About the Multi-Head Linear Restricted Chase Termination

The story of relocating to an address within the restricted chase borders

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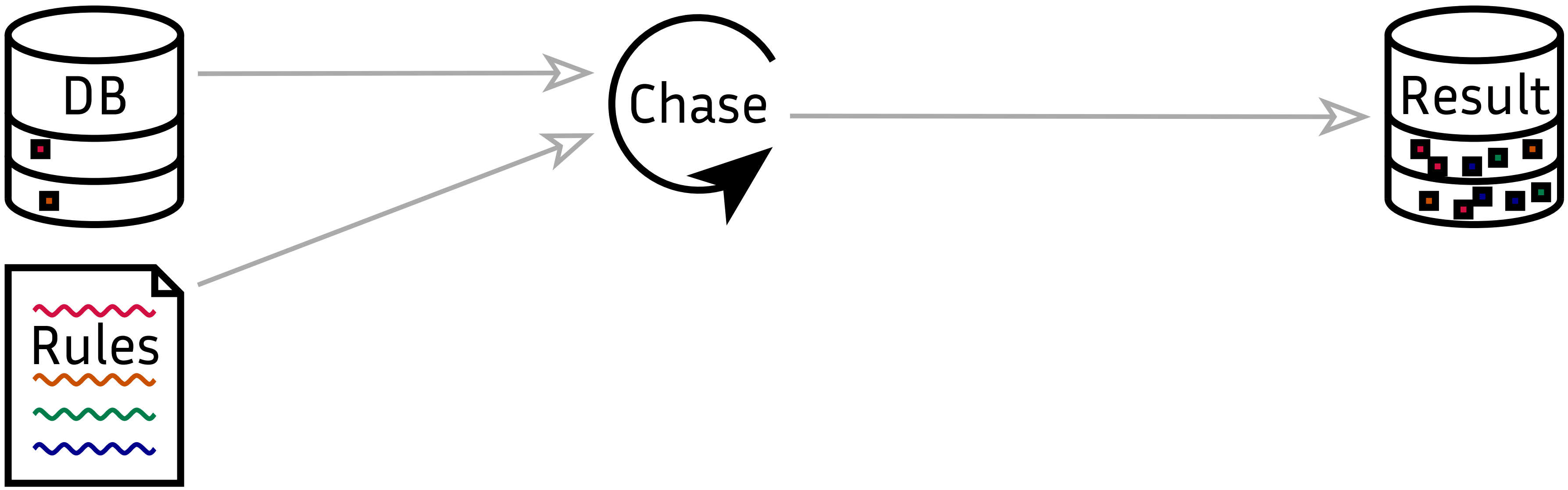
Technische Universität Dresden International Center for Computational Logic Inria Uniwersytet Wrocławski

The problem of chase termination



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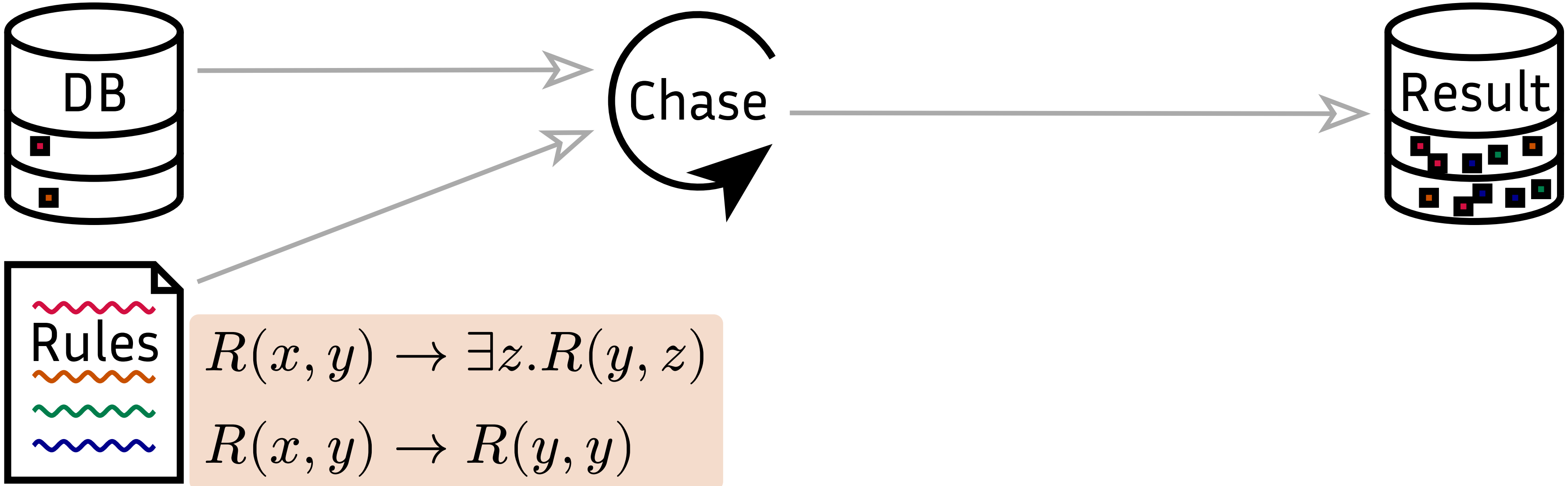

