TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

ABSTRACT ARGUMENTATION

Introduction to Formal Argumentation II

*slides adapted from Stefan Woltran's lecture on Abstract Argumentation

Sarah Gaggl

ICCL Summer School 2016

Outline

Semantics

Naive Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a naive extension of F, if

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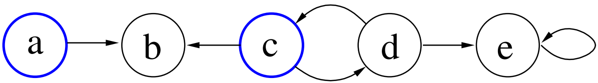

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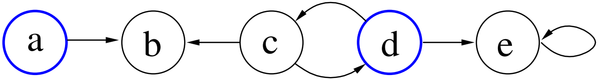

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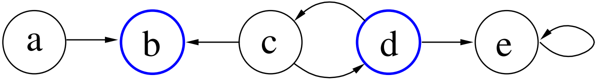

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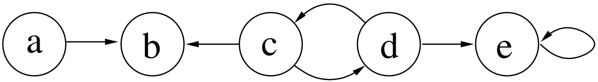

$naive(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \cancel{\{a\}}, \cancel{\{b\}}, \cancel{\{c\}}, \cancel{\{d\}}, \cancel{\emptyset}\}$

Semantics (ctd.)

Grounded Extension [Dung, 1995]

Given an AF $F = (A, R)$. The unique grounded extension of F is defined as the outcome S of the following “algorithm”:

1. put each argument $a \in A$ which is not attacked in F into S; if no such argument exists, return S;
2. remove from F all (new) arguments in S and all arguments attacked by them (together with all adjacent attacks); and continue with Step 1.

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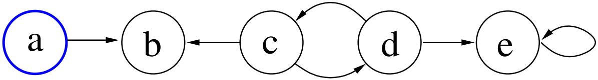

$ground(F) = \{\{a\}\}$

Semantics (ctd.)

Complete Extension [Dung, 1995]

Given an AF (A, R). A set $S \subseteq A$ is complete in F, if

- S is admissible in F
- each $a \in A$ defended by S in F is contained in S
  - Recall: $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

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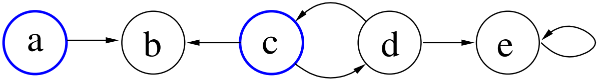

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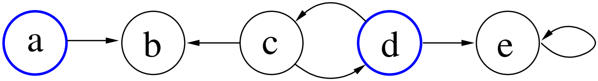

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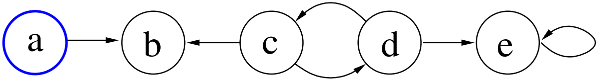

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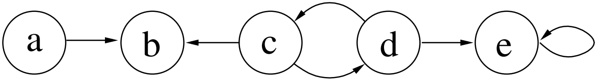

$comp(F) = \{\{a, c\}, \{a, d\}, \{a\}, \sout{\{c\}, \{d\}, \emptyset}\}$

Semantics (ctd.)

Properties of the Grounded Extension

For any AF F, the grounded extension of F is the subset-minimal complete extension of F.

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For any AF F, the grounded extension of F is the subset-minimal complete extension of F.

Remark

Since there exists exactly one grounded extension for each AF F, we often write $\mathit{ground}(F) = S$ instead of $\mathit{ground}(F) = \{S\}$.

Semantics (ctd.)

Preferred Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of F, if

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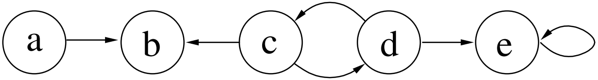

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Semantics (ctd.)

Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of F, if

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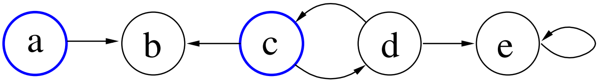

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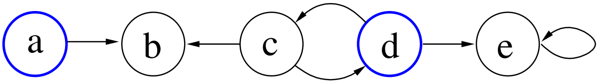

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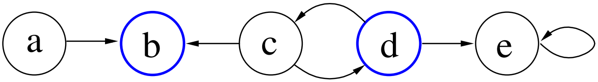

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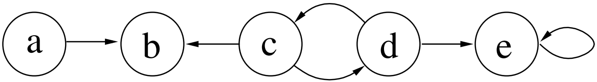

$\mathit{stable}(F) = \{$ ~~$\{a, c\}$~~, $\{a, d\}$, ~~$\{b, d\}$~~, ~~$\{a\}$~~, ~~$\{b\}$~~, ~~$\{c\}$~~, ~~$\{d\}$~~, ~~$\emptyset$~~, $\}$

Semantics (ctd.)

Some Relations

For any AF F the following relations hold:

1. Each stable extension of F is admissible in F
2. Each stable extension of F is also a preferred one
3. Each preferred extension of F is also a complete one

Semantics (ctd.)

Semi-Stable Extensions [Caminada, 2006]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a semi-stable extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S^+ \not\subset T^+$
    - for $S \subseteq A$, define $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$

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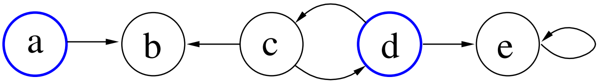

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Semantics (ctd.)

Stage Extensions [Verheij, 1996]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stage extension of F, if

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Ideal Extension [Dung, Mancarella & Toni 2007]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is an ideal extension of F, if

- S is admissible in F and contained in each preferred extension of F
- there is no $T \supset S$ admissible in F and contained in each of $pref(F)$

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Eager Extension [Caminada, 2007]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is an eager extension of F, if

- S is admissible in F and contained in each semi-stable extension of F
- there is no $T \supset S$ admissible in F and contained in each of $\mathit{semi}(F)$

Semantics (ctd.)

Properties of Ideal Extensions

For any AF F the following observations hold:

1. there exists exactly one ideal extension of F
2. the ideal extension of F is also a complete one

The same results hold for the eager extension and similar variants [Dvořák et al., 2011].

Semantics (ctd.)

Resolution-based grounded Extensions [Baroni,Giacomin 2008]

A resolution β of an AF $F = (A, R)$ contains exactly one of the attacks (a, b), (b, a) for each pair $a, b \in A$ with $\{(a, b), (b, a)\} \subseteq R$.

A set $S \subseteq A$ is a resolution-based grounded extension of F, if

- there exists a resolution β such that $ground((A, R \setminus \beta)) = S$
- and there is no resolution β' such that $ground((A, R \setminus \beta')) \subset S$

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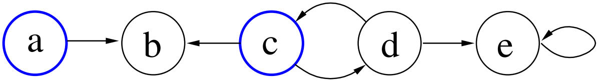

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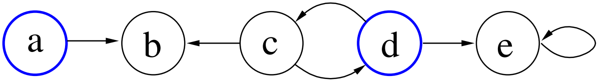

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cf2 Semantics [Baroni, Giacomin & Guida 2005]

Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, there exists a path from b to a. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of F.

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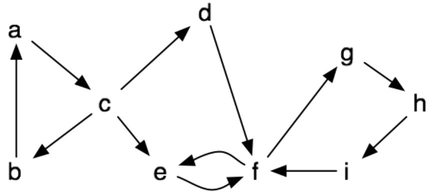

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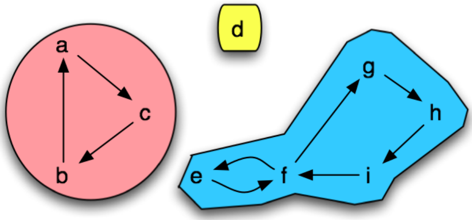

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Definition (Reachability)

Let $F = (A, R)$ be an AF, B a set of arguments, and $a, b \in A$. We say that b is reachable in F from a modulo B, in symbols $a \Rightarrow_F^B b$, if there exists a path from a to b in $F|_B$.

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Definition ($\Delta_{F,S}$)

For an AF $F = (A, R)$, $D \subseteq A$, and a set S of arguments,

$$\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b, a) \in R, a \not\Rightarrow_F^{A \setminus D} b\}.$$

By $\Delta_{F,S}$, we denote the lfp of $\Delta_{F,S}(\emptyset)$.

