Syllogistic Reasoning
under the Weak Completion Semantics

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Summer School 2015

September 2015

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Reasoning Towards An Appropriate Logical Form

<table>
<thead>
<tr>
<th>Mood</th>
<th>NL</th>
<th>FOL</th>
<th>Short</th>
</tr>
</thead>
<tbody>
<tr>
<td>Affirmative universal (A)</td>
<td>All a are b.</td>
<td>$\forall X (a(X) \rightarrow b(X))$</td>
<td>Aab</td>
</tr>
<tr>
<td>Affirmative existential (I)</td>
<td>Some a are b.</td>
<td>$\exists X (a(X) \land b(X))$</td>
<td>lab</td>
</tr>
<tr>
<td>Negative universal (E)</td>
<td>No a are b.</td>
<td>$\forall X (a(X) \rightarrow \neg b(X))$</td>
<td>Eab</td>
</tr>
<tr>
<td>Negative existential (O)</td>
<td>Some a are not b.</td>
<td>$\exists X (a(X) \land \neg b(X))$</td>
<td>Oab</td>
</tr>
</tbody>
</table>

- We assume that humans understand quantifiers with existential import, i.e.,
  
  *For all implies there exists.*

- The second and the third mood, I and E, each implies two facts about something, e.g. about some constant $o$. I.e. the example for I can be represented as

  \[
  P_I = \{ a(o) \leftarrow T, \ b(o) \leftarrow T \}
  \]

- The consequence in the third mood, E, is the negation of $b(X)$. The program representing the example for E together with the existential import, is

  \[
  P_E = \{ b'(X) \leftarrow a(X), \ b(X) \leftarrow \neg b(X), \ a(o) \leftarrow T \}
  \]
Three Examples‡

‡Experimental Results from (Khemlani and Johnson-Laird, 2012)
Syllogism IE1

**Some A are B**

**No B are C**

What follows?

- All A are C
- No A are C
- Some A are C
- Some A are not C
- All C are A
- No C are A
- Some C are A
- Some C are not A
- NVC
Syllogism: IE1

\[ \mathcal{P}_{IE1} = \{ a(o_1) \leftarrow \top, b(o_1) \leftarrow \top, c'(X) \leftarrow b(X), c(X) \leftarrow \neg c'(X), b(o_2) \leftarrow \top \} \]

Its weak completion is

\[
\begin{align*}
    a(o_1) & \iff \top \\
    b(o_1) & \iff \top \\
    c'(o_1) & \iff b(o_1) \\
    c(o_1) & \iff \neg c'(o_1)
\end{align*}
\begin{align*}
    b(o_2) & \iff \top \\
    c'(o_2) & \iff b(o_2) \\
    c(o_2) & \iff \neg c'(o_2)
\end{align*}
\]

Its least model is

\[
\langle \{ a(o_1), b(o_1), c'(o_1), b(o_2), c'(o_2) \}, \{ c(o_1), c(o_2) \} \rangle
\]

which entails \textbf{Oac} and \textbf{Eac}, both correspond to the majority's conclusions.
Syllogism AA4

All b are a
All b are c

What follows?

All A are C  No A are C  Some A are C  Some A are not C
All C are A  No C are A  Some C are A  Some C are not A  NVC
Syllogism AA4

\[ P_{AA4} = \{ a(X) \leftarrow b(X), b(o_1) \leftarrow \top, c(X) \leftarrow b(X), b(o_2) \leftarrow \top \} \]

Its weak completion is:

\[
\begin{align*}
    a(o_1) & \iff b(o_1) \\
    c(o_1) & \iff b(o_1) \\
    b(o_1) & \iff \top
\end{align*}
\]

Its least model is

\[ \langle \{ b(o_1), c(o_1), a(o_1), b(o_2), c(o_2), a(o_2) \}, \emptyset \rangle \]

and entails all valid conclusions, Iac and Ica, and the invalid conclusions Aac and Aca. Aac and NVC correspond to the majority’s conclusions.
### Syllogism EA3

**No** A are B  
**All** C are B

What follows?

<table>
<thead>
<tr>
<th>All A are C</th>
<th>No A are C</th>
<th>Some A are C</th>
<th>Some A are not C</th>
</tr>
</thead>
<tbody>
<tr>
<td>All C are A</td>
<td>No C are A</td>
<td>Some C are A</td>
<td>Some C are not A</td>
</tr>
</tbody>
</table>
Syllogism EA3

\[ \mathcal{P}_{EA3} = \{ b'(X) \leftarrow a(X), b(X) \leftarrow \neg b'(X), a(o_1) \leftarrow \top, b(X) \leftarrow c(X), c(o_2) \leftarrow \top \} \]

Its weak completion is

\[
\begin{align*}
b'(o_1) & \iff a(o_1) & b'(o_2) & \iff a(o_2) \\
b(o_1) & \iff \neg b'(o_1) \lor c(o_1) & b(o_2) & \iff \neg b'(o_2) \lor c(o_2) \\
a(o_1) & \iff \top & c(o_2) & \iff \top
\end{align*}
\]

Its least model is

\[
\langle \{ a(o_1), b'(o_1), c(o_2), b(o_2) \}, \{ b(o_1) \} \rangle
\]

which does not entail any of the valid conclusions, neither the majority’s response.