Foundations of Constraint Programming
Tutorial 2 (on November 4th)

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Exercise 2.1:
Consider the following two CSPs

\[ P_1 := \langle x + y \leq z, 4 \leq z < 6; x, y, z \in [2..6] \rangle \]
\[ P_2 := \langle a < z, x + y = a, z \geq 5; a \in [4..6], x, y, z \in [2..6] \rangle \]

a) Assume the order \( X = a, x, y, z \) between variables. Represent each constraint \( C \) of \( P_1 \) and \( P_2 \) as set of projections \( d[Y] \), where \( d \in [4..6] \times [2..6]^3 \) and \( Y \) is the subsequence of \( X \) which exactly contains the variables mentioned in \( C \) (cf. Slide 3, Lecture 2).

b) Give all solutions to \( P_1 \) and \( P_2 \).

c) Are \( P_1 \) and \( P_2 \) equivalent? Are they equivalent with respect to some subsequence of \( X = a, x, y, z \)?

Exercise 2.2:
Consider the following Boolean constraints (see also Slide 22, Lecture 2):

\[ i_1 \land o_2 = y_1 \]
\[ i_2 \land o_1 = y_2 \]
\[ \neg y_1 = o_1 \]
\[ \neg y_2 = o_2 \]

For the above constraints show two successful derivations using the Boolean constraint propagation rules given on Slides 23-24 (Lecture 2). For each derivation step you should underline the selected constraint and give the used rule.

The initial CSPs are:

a) \( (i_1 \land o_2 = y_1, i_2 \land o_1 = y_2, \neg y_1 = o_1, \neg y_2 = o_2; i_1 = 0, i_2 = 1) \)

b) \( (i_1 \land o_2 = y_1, i_2 \land o_1 = y_2, \neg y_1 = o_1, \neg y_2 = o_2; o_2 = 1, i_1 = 1) \)
Exercise 2.3:
Consider the CSP from Slide 33, Lecture 2:

\[ x \cdot y = z; x \in [1..20], y \in [9..11], z \in [155..161] \]

Transform this CSP using the three Multiplication Rules from Slide 32 until you reach a fixed point. Give the selected constraint and the used rule for each derivation step.