrow & \neg c'(o_2) \end{array}$$

Its least model is

$$\langle \{a(o_1), b(o_1), c'(o_1), b(o_2), c'(o_2)\}, \{c(o_1), c(o_2)\} \rangle$$

which entails Oac and Eac, both correspond to the majority's conclusions.

Syllogism AA4

All B are A
All B are C

What follows?

All A are C	No A are C	Some A are C	Some A are not C	
All C are A	No C are A	Some C are A	Some C are not A	NVC

Syllogism AA4

$$\mathcal{P}_{AA4} \quad = \quad \{a(X) \leftarrow b(X), b(o_1) \leftarrow \top, c(X) \leftarrow b(X), b(o_2) \leftarrow \top\}$$

Its weak completion is:

$$\begin{array}{lcl} a(o_1) & \leftrightarrow & b(o_1) \\ c(o_1) & \leftrightarrow & b(o_1) \\ b(o_1) & \leftrightarrow & \top \end{array} \qquad \begin{array}{lcl} a(o_2) & \leftrightarrow & b(o_2) \\ c(o_2) & \leftrightarrow & b(o_2) \\ b(o_2) & \leftrightarrow & \top \end{array}$$

Its least model is

$$\langle\{b(o_1), c(o_1), a(o_1), b(o_2), c(o_2), a(o_2)\}, \emptyset\rangle$$

and entails all valid conclusions, Iac and Ica, and the invalid conclusions Aac and Aca. Aac and NVC correspond to the majority's conclusions.

Syllogism EA3

No A are B
All C are B

What follows?

All A are C	No A are C	Some A are C	Some A are not C	
All C are A	No C are A	Some C are A	Some C are not A	NVC

Syllogism EA3

$$\mathcal{P}_{EA3} \quad = \quad \{b'(X) \leftarrow a(X), b(X) \leftarrow \neg b'(X), a(o_1) \leftarrow \top, b(X) \leftarrow c(X), c(o_2) \leftarrow \top\}$$

Its weak completion is

$$\begin{array}{lcl@{\qquad\qquad}lcl} b'(o_1) & \leftrightarrow & a(o_1) & b'(o_2) & \leftrightarrow & a(o_2) \\ b(o_1) & \leftrightarrow & \neg b'(o_1) \vee c(o_1) & b(o_2) & \leftrightarrow & \neg b'(o_2) \vee c(o_2) \\ a(o_1) & \leftrightarrow & \top & c(o_2) & \leftrightarrow & \top \end{array}$$

Its least model is

$$\langle \{a(o_1), b'(o_1), c(o_2), b(o_2)\}, \{b(o_1)\} \rangle$$

which does not entail any of the valid conclusions, neither the majority's response.