Reasoning in $\mathcal{SHIQ}$ with Axiom- and Concept-Level Standpoint Modalities

Lucía Gómez Álvarez, Sebastian Rudolph

Motivation

Multiperspective Reasoning

Applications in Knowledge Integration

Applications in Knowledge Integration

Diverse Knowledge Sources

Applications in Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

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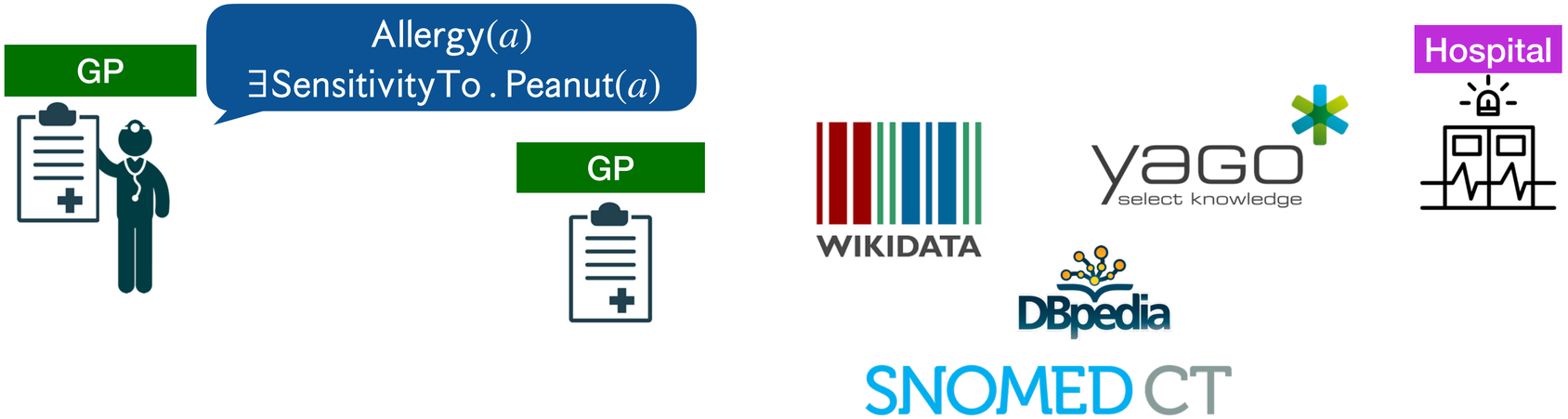

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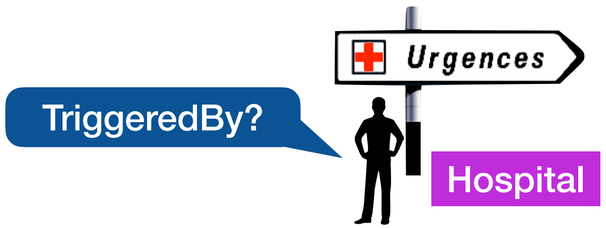
Non-trivial combinations of the huge diversity of knowledge sources available
Knowledge sources embed the perspectives of their creators!



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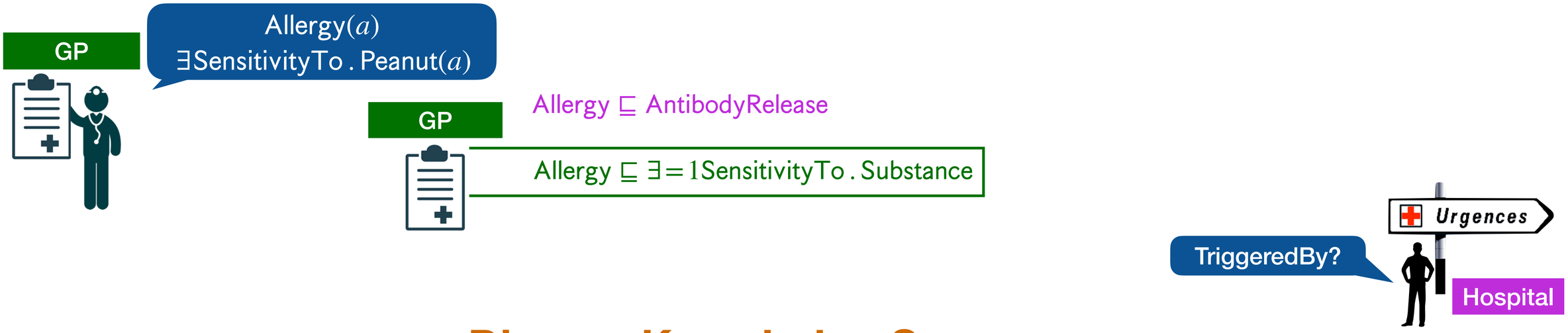

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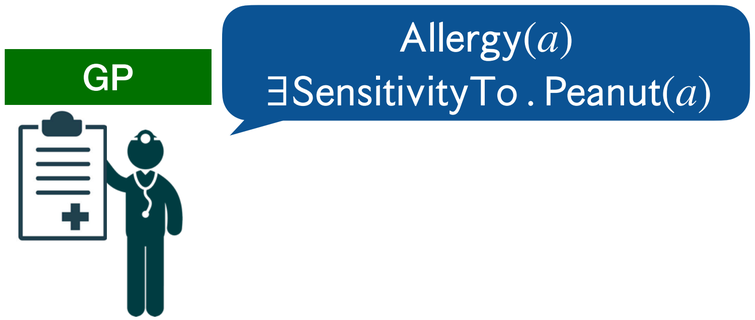
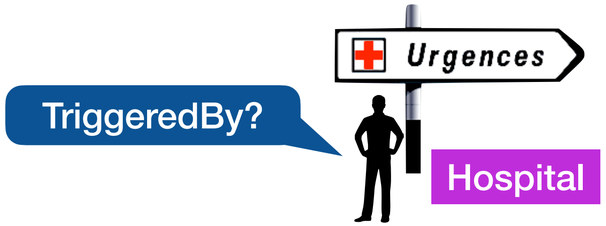


Allergy ⊑ AntibodyRelease

Allergy ⊑ ∃=1SensitivityTo . Substance

AntibodyRelease ⊓ ∃SensitivityTo . Substance ⊑ ⊥



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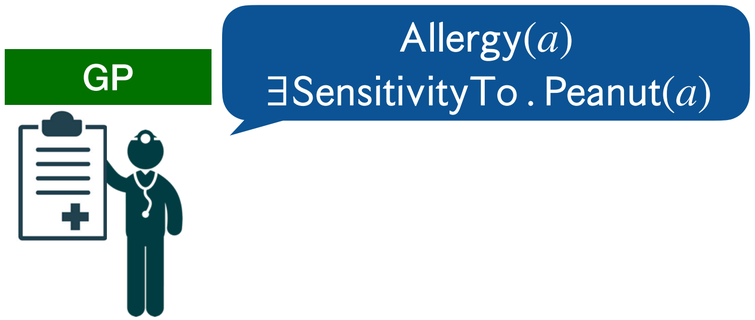

$\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}$

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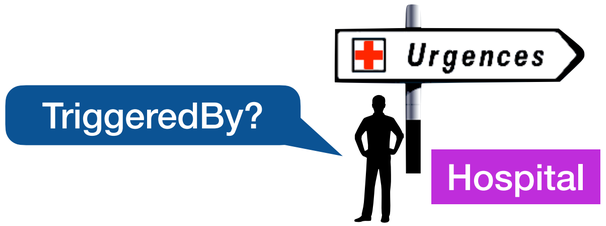
$\mathsf{AntibodyRelease} \sqcap \exists\mathsf{SensitivityTo} . \mathsf{Substance} \sqsubseteq \bot$



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Challenge: Integration

GP

$\mathrm{Allergy}(a)$
$\exists \mathrm{SensitivityTo} \,.\, \mathrm{Peanut}(a)$

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TriggeredBy?

Hospital

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Multiperspective Ontology Management

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Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

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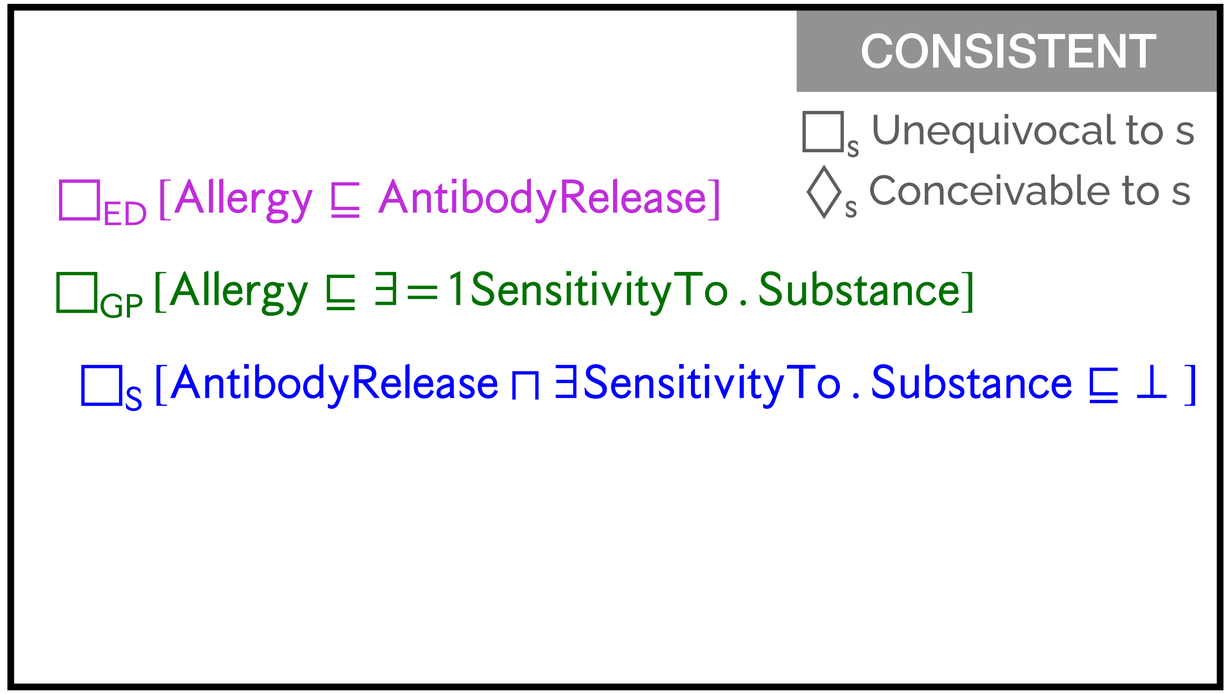