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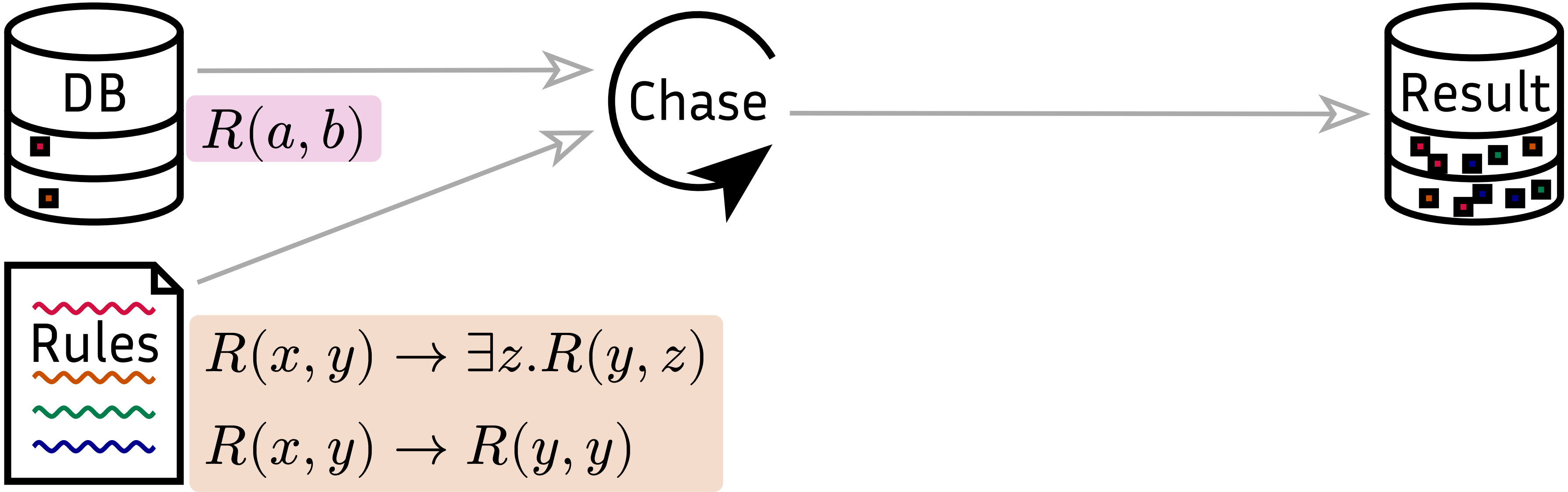

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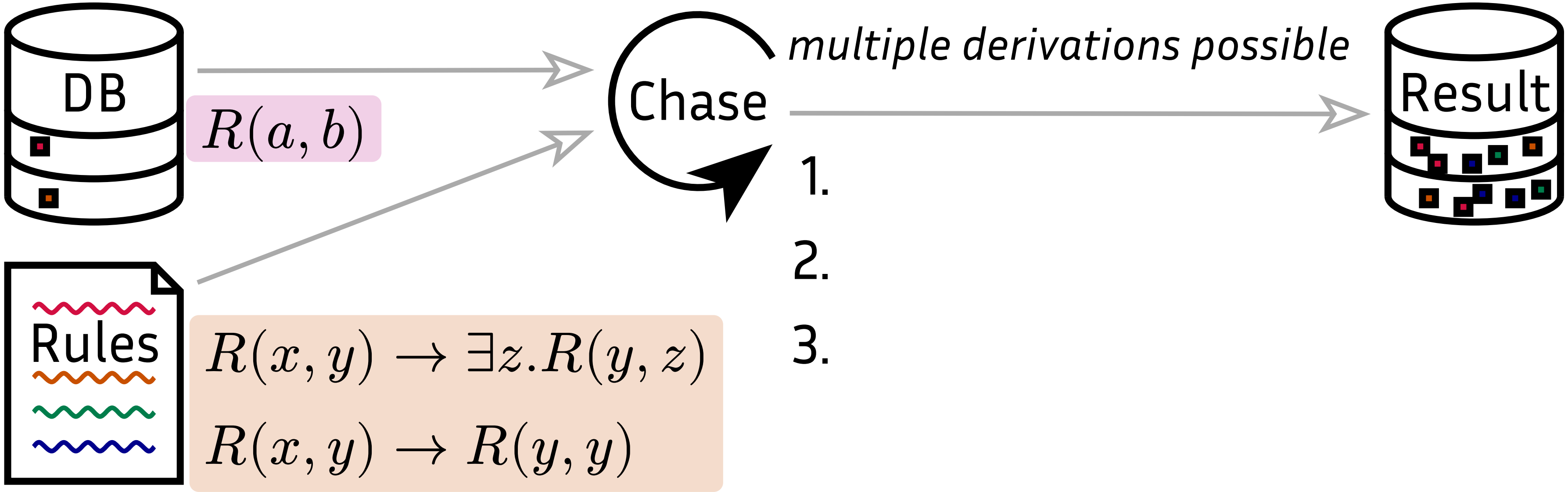

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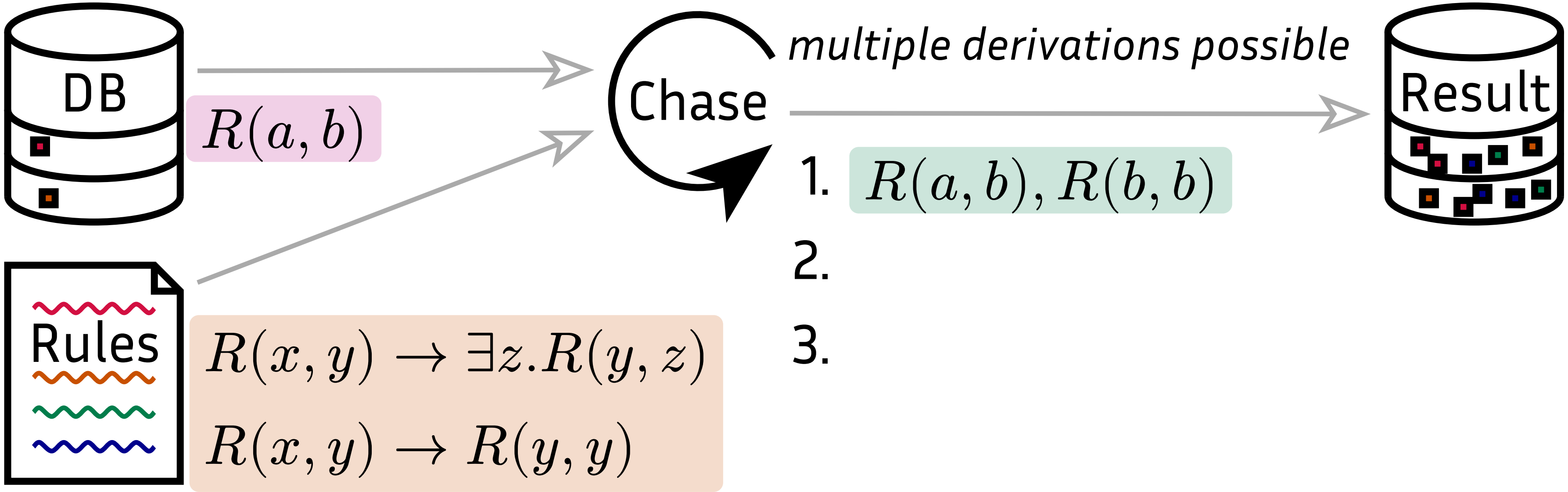