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cf2 Extensions [G & Woltran 2010]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of F, if

- S is conflict-free in F
- and $S \in naive([[F - \Delta_{F,S}]])$.

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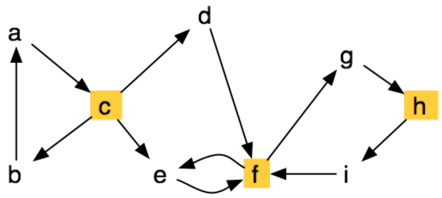
$S = \{c, f, h\}$, $S \in cf(F)$.



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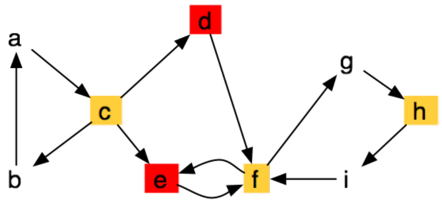
$S = \{c, f, h\},\ S \in cf(F)$, $\Delta_{F,S}(\emptyset) = \{d, e\}$.



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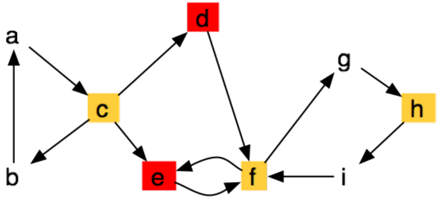

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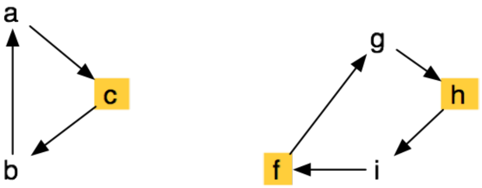
$S = \{c, f, h\}$, $S \in cf(F)$, $\Delta_{F,S} = \{d, e\}$, $S \in naive([[F - \Delta_{F,S}]])$.



Outline

Relations between Semantics

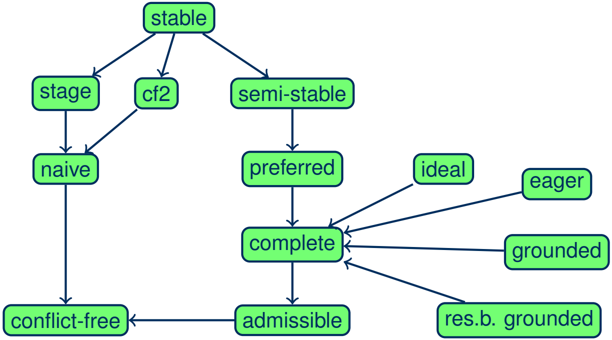


Figure: An arrow from semantics σ to semantics τ encodes that each σ-extension is also a τ-extension.

Outline

Exercises

1. Give an AF F such that $\mathit{stable}(F) = \emptyset$ and $\mathit{semi}(F) \neq \{\emptyset\}$.
2. Show that the following statement holds for any AF F.
   If $\mathit{stable}(F) \neq \emptyset$ then $\mathit{stable}(F) = \mathit{semi}(F) = \mathit{stage}(F)$.
3. Select three different semantics $\sigma, \sigma', \sigma''$ out of $\{\mathit{pref}, \mathit{ideal}, \mathit{semi}, \mathit{eager}, \mathit{ground}, \mathit{stable}\}$ of your choice and provide three pairs of AFs such that
   - $\sigma(F_1) = \sigma(G_1)$ but $\sigma'(F_1) \neq \sigma'(G_1)$
   - $\sigma'(F_2) = \sigma'(G_2)$ but $\sigma''(F_2) \neq \sigma''(G_2)$
   - $\sigma''(F_3) = \sigma''(G_3)$ but $\sigma(F_3) \neq \sigma(G_3)$

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On the resolution-based family of abstract argumentation semantics and its grounded instance.
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