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CONSISTENT

$\Box_s$ Unequivocal to s

$\Diamond_s$ Conceivable to s

$\Box_{ED}$ [Allergy $\sqsubseteq$ AntibodyRelease]

$\Box_{GP}$ [Allergy $\sqsubseteq \exists{=}1$SensitivityTo . Substance]

$\Box_S$ [AntibodyRelease $\sqcap \exists$SensitivityTo . Substance $\sqsubseteq \bot$]

(ED $\cup$ GP) $\preceq$ S

(L and H inherit the axioms of S)

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$\Box_{GP}\,[\text{Allergy} \sqsubseteq \exists{=}1\text{SensitivityTo} \,.\, \text{Substance}]$

$\Box_S\,[\text{AntibodyRelease} \sqcap \exists\text{SensitivityTo} \,.\, \text{Substance} \sqsubseteq \bot\,]$

$(\text{ED} \cup \text{GP}) \preceq \text{S}$ (L and H inherit the axioms of S)

$\Box_{ED}\,[\text{Allergy} \sqsubseteq \exists\text{TriggeredBy} \,.\, \Diamond_{GP}(\text{Allergy})]$

Standpoint $\mathcal{SHIQ}$

The description logic $\mathcal{SHIQ}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid A \mid \neg C \mid C_1 \sqcap C_2 \mid \exists r \,.\, C \mid \exists \leq n r \,.\, C$$

With $A \in N_C, r \in N_R$

$\mathsf{Substance} \quad \exists \mathsf{sensitiveTo} \,.\, \mathsf{Nut}$

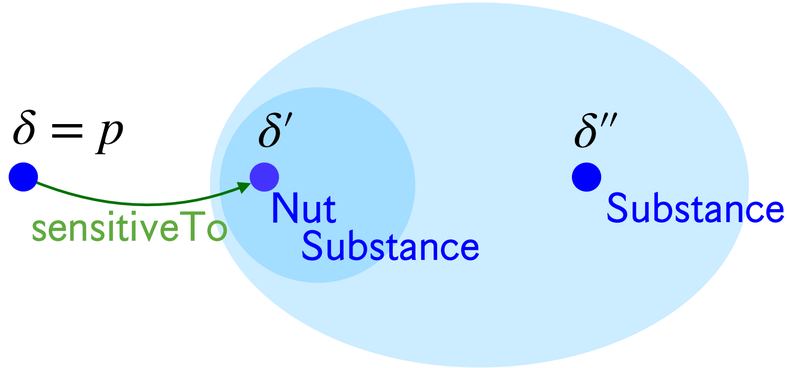
$\mathsf{Substance} \sqcap \neg \mathsf{Dangerous} \quad \exists \leq 1 \mathsf{sensitiveTo} \,.\, \mathsf{Substance}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D, \quad R \sqsubseteq R', \quad R \circ R \sqsubseteq R$
- Assertions: $C(a), \quad r(a, b)$

$$\big(\mathsf{Peanut} \sqsubseteq \mathsf{Substance}\big) \wedge \neg \big(\exists \mathsf{sensitiveTo} \,.\, \mathsf{Peanut}\big)(p)$$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



Towards Standpoint-$\mathcal{SHIQ}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual

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Substance	$\exists$sensitiveTo . Nut
Substance $\sqcap \neg$Dangerous	$\exists \leq 1$sensitiveTo . Substance

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$(\text{Peanut} \sqsubseteq \text{Substance})$	$(\exists \text{sensitiveTo} \,.\, \Diamond_P \text{Nut})(p)$

Towards Standpoint-$\mathcal{SHIQ}$

Vocabulary $\langle N_C, N_R, N_I, N_S \rangle$ of concept, role, individual and standpoint names

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Vocabulary $\langle N_C, N_R, N_I, N_S \rangle$ of concept, role, individual and standpoint names, $* \in N_S$ (universal standpoint).

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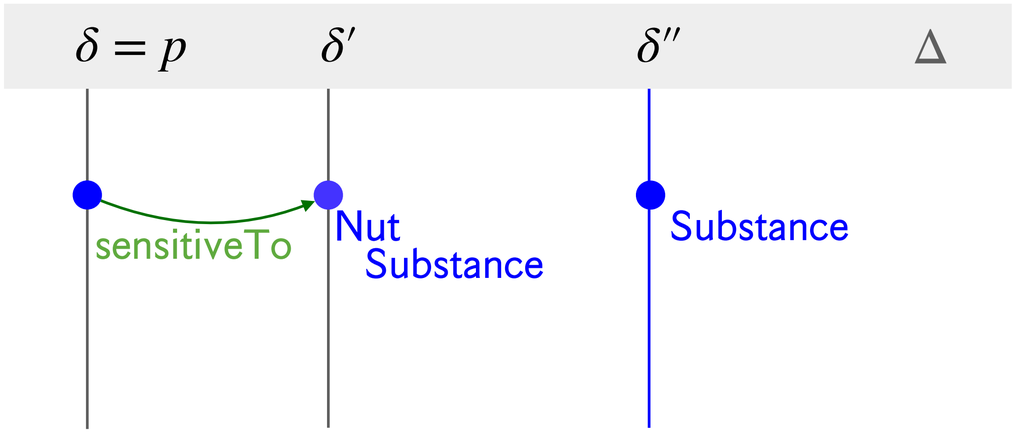

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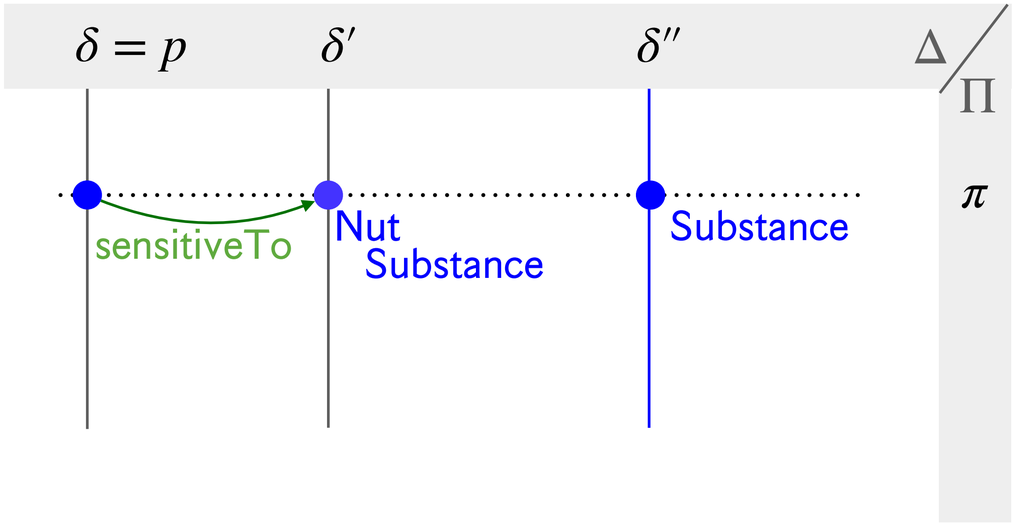

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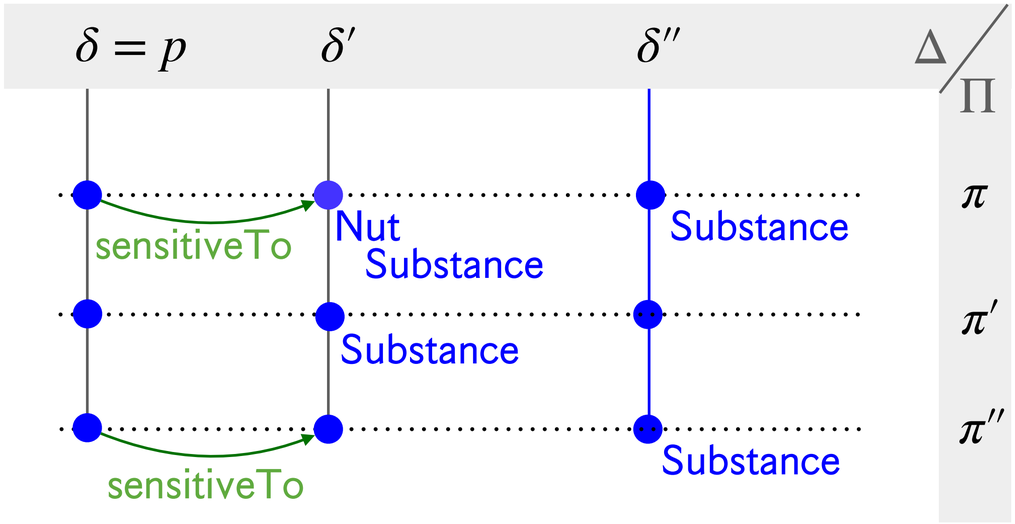

Towards Standpoint-$\mathcal{SHIQ}$

Vocabulary $\langle N_C, N_R, N_I, N_S \rangle$ of concept, role, individual and standpoint names, $* \in N_S$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid A \mid \neg C \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists{\leq}nr\,.\,C \mid \odot_s C$$

With $A \in N_C, r \in N_R, s \in N_S, \odot \in \{\Box, \Diamond\}$.