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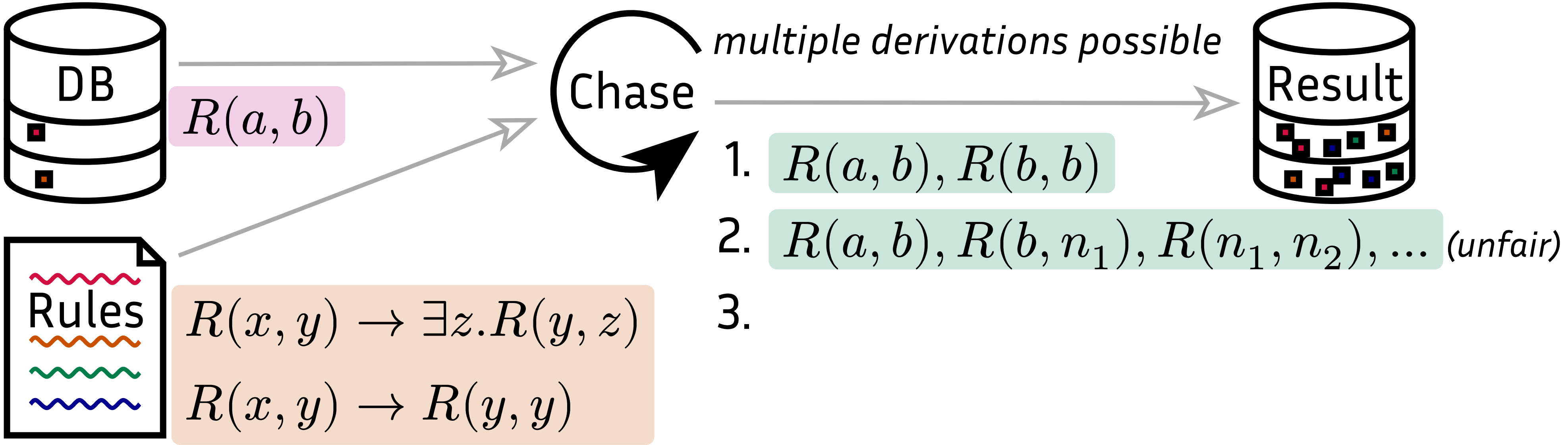

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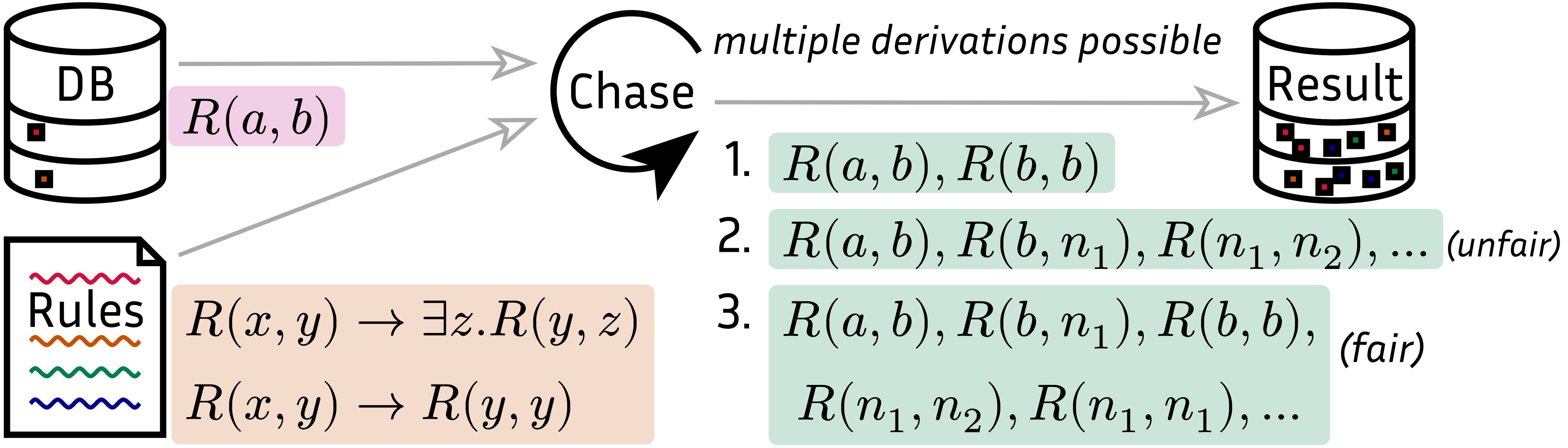

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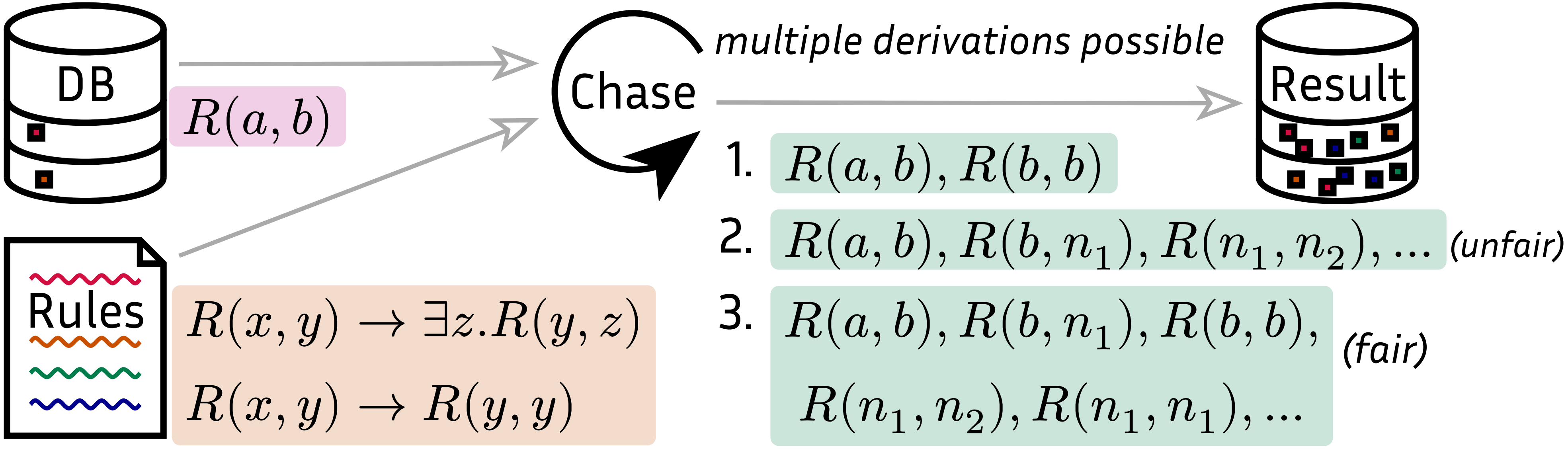

Universal Chase Termination asks:
Given a rule set $\mathcal{T}$, does every *fair* restricted chase derivation on $(\mathcal{T}, \mathbb{I})$ terminate for every initial fact set $\mathbb{I}$?

