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Algorithmic Game Theory

Summer Term 2026

Exercises 12

23/07/2026

Problem 1.

Consider the 3-person cooperative game with $v(C) = 2$ if $|C| \geq 2$ and $v(C) = 0$ otherwise. Show that $S = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a stable set of this game.

Problem 2.

Show that the game from Problem 1 has (at least) one other stable set.

Hint: Consider the stable set given for “Hospitals and X-Ray Machines” in the lecture.

Problem 3.

Consider the following cooperative game: $P = \{1, 2, 3\}$ and

$v(\{1\}) = 2$	$v(\{2\}) = 4$	$v(\{3\}) = 6$
$v(\{1, 2\}) = 6$	$v(\{1, 3\}) = 8$	$v(\{2, 3\}) = 12$
$v(\{1, 2, 3\}) = 14$		

Do the following:

- (a) Compute each player’s marginal contributions to coalitions.
- (b) Check whether there exists a dummy player.

Problem 4.

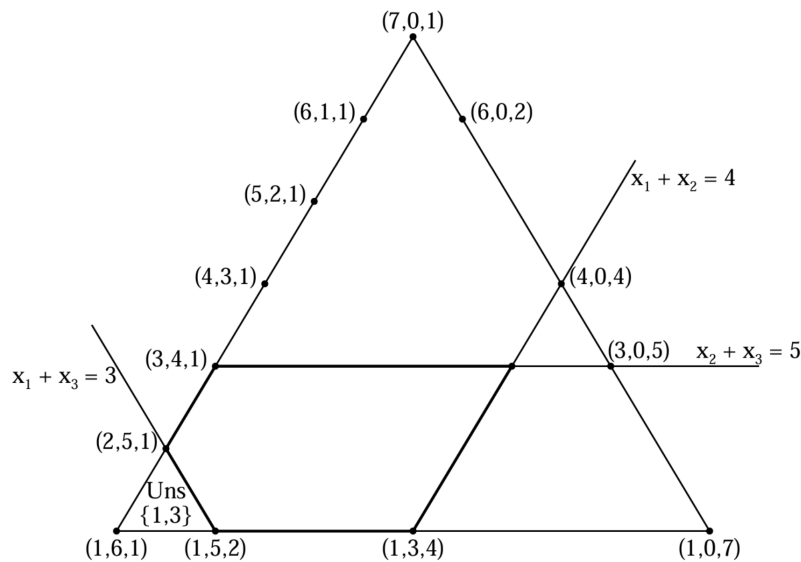
Consider the following cooperative game: $P = \{1, 2, 3\}$ and

$$\begin{array}{lll} v(\emptyset) = 0 & & \\ v(\{1\}) = 1 & v(\{2\}) = 0 & v(\{3\}) = 1 \\ v(\{1, 2\}) = 4 & v(\{1, 3\}) = 3 & v(\{2, 3\}) = 5 \\ v(\{1, 2, 3\}) = 8 & & \end{array}$$

Note that the imputations are the points (x_1, x_2, x_3) such that

- $x_1 + x_2 + x_3 = 8$ and
- $x_1 \geq 1, x_2 \geq 0, x_3 \geq 1$.

The set of all possible imputations is illustrated in the following triangle where we denote those imputations as *unstable* (abbreviated as *Uns* in the picture) that are not coalitionally rational.



Answer the following questions:

- Which of the imputations are not coalitionally rational? A first set of such unstable imputations is already given through the coalition $\{1, 3\}$.
- Identify the core of the game based on the unstable imputations.
- Is the vector of Shapley values contained in the core? For this exercise, compute the Shapley values based on the original formula.