Substance $\quad \exists \text{sensitiveTo}\,.\,\text{Nut} \quad \Diamond_S \text{Dangerous}$

$\text{Substance} \sqcap \neg\text{Dangerous} \quad \exists \leq 1 \text{sensitiveTo}\,.\,\text{Substance}$

Formulas are $\odot_s (\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

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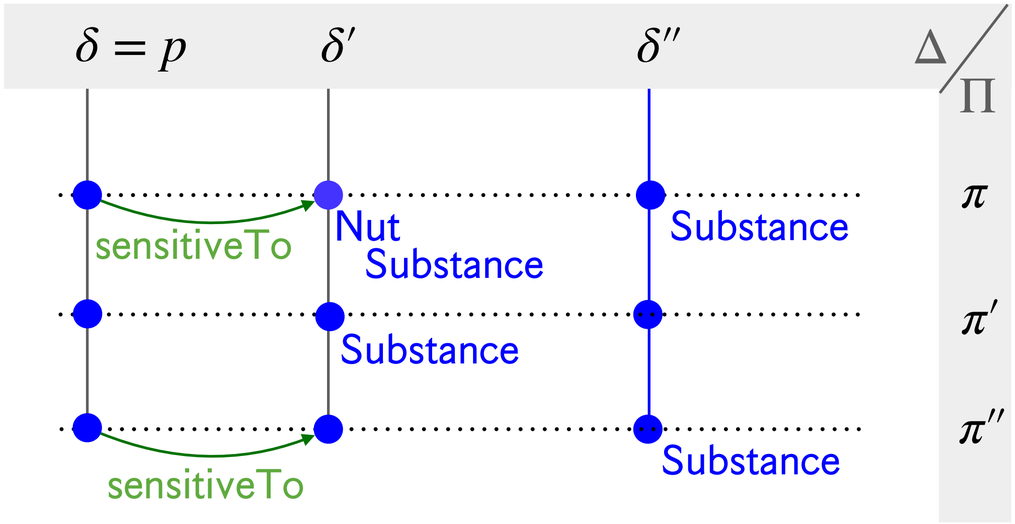

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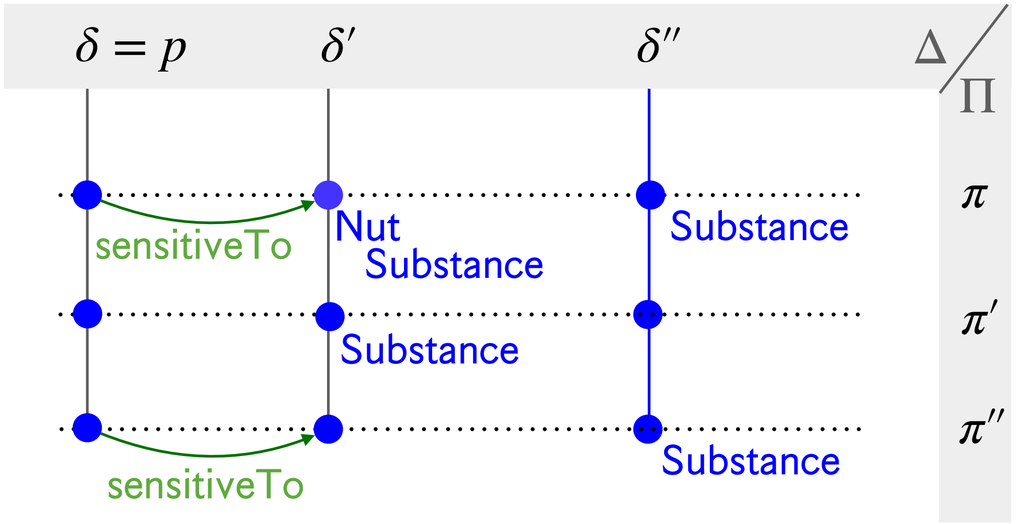

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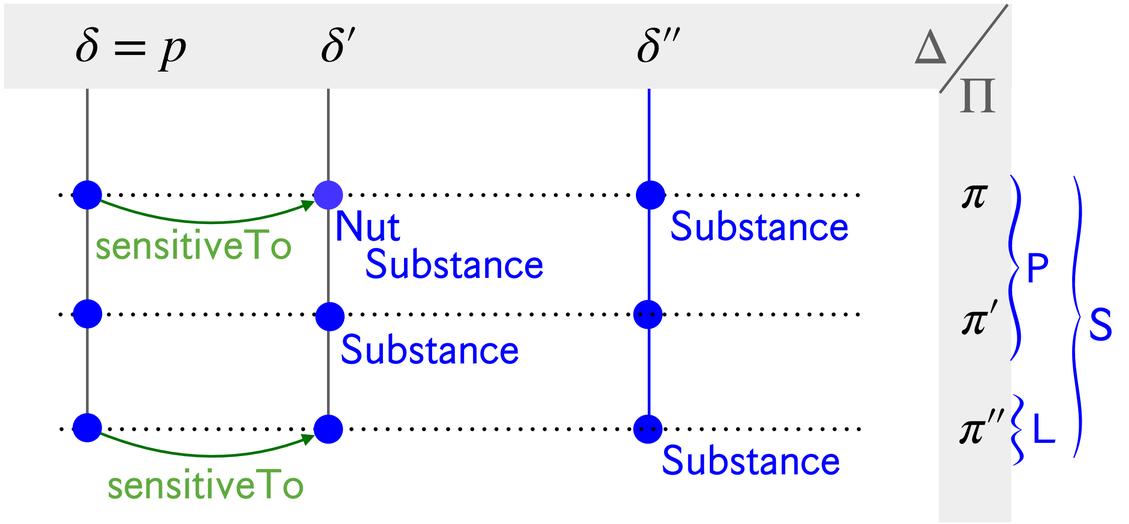

Reasoning with Standpoint DLs

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- Nominals break the small model property

Small Models for Standpoint $\mathcal{SHIQ}$

Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

Normalisation:

- Sharpenings not using 0:
  - $s' \preceq s$ $\quad$ $s_1 \cap s_2 \preceq s$
- GCIs:
  - $\Box_s(\top \sqsubseteq C)$ $\quad$ with C in NNF
- Other modalised axioms :
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Notice that $\Diamond_s$ only occurs at the concept level

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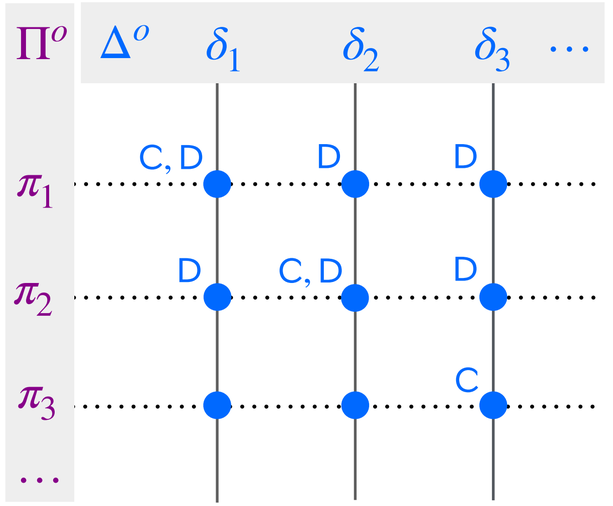
$$\mathcal{K} = \{ \Box_\star (\top \sqsubseteq \mathsf{D}), \; \Box_\star (\top \sqsubseteq \Diamond_\star \mathsf{C}) \}$$



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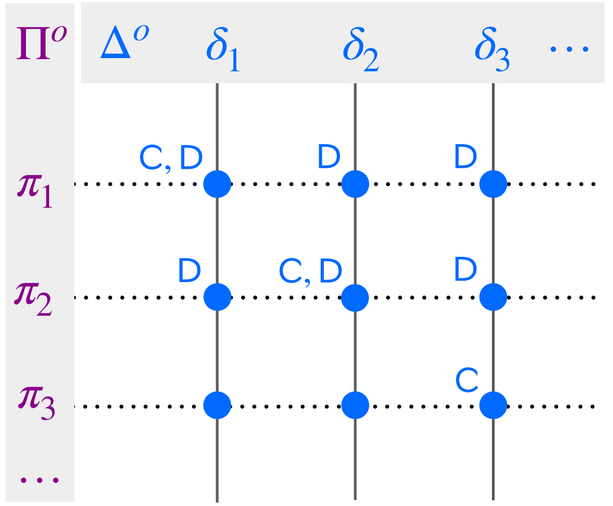

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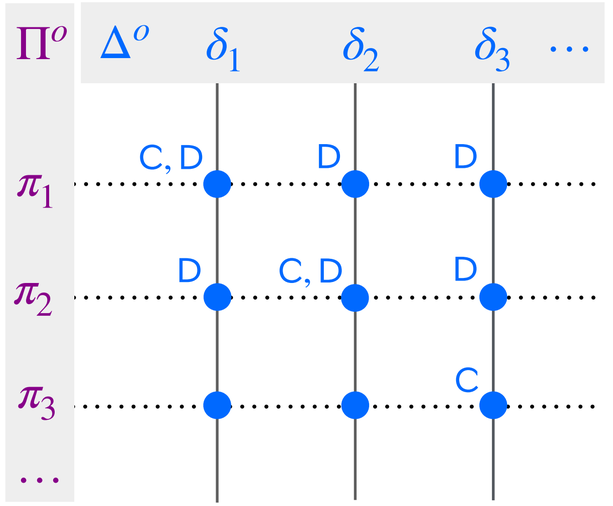

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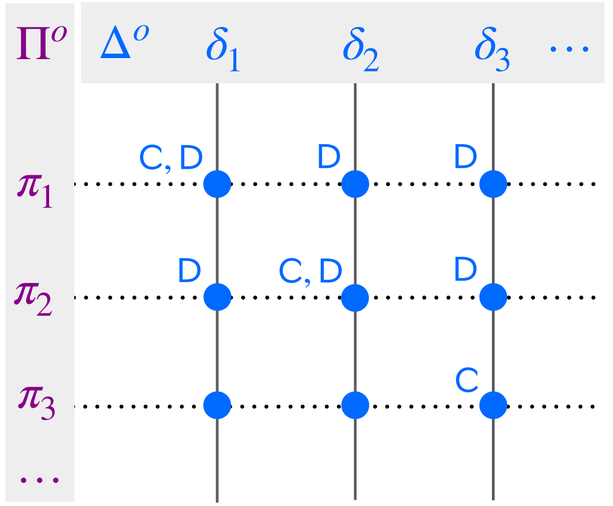

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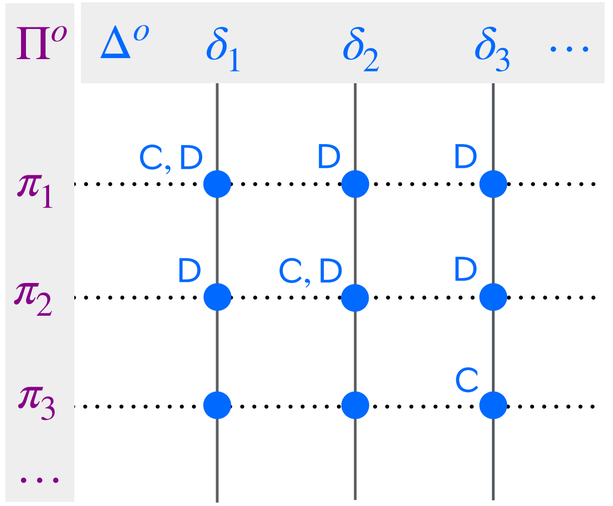

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  - How do we pick witnesses?