Why is the single-head case simpler? 🤙

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$$R(x,y,y) \to \exists z. R(x,z,y) \wedge R(z,y,y)$$
$$R(x,y,z) \to R(z,z,z)$$

A *single* application of second rule blocks *all* applications of the first!

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Aim: The properties of an ω*-sensitive path derivation*
can be massaged to be easily expressible in MSOL.

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A derivation is a trigger sequence and corresponds to a forest sequence.

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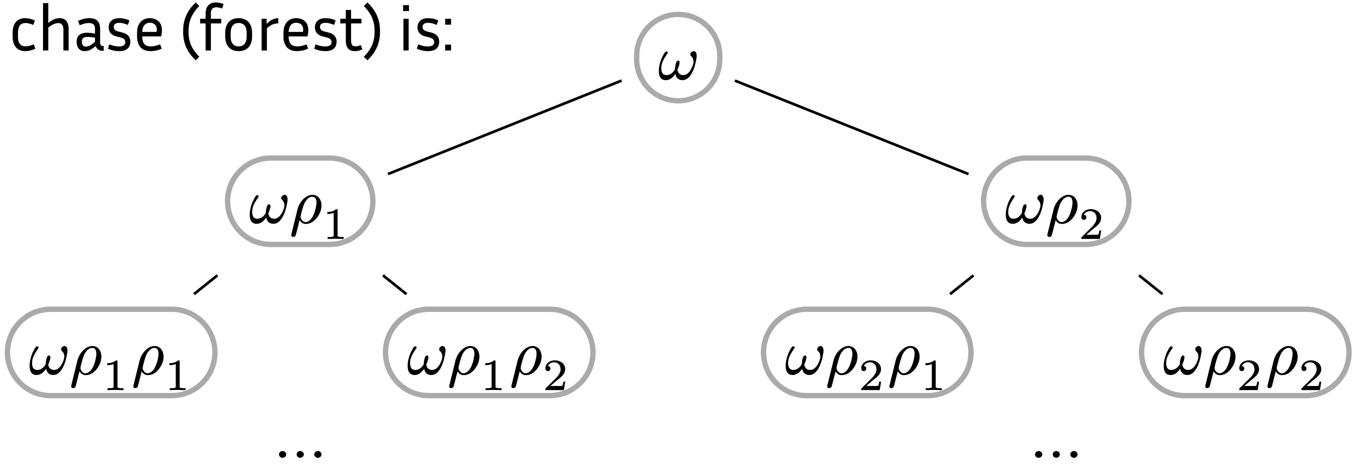

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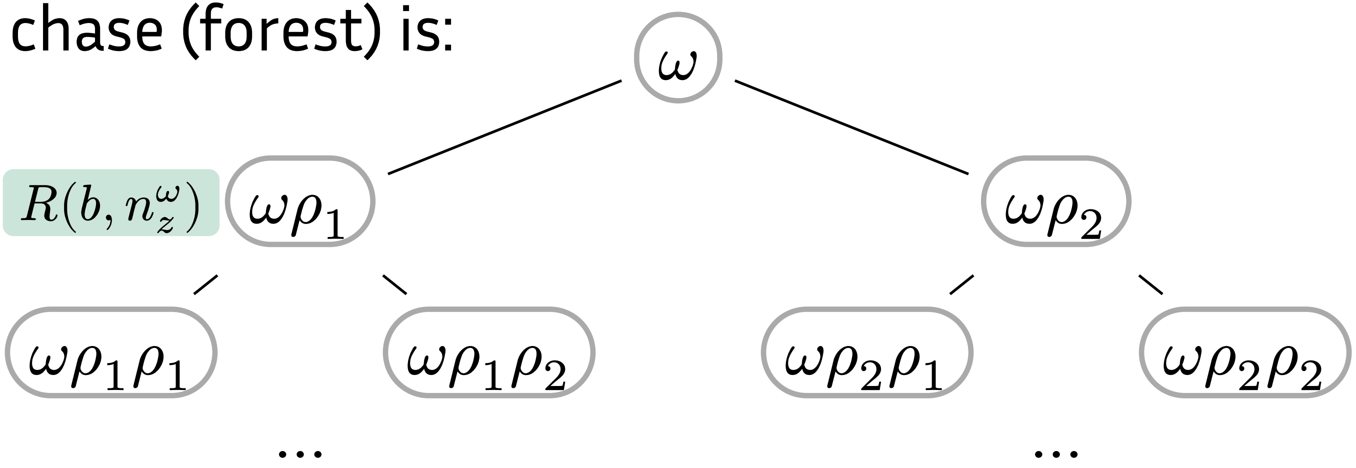

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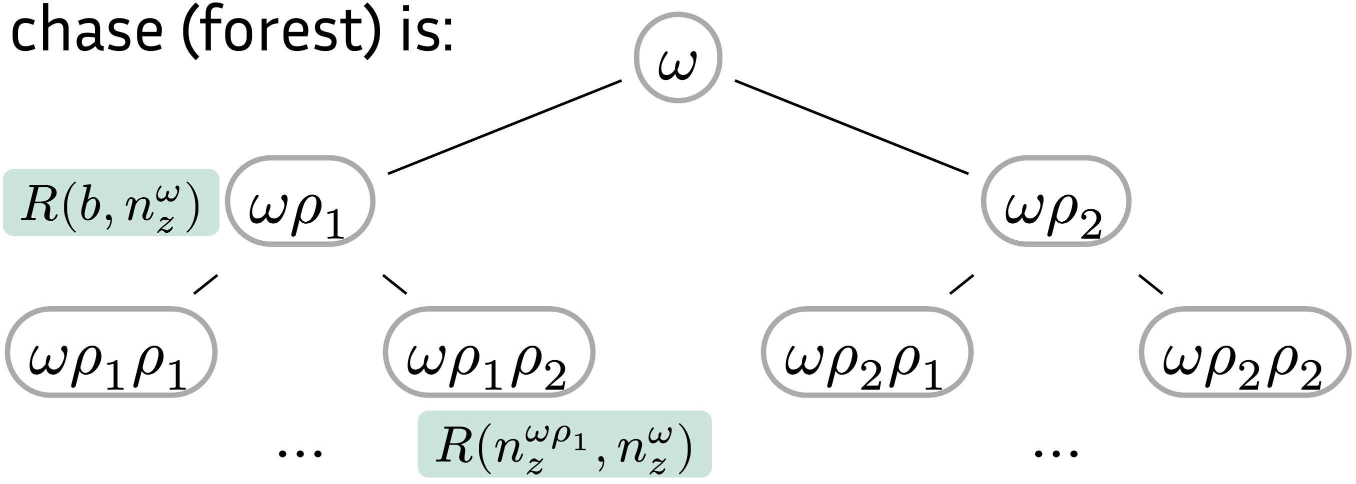

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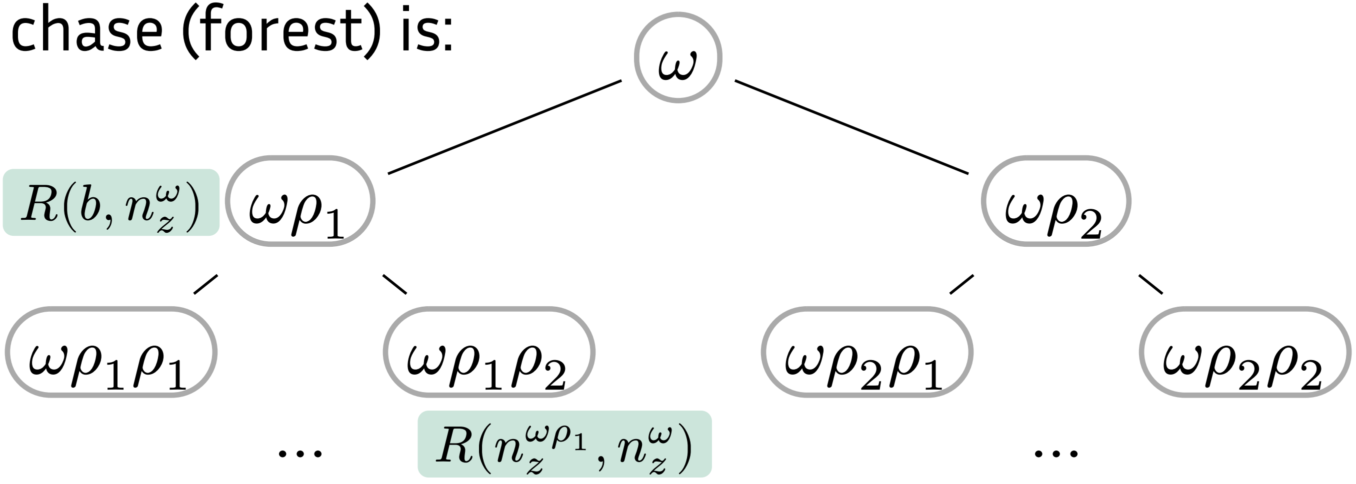

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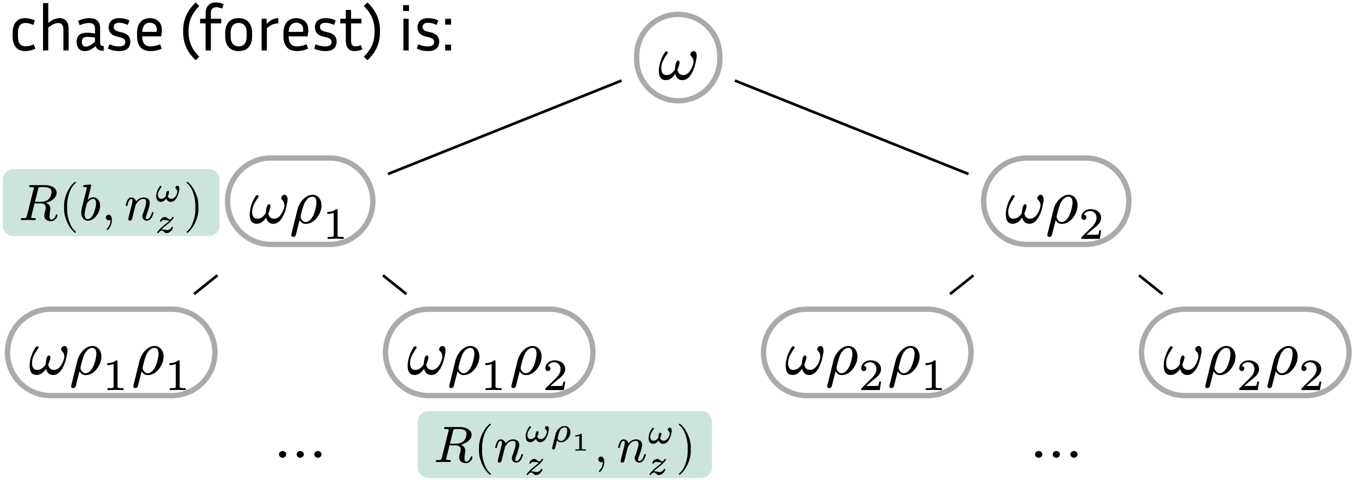

