TU Dresden, Fakultät Informatik
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Complexity Theory

Exercise 1: Mathematical Foundations, Decidability, and Recognisability

Exercise 1.1. Show the following claims.

1. $|\mathbb{N}| = |\mathbb{Z}|$.
2. $|\mathbb{N}| \neq |\mathbb{R}|$.

Exercise 1.2. Show the following claims.

1. There exist non-regular languages.
2. There exist undecidable languages.
3. There exist non-Turing-recognizable languages.

Exercise 1.3. Let $G = \{V, E\}$ be a simple undirected graph such that $|V| \geq 2$ (i. e., no self-loops). Show that G contains two or more nodes that have equal degree. That is, show that that there is a pair of nodes that occur in the same number of edges.

$*$ **Exercise 1.4.** Show that the class of Turing-recognizable languages is closed under homomorphisms.

Exercise 1.5. A *Turing machine with two-sided unbounded tape* is a single-tape Turing machine where the tape is unbounded on both sides. Argue that such machines can be simulated by ordinary Turing machines.

Exercise 1.6. Let $\mathsf{E}_{\mathsf{TM}} = \{\, \langle M \rangle \mid M \text{ is a TM such that } \mathcal{L}(M) = \emptyset \,\}$. Show that $\overline{\mathsf{E}_{\mathsf{TM}}}$ is Turing-recognizable.

Exercise 1.7. Let C be a language. Prove that C is Turing-recognizable if and only if a decidable language D exists such that $C = \{\, x \mid \exists y. \langle x, y \rangle \in D \,\}$.