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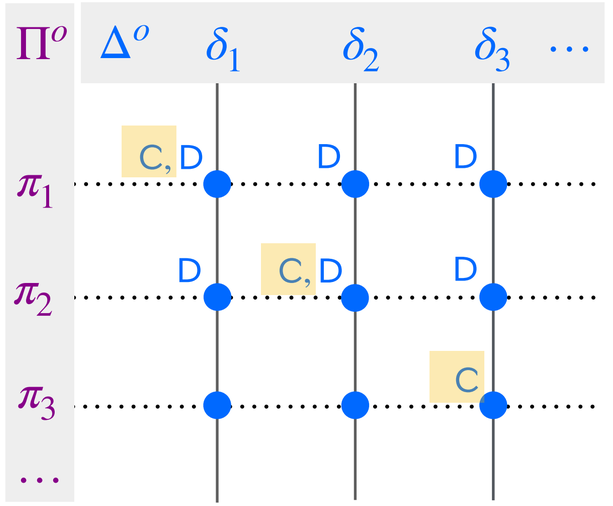

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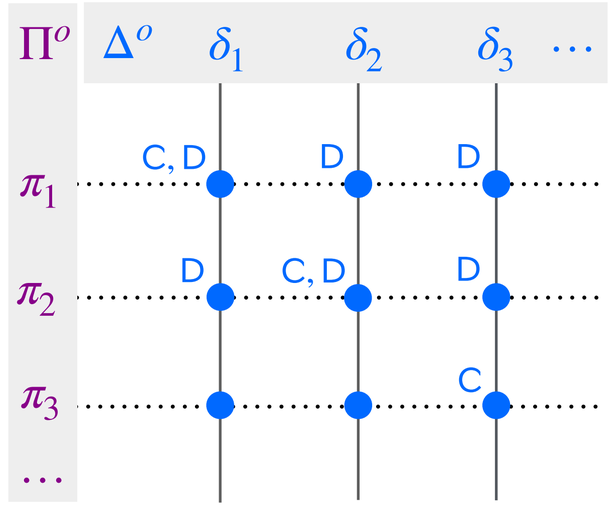

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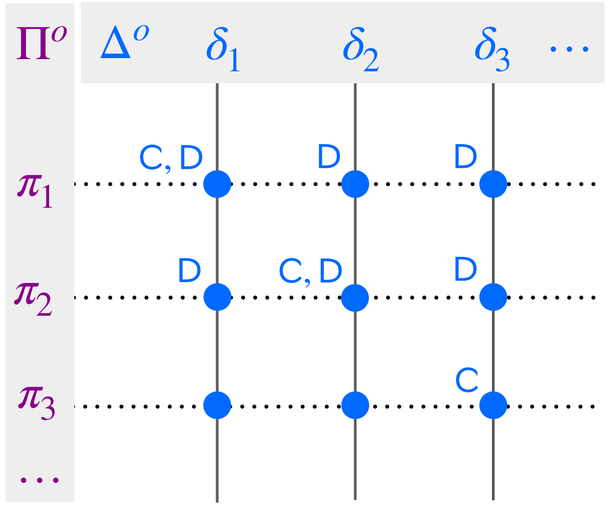

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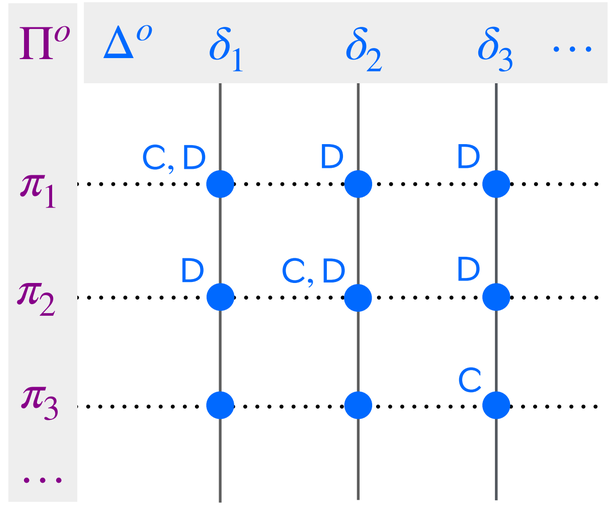

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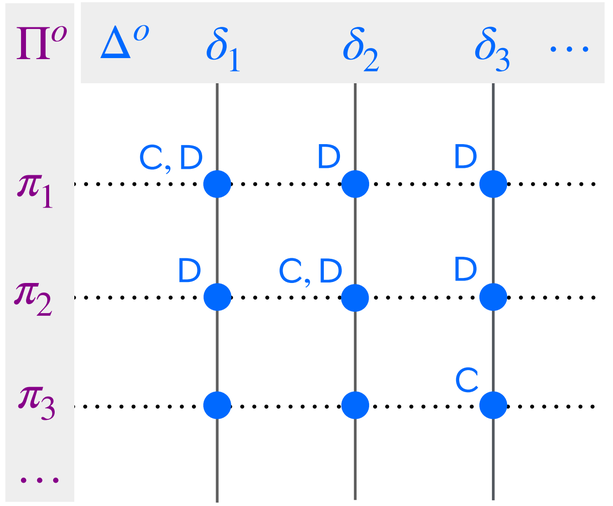

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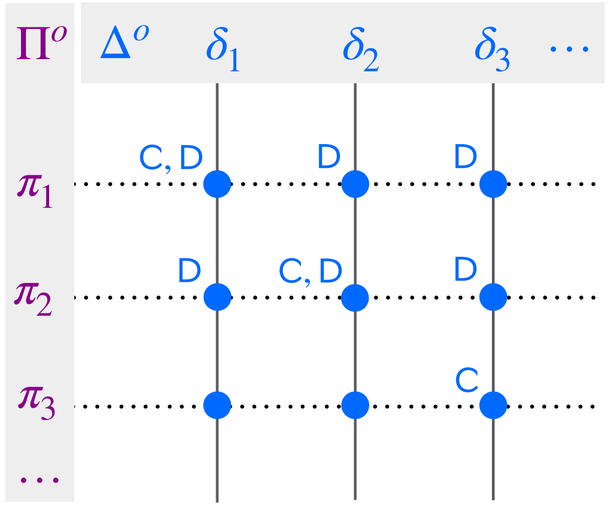

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For witnessing $\Diamond_{\mathrm{s}} C$ membership:

- A witness for each **named individual** a
- For **unnamed individuals**:
  - ➡ Squeeze infinite copies of all s precisifications
  - ➡ Arrange them so for each δ, either $\pi^{0}_{\mathrm{s},C}$ or $\pi^{1}_{\mathrm{s},C}$ deals with the $\Diamond_{\mathrm{s}} C$ membership

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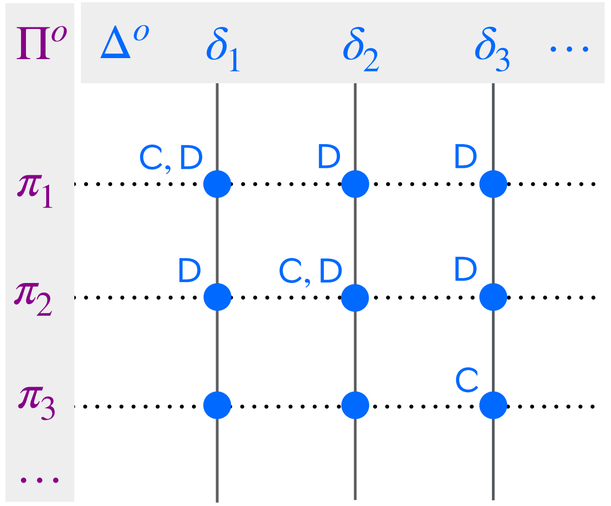

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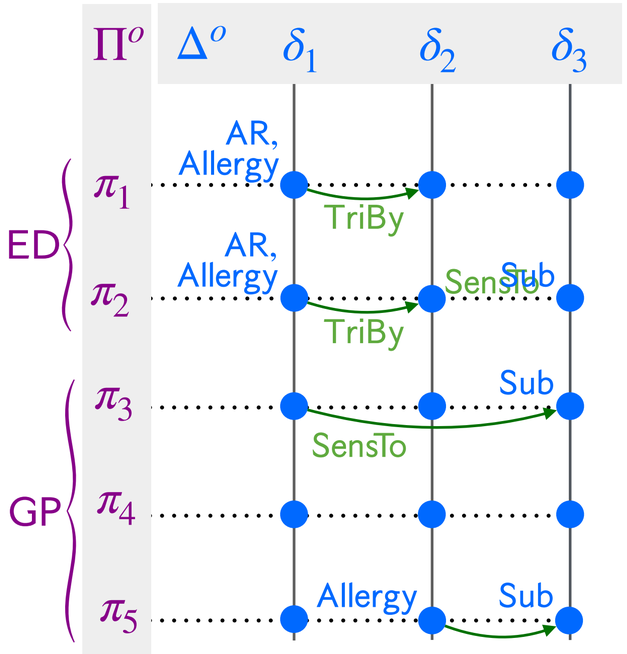
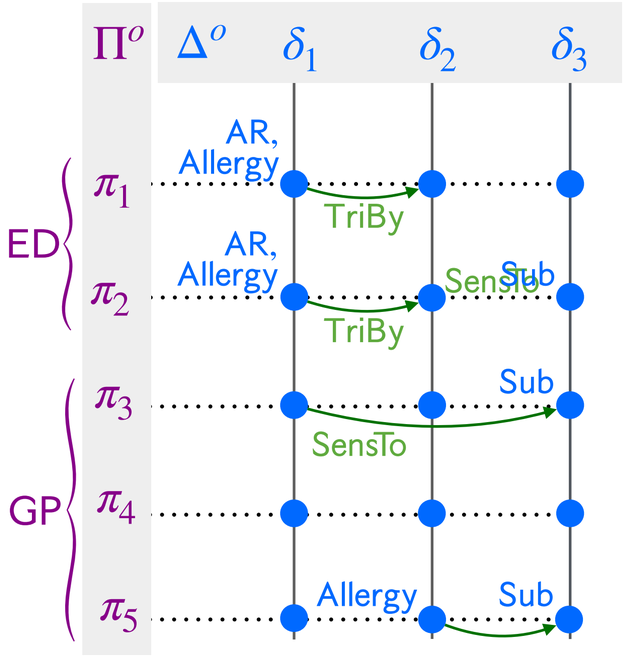
$$\Box_{\mathsf{GP}}\,(\mathsf{Allergy} \sqsubseteq = 1\,\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),$$
$$\Box_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \exists\,\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\}$$



Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\mathcal{K} = \{\ \Box_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),$$
$$\Box_{\mathsf{GP}}\,(\mathsf{Allergy} \sqsubseteq = 1\,\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),$$
$$\Box_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\}$$



Δ

Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\mathcal{K} = \{\ \Box_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),$$
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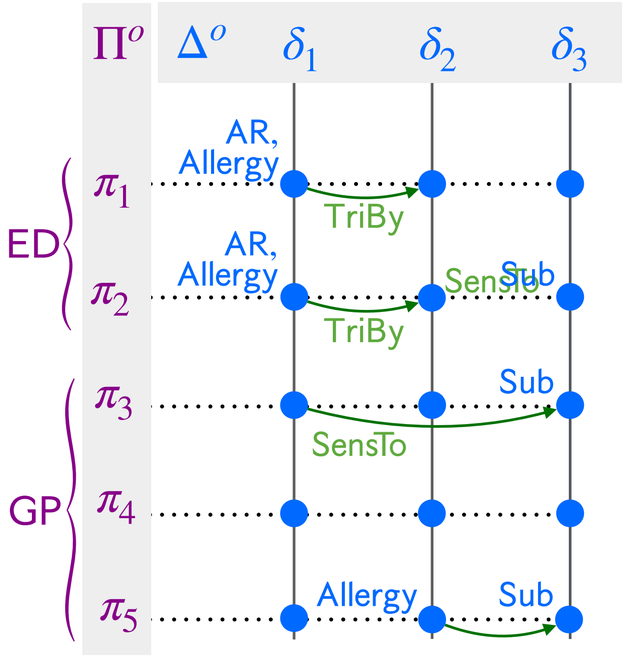
$$\Box_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\}$$



Δ $<\delta_1,0>$ $<\delta_2,0>$ $<\delta_3,0>$

Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\mathcal{K} = \{\ \Box_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),$$
$$\Box_{\mathsf{GP}}\,(\mathsf{Allergy} \sqsubseteq = 1\,\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),$$
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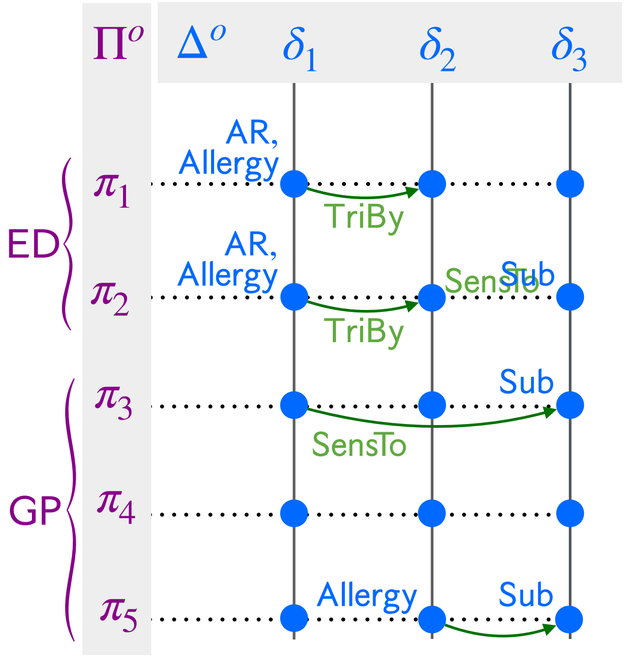
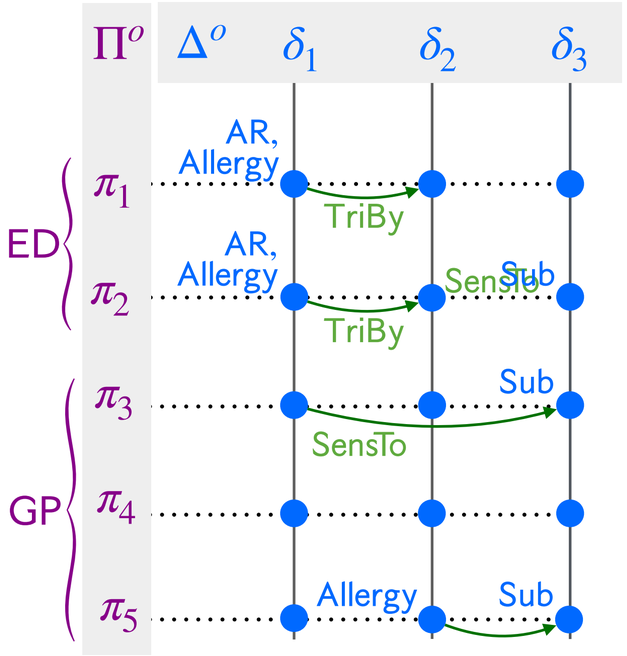


Δ $<\delta_1,0>$ $<\delta_2,0>$ $<\delta_3,0>$ $<\delta_1,1>$ $<\delta_2,1>$ $<\delta_3,1>$

Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$\mathcal{K} = \{ \Box_{\text{ED}}(\text{Allergy} \sqsubseteq \text{AntibodyRelease}),$
$\Box_{\text{GP}}(\text{Allergy} \sqsubseteq = 1\,\text{SensitivityTo}\,.\,\text{Substance}),$
$\Box_{\text{ED}}(\text{Allergy} \sqsubseteq \exists\,\text{TriggeredBy}\,.\,\Diamond_{\text{GP}}\text{Allergy})\}$



Δ $<\delta_1,0>$ $<\delta_2,0>$ $<\delta_3,0>$ $<\delta_1,1>$ $<\delta_2,1>$ $<\delta_3,1>$ $<\delta_1,2>$ $<\delta_2,2>$ $<\delta_3,2>$

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$$\mathcal{K} = \{\ \Box_{\mathsf{ED}}\,(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),$$
$$\Box_{\mathsf{GP}}\,(\mathsf{Allergy} \sqsubseteq\!= 1\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),$$
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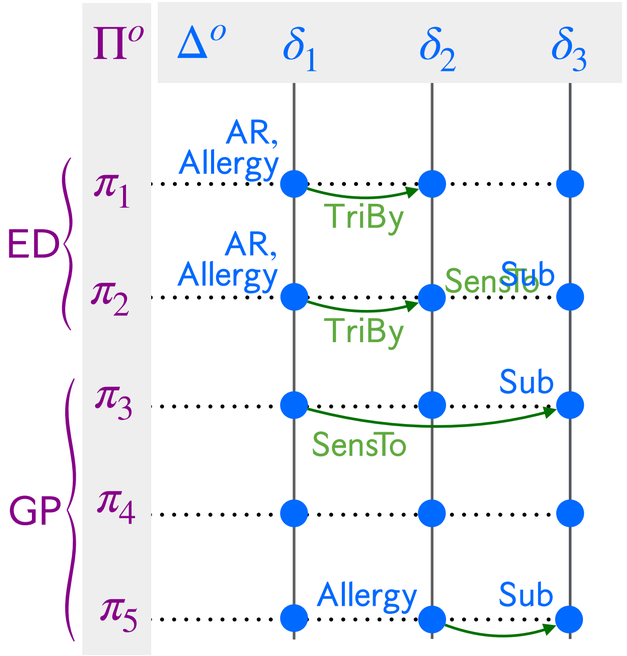

Δ $<\delta_1,0>$ $<\delta_2,0>$ $<\delta_3,0>$ $<\delta_1,1>$ $<\delta_2,1>$ $<\delta_3,1>$ $<\delta_1,2>$ $<\delta_2,2>$ $<\delta_3,2>$ $<\delta_1,3>$ $<\delta_2,3>$ $<\delta_3,3>$

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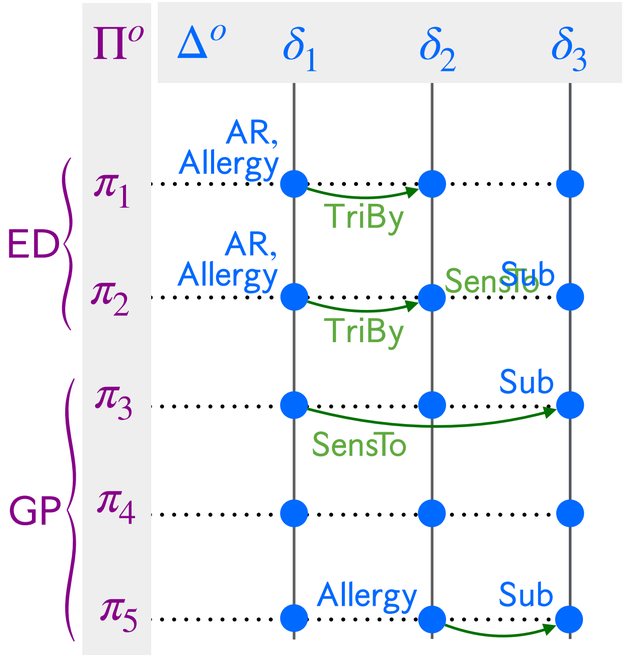

Δ $<\delta_1,0>$ $<\delta_2,0>$ $<\delta_3,0>$ $<\delta_1,1>$ $<\delta_2,1>$ $<\delta_3,1>$ $<\delta_1,2>$ $<\delta_2,2>$ $<\delta_3,2>$ $<\delta_1,3>$ $<\delta_2,3>$ $<\delta_3,3>\ldots$