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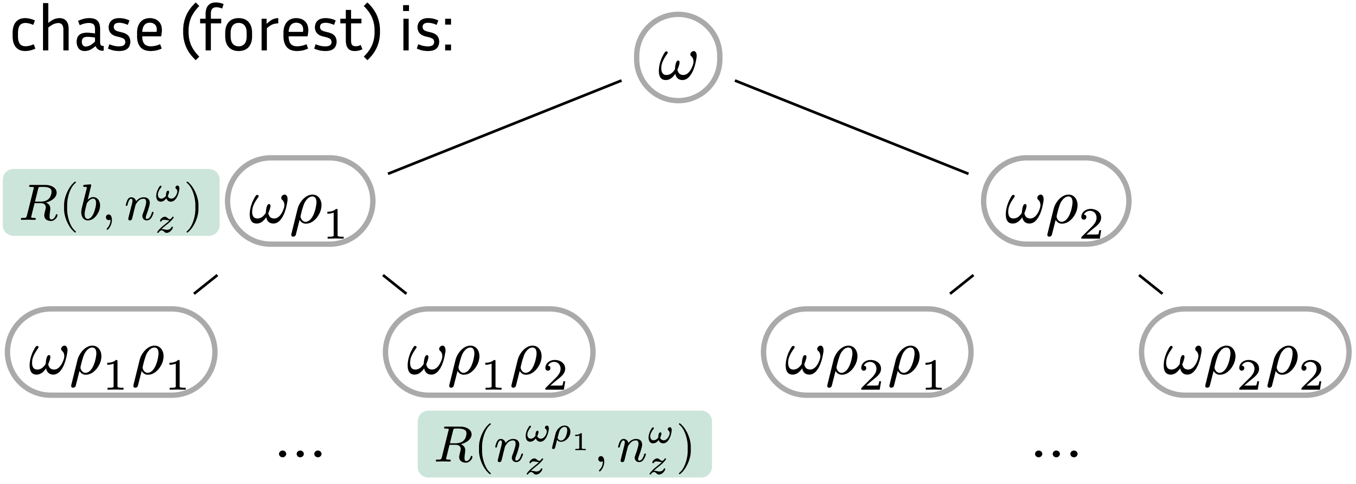

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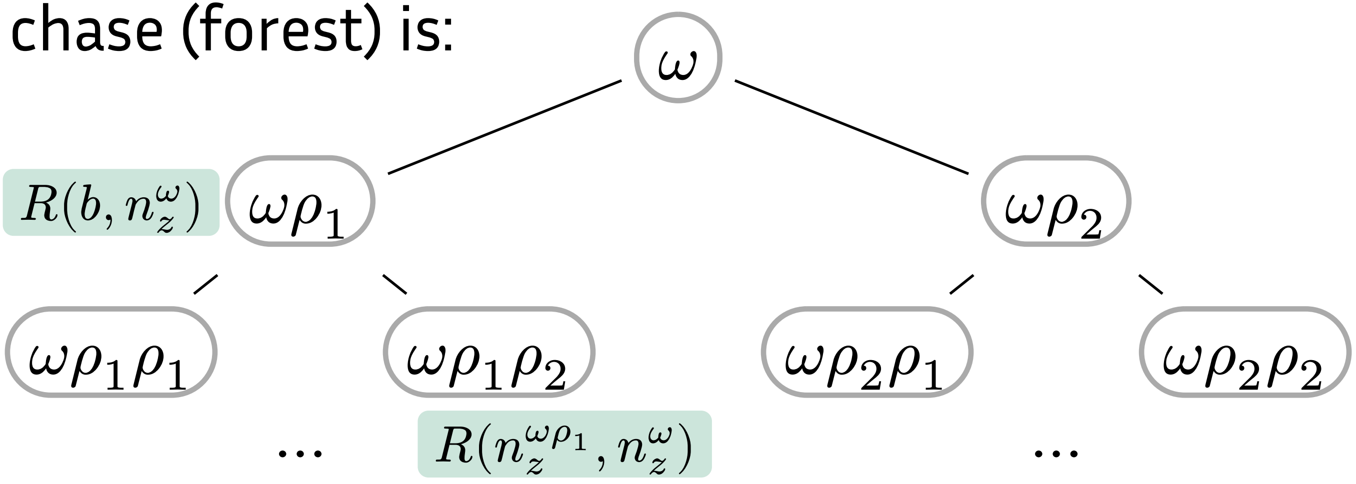

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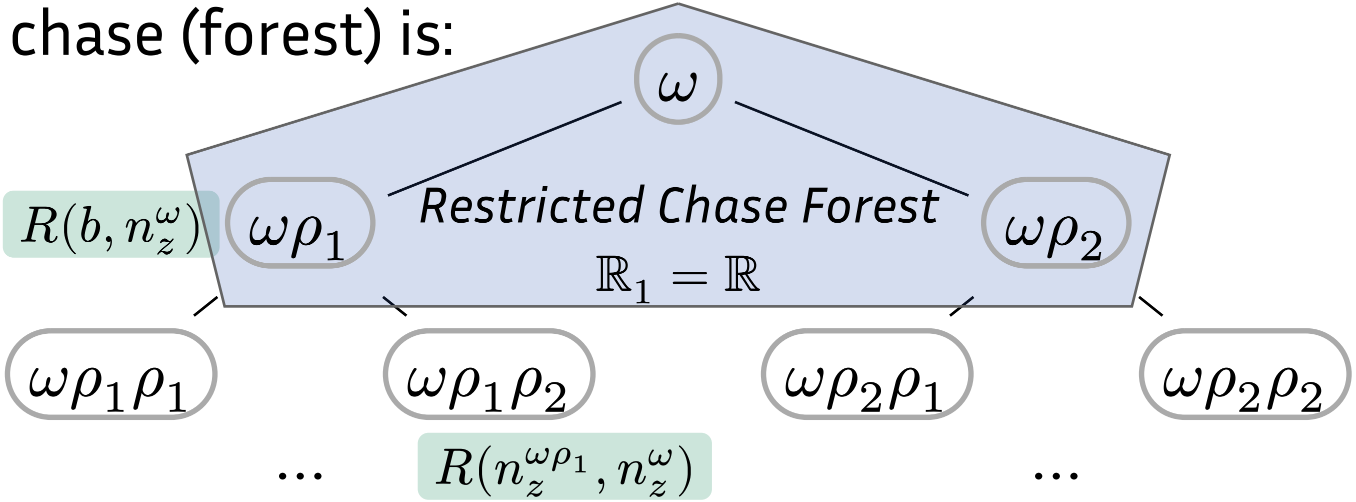

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Issue: How to make the oblivious triggers part of the restricted derivation without blocking triggers that were not blocked before?

