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Knowledge Representation and Reasoning

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Exercises 2

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Problem 1.

Consider the following set of formulae:

1. $A \vee \neg A$
2. $(\neg A \vee B) \wedge A \wedge (\neg C \vee A \vee B) \wedge \neg B$
3. $(A \vee B) \wedge (A \vee \neg B \vee \neg C) \wedge (\neg A \vee B \vee \neg C)$

Decide which of these formulae are Horn-formulae.

Problem 2.

Consider the following knowledge bases (1) in propositional logic and (2) in first-order logic:

Knowledge Base (1):

- $Cinema \vee SwimmingPool$
- $Cinema \rightarrow Fun$
- $SwimmingPool \vee Sleeping_In \rightarrow Fun$

Knowledge Base (2):

- $\forall x(GradStudent(x) \rightarrow Student(x))$
- $\forall x(Student(x) \rightarrow HardWorker(x))$
- $GradStudent(Lisa)$

Use Resolution to:

1. Show $KB_1 \models Fun$.
2. Show $KB_1 \not\models Swimming\ Pool$.
3. Decide whether $KB_2 \models HardWorker(Lisa)$ holds.

Problem 3.

Assume that $Parent(x, y)$ is true iff (if and only if) x is a parent of y , $Male(x)$ is true iff x is male and $Female(x)$ is true iff x is female. Write down Datalog rules that specify membership conditions

for the binary familial relationship predicates Father, Mother, Son, Daughter, Sibling, Brother, Sister, Grandparent, Uncle, and Cousin. For example:

$$\text{Parent}(x, y) \wedge \text{Male}(x) \rightarrow \text{Father}(x, y)$$

Make the specification as precise as possible; e.g., in the case of Father, also add:

$$\begin{aligned} \text{Father}(x, y) &\rightarrow \text{Parent}(x, y) \\ \text{Father}(x, y) &\rightarrow \text{Male}(x) \end{aligned}$$

Try to be more succinct by reusing the specified predicates; e.g., use Sibling in the specification of Brother and Sister. Explain the nature of the problem in any case where expressivity limitations of Datalog make exact specification impossible.

Problem 4.

Consider the Datalog knowledge base $\mathcal{K} = \langle \mathcal{R}, \mathcal{F} \rangle$ where \mathcal{F} contains the following facts:

$$\mathcal{F} = \{\text{Father}(\text{john}, \text{mary}), \text{Mother}(\text{mary}, \text{peter}), \text{Father}(\text{john}, \text{david})\}$$

and \mathcal{R} contains the following rules:

$$\begin{aligned} \text{Parent}(x, y) \wedge \text{Parent}(y, z) &\rightarrow \text{GrandParent}(x, z) && (1) \\ \text{Parent}(x, y) \wedge \text{Parent}(x, z) &\rightarrow \text{Sibling}(y, z) && (2) \\ \text{Father}(x, y) &\rightarrow \text{Parent}(x, y) && (3) \\ \text{Mother}(x, y) &\rightarrow \text{Parent}(x, y) && (4) \end{aligned}$$

Do the following:

1. Using backward-chaining show that John is a grand parent of Peter.
2. Using forward-chaining show that Mary and David are siblings.