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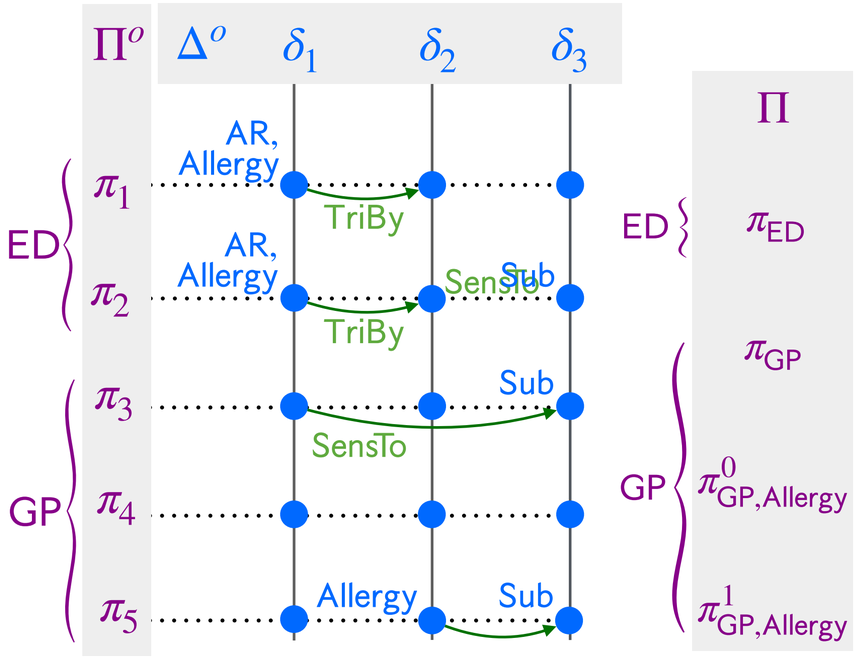
$$\Box_{\text{ED}}(\text{Allergy} \sqsubseteq \exists \text{TriggeredBy} \,.\, \Diamond_{\text{GP}}\text{Allergy})\}$$



Δ $<\delta_1,0>$ $<\delta_2,0>$ $<\delta_3,0>$ $<\delta_1,1>$ $<\delta_2,1>$ $<\delta_3,1>$ $<\delta_1,2>$ $<\delta_2,2>$ $<\delta_3,2>$ $<\delta_1,3>$ $<\delta_2,3>$ $<\delta_3,3>\ldots$

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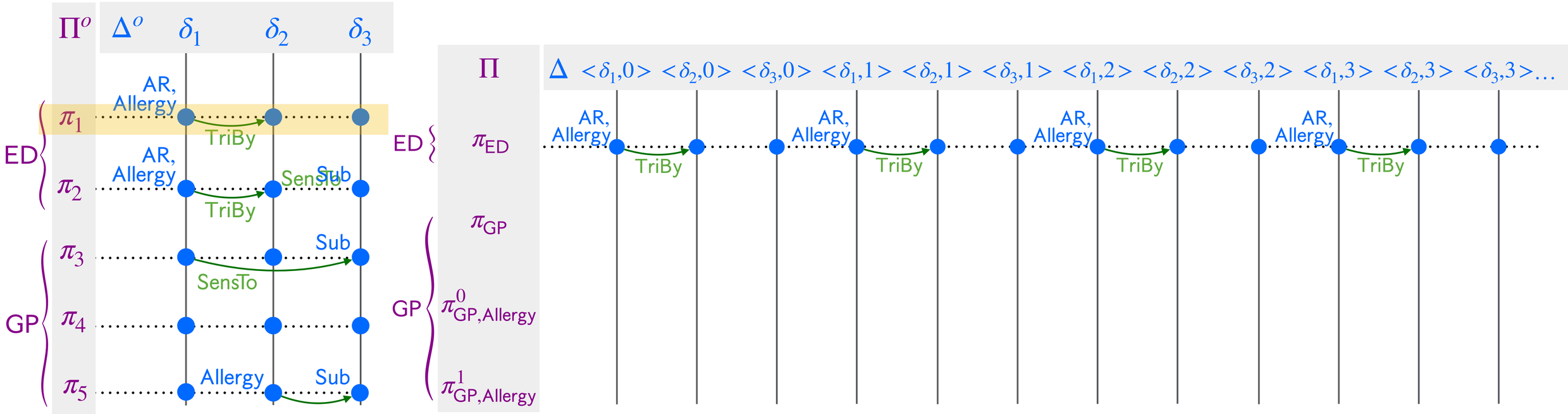

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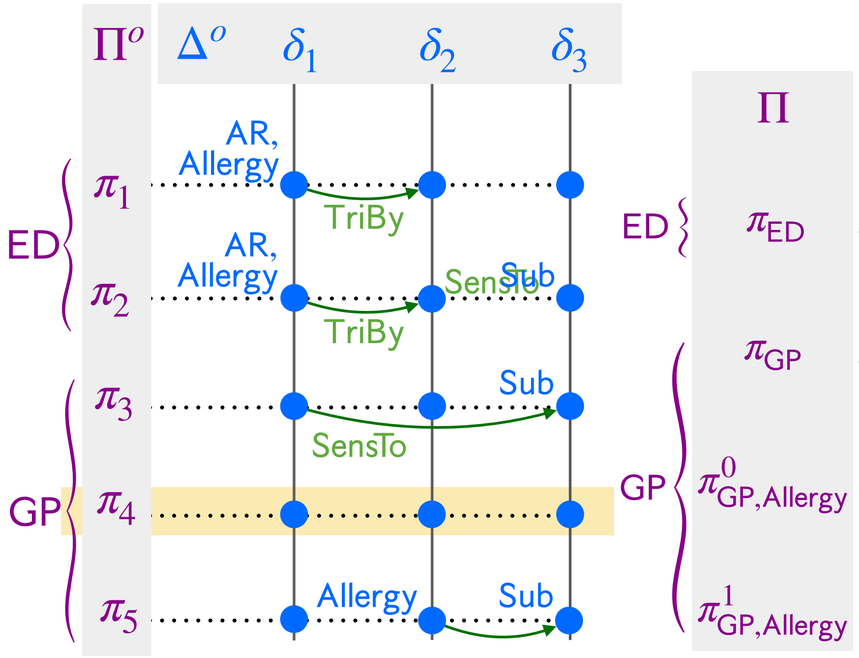
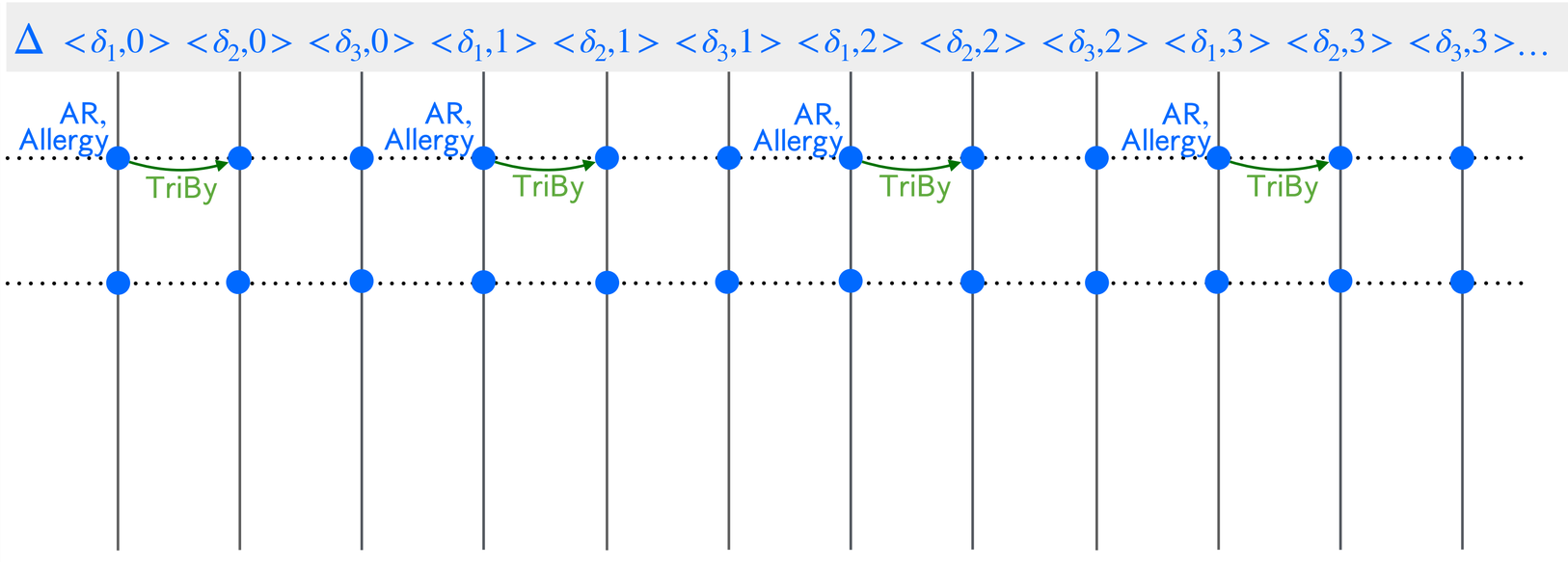
$$\Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
π_{ED}	δ_1	$\pi_1, 0$	$\pi_1, 1$	$\pi_1, 2$	$\pi_1, 3$	
	δ_2	$\pi_1, 0$	$\pi_1, 1$	$\pi_1, 2$	$\pi_1, 3$	
	δ_3	$\pi_1, 0$	$\pi_1, 1$	$\pi_1, 2$	$\pi_1, 3$	
π_{GP}	δ_1	$\pi_4, 0$	$\pi_4, 1$	$\pi_4, 2$	$\pi_4, 3$	
	δ_2	$\pi_4, 0$	$\pi_4, 1$	$\pi_4, 2$	$\pi_4, 3$	
	δ_3	$\pi_4, 0$	$\pi_4, 1$	$\pi_4, 2$	$\pi_4, 3$	