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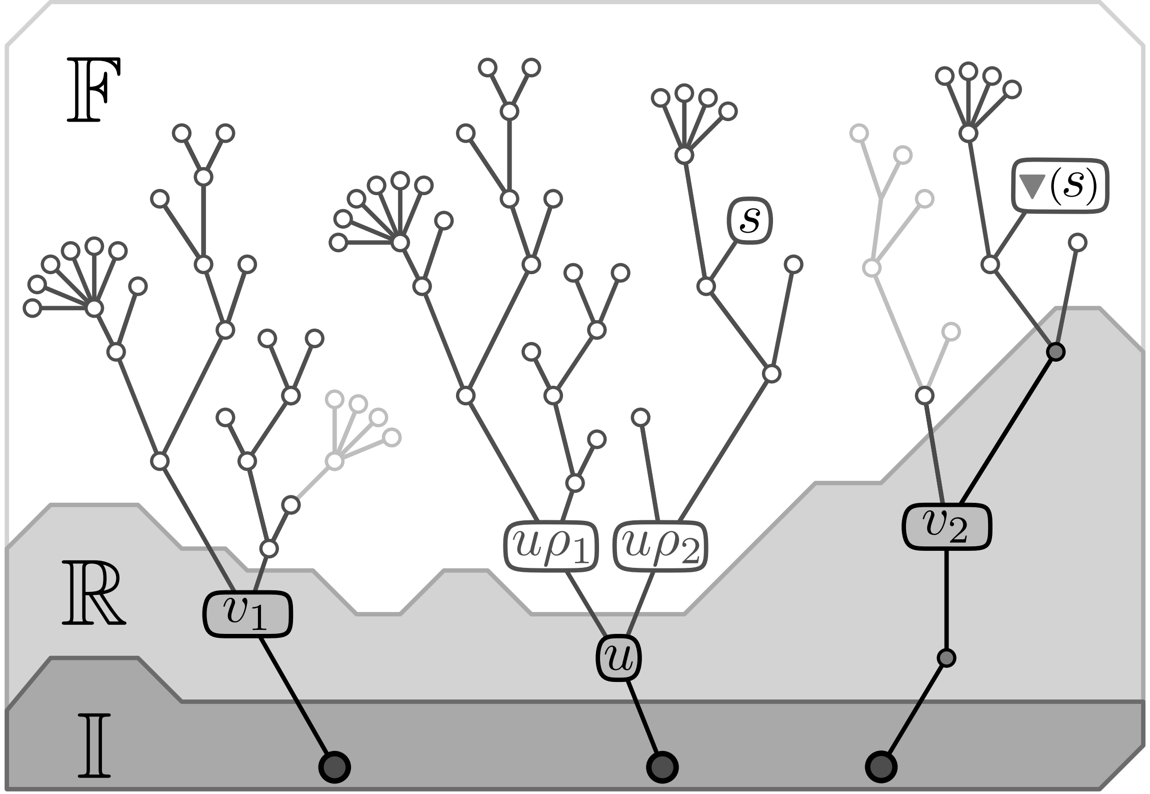

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Idea: "$\rightarrow$" - $\mathbb{R}$ must contain an infinite path. Extract the trigger seq. as $\mathcal{P}$.
"$\leftarrow$" - Set $\mathcal{R}$ to the non-blocked triggers of $\mathcal{M}$ and pick $\mathbb{I} = \{\omega\}$.

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MSOL Massage 💆

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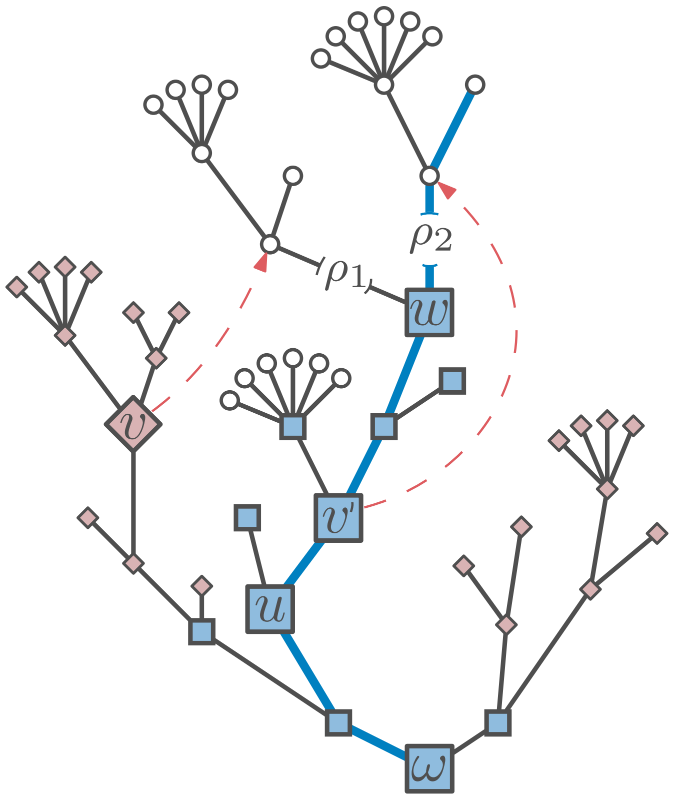
For each address $u \in \mathbb{P}^0$, define
$\mathrm{bfr}(u) = \{v \mid |v| \leq |u|\}$ and
$\mathrm{fragile}(u)$ as the $w \in \mathbb{P}^0$ after u where π_w is blocked by addresses not depending on w.



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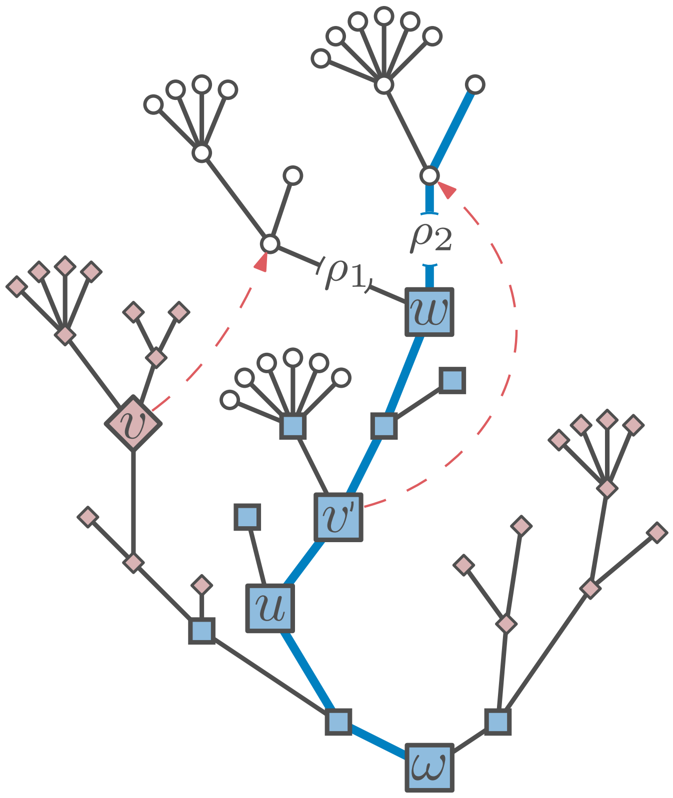