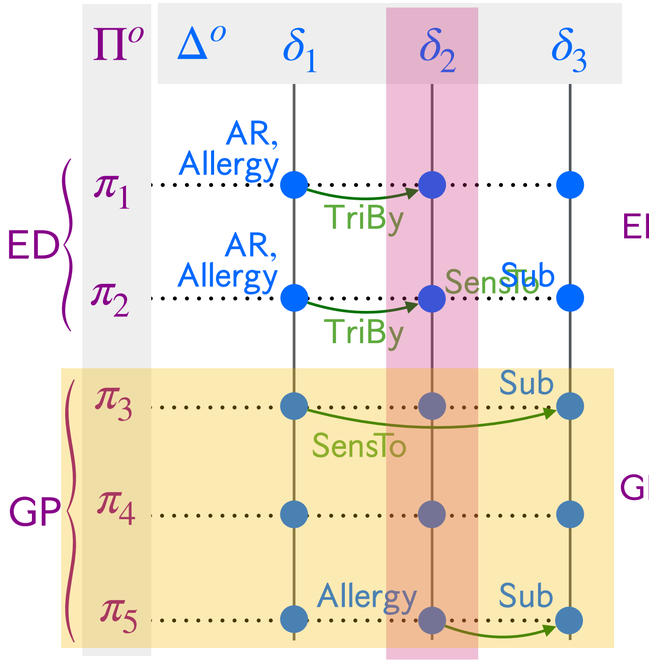


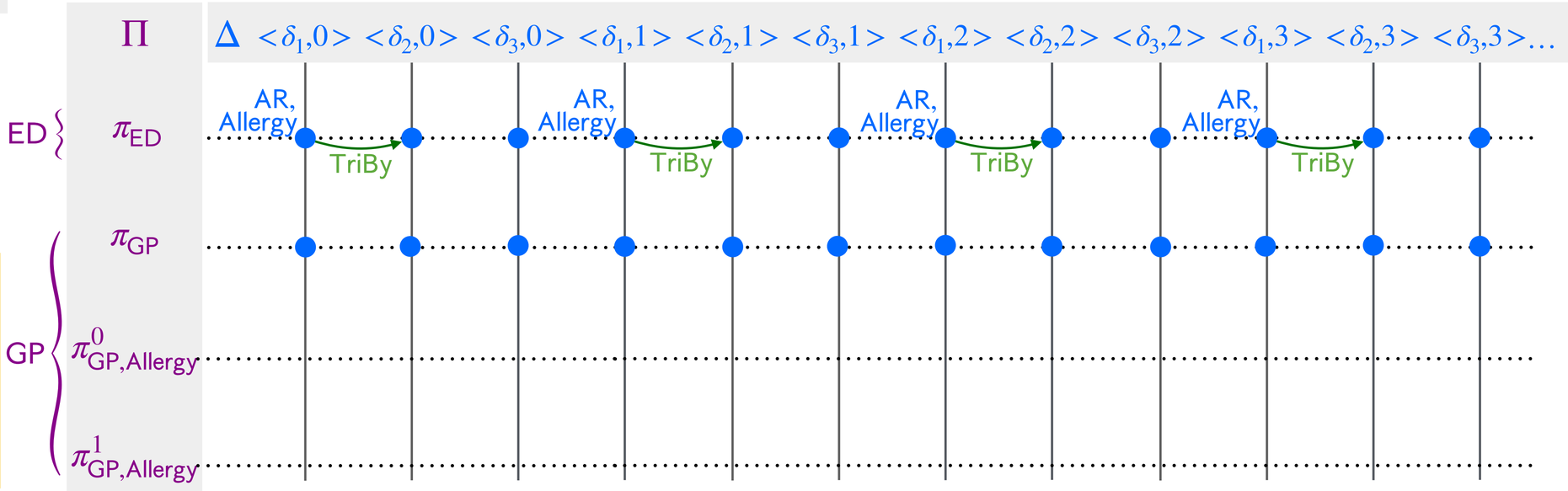


Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\mathcal{K} = \{\ \Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),$$
$$\Box_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq = 1\,\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),$$
$$\Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	

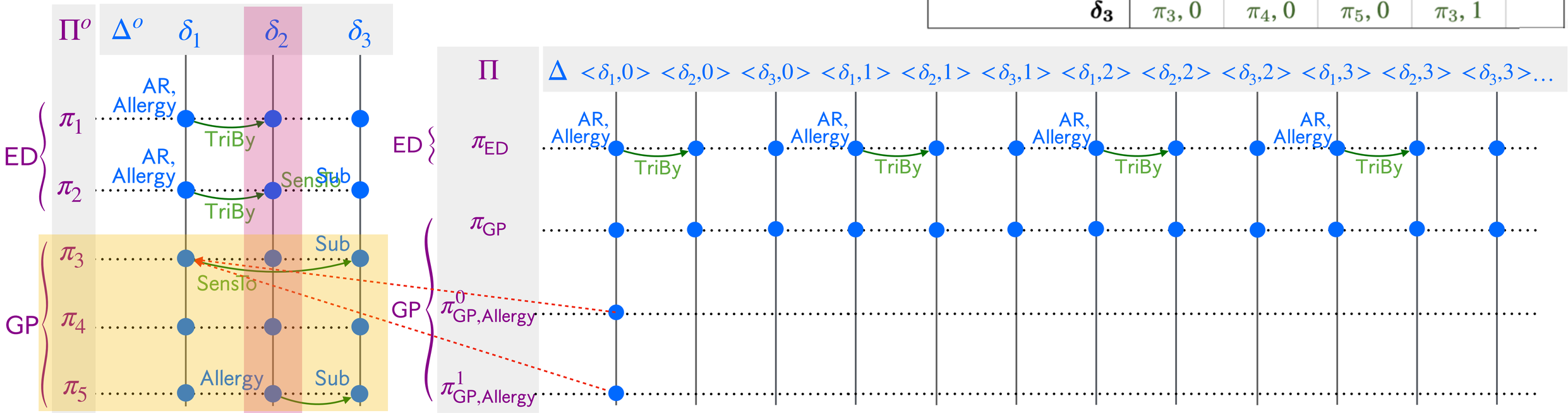




Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

$$\mathcal{K} = \{\ \Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \mathsf{AntibodyRelease}),$$
$$\Box_{\mathsf{GP}}(\mathsf{Allergy} \sqsubseteq = 1\,\mathsf{SensitivityTo}\,.\,\mathsf{Substance}),$$
$$\Box_{\mathsf{ED}}(\mathsf{Allergy} \sqsubseteq \exists\mathsf{TriggeredBy}\,.\,\Diamond_{\mathsf{GP}}\mathsf{Allergy})\}$$

Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	



Small model property for $\mathbb{S}_{\mathcal{SHIQ}}$

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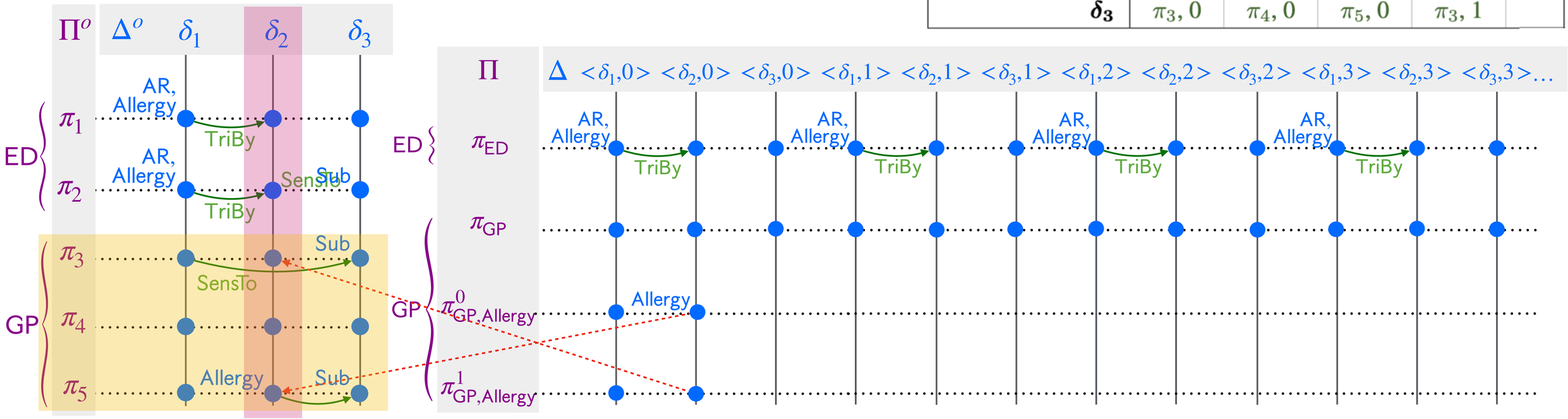
Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	



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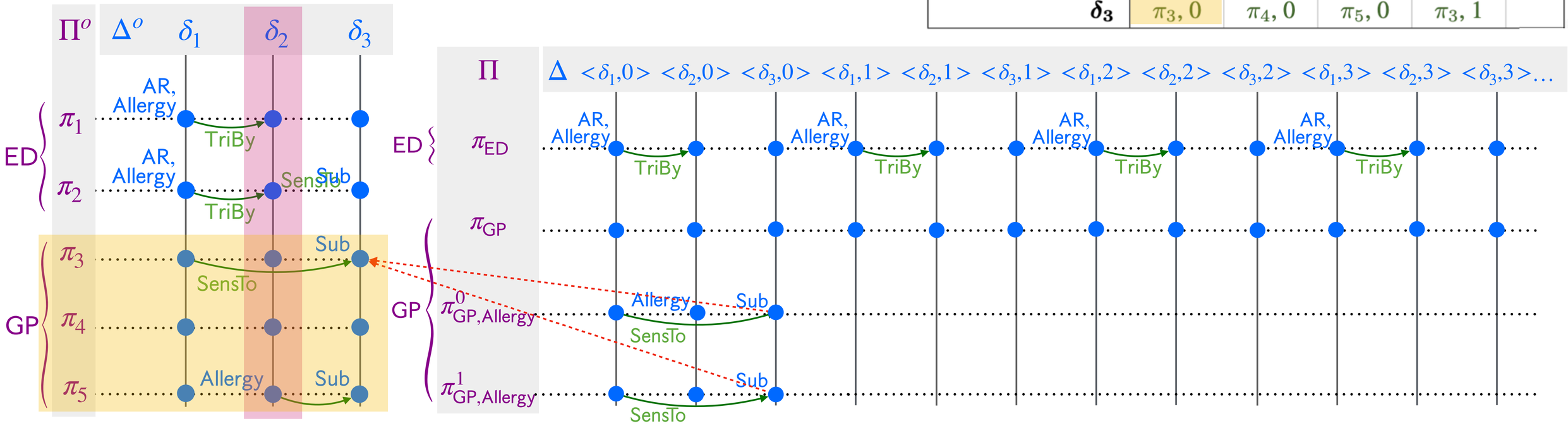
Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	



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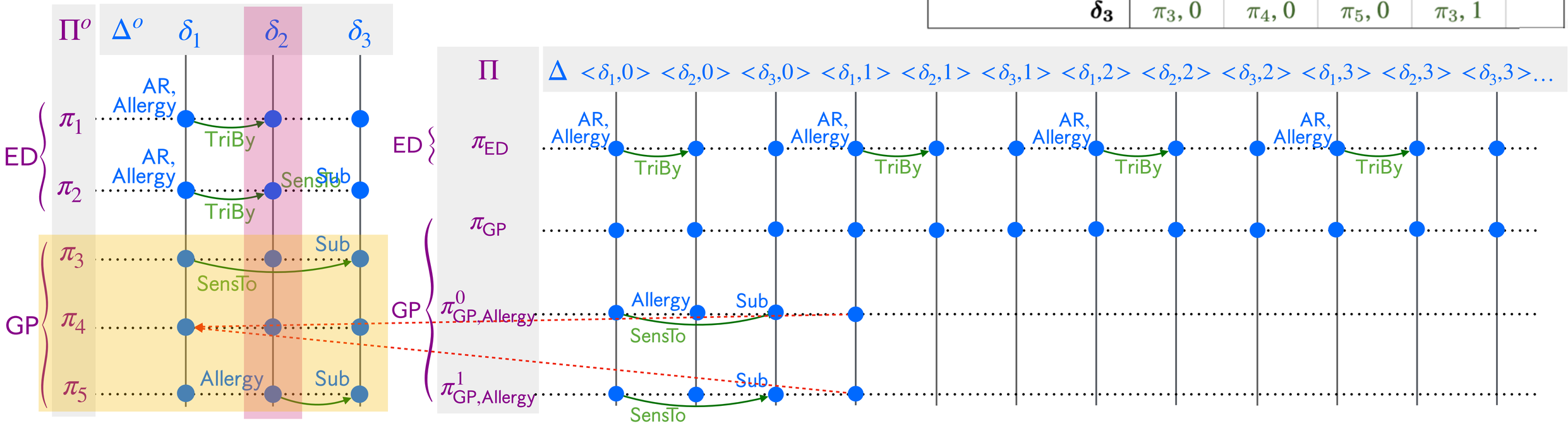
Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	



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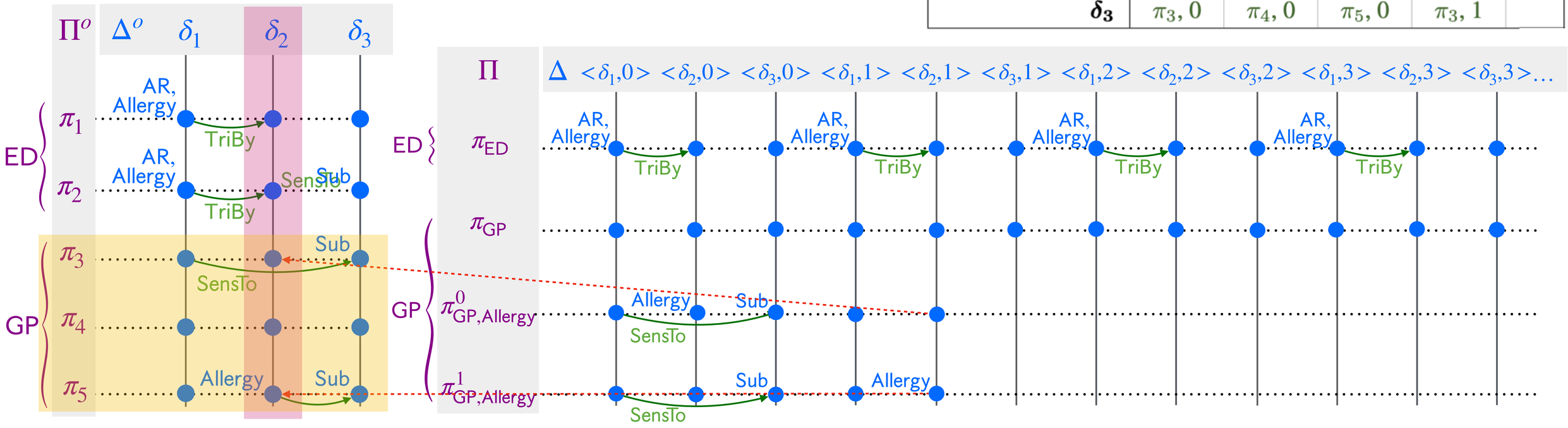
Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	



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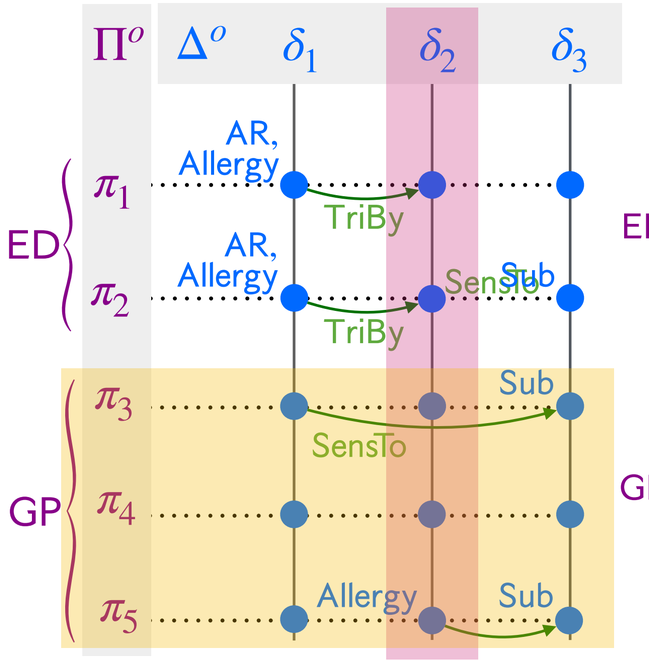
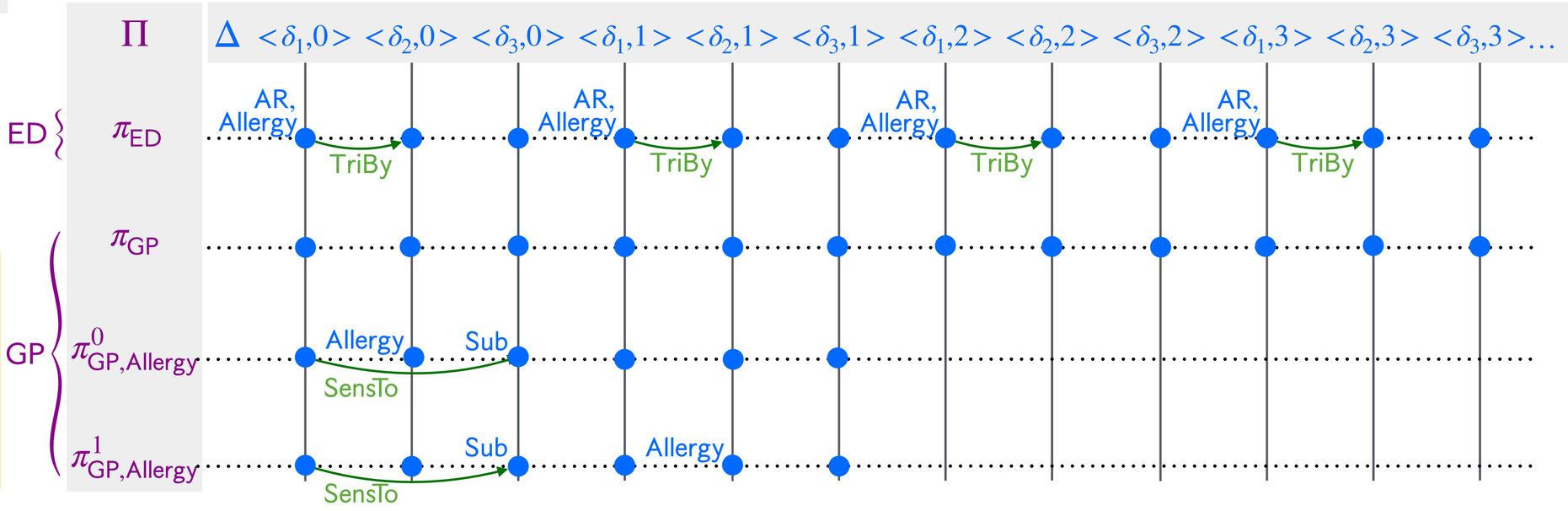
$\pi^0_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	





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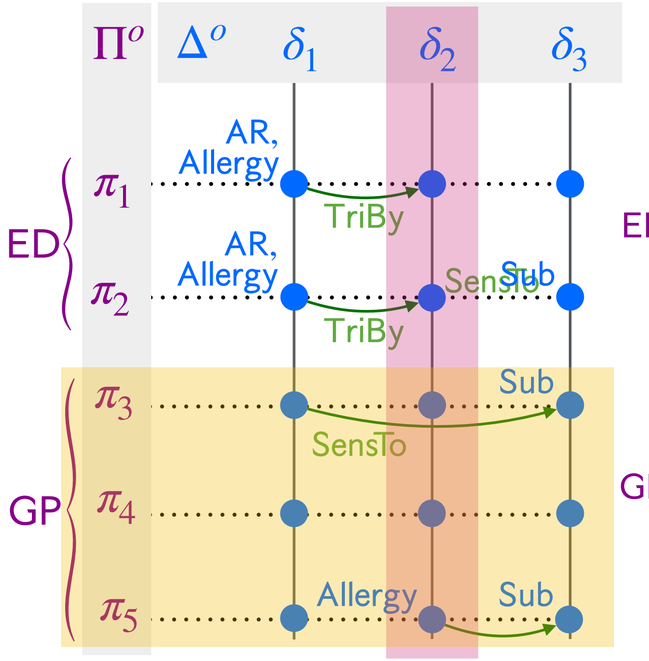