Lemma: There exists an $\mathcal{M}$ with $(\mathcal{M}, \mathcal{P})$ an ω-sensitive derivation *iff* for each $u \in \mathbb{P}^0$

1. π_u is not blocked by teams in $\text{bfr}(u)$ and
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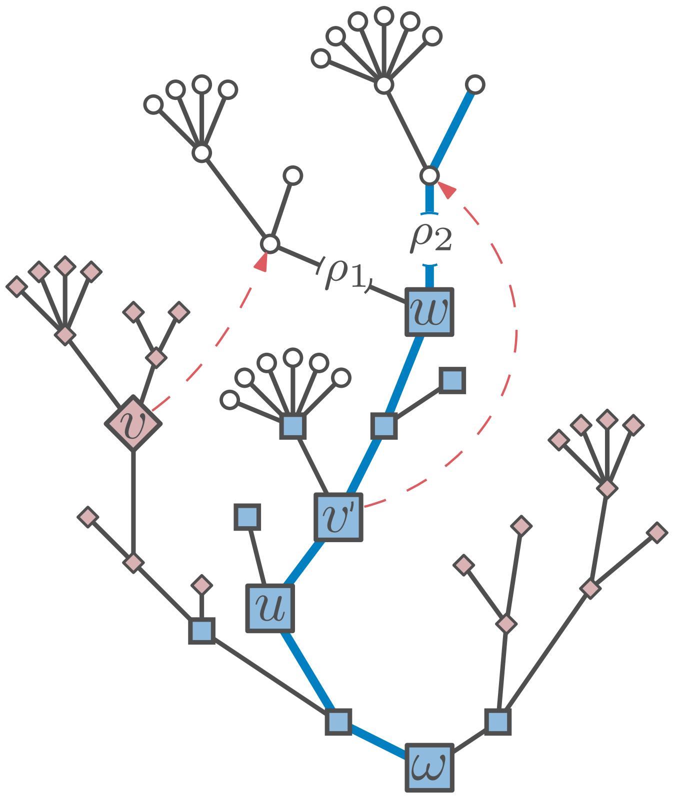

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These properties are MSOL-definable!

Summary 📋

Universal Chase Termination is decidable for linear multi-head existential rules.

By stepwise reduction to MSOL satisfiability over infinite trees of bounded degree through:

1. *mixed derivations* (embedding restricted derivation into oblivious derivation),
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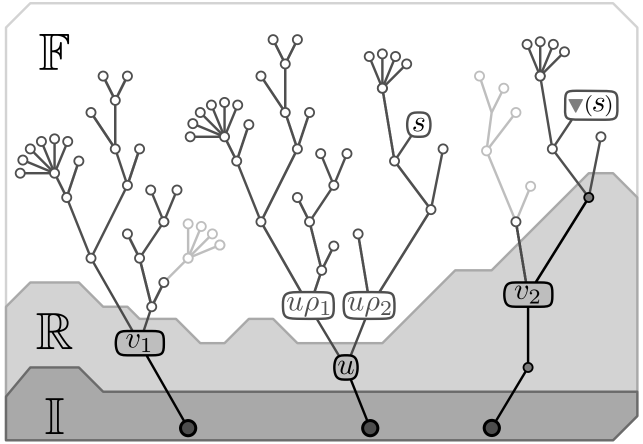
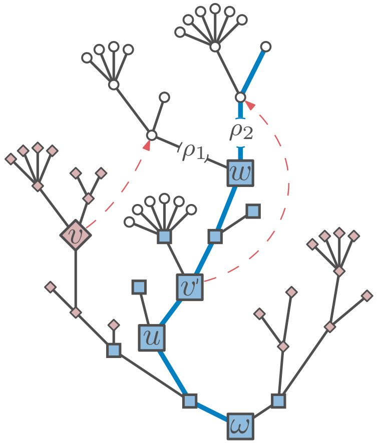
3. casting into MSOL-definable path properties.





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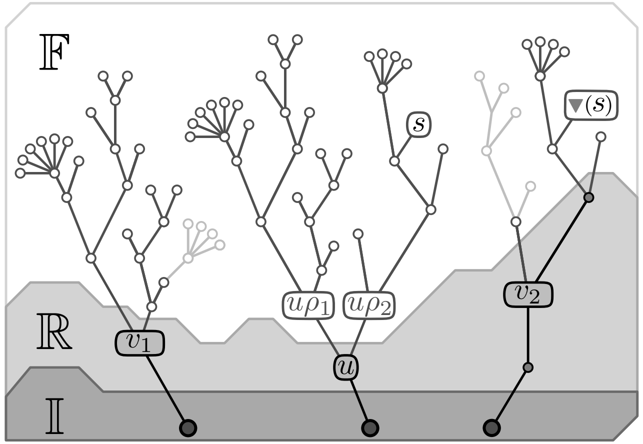
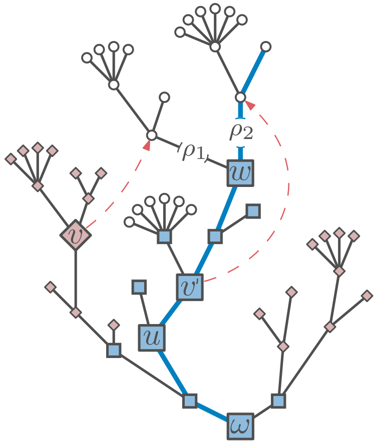
Open: What about guarded multi-head rules?





References

Gogacz T, Marcinkowski J, Pieris A (2023) Uniform Restricted Chase Termination. SIAM J Comput 52:641–683. https://doi.org/10.1137/20M1377035

Leclère M, Mugnier M-L, Thomazo M, Ulliana F (2019) A Single Approach to Decide Chase Termination on Linear Existential Rules. In: Barceló P, Calautti M (eds) 22nd International Conference on Database Theory, ICDT 2019, March 26-28, 2019, Lisbon, Portugal. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, pp 18:1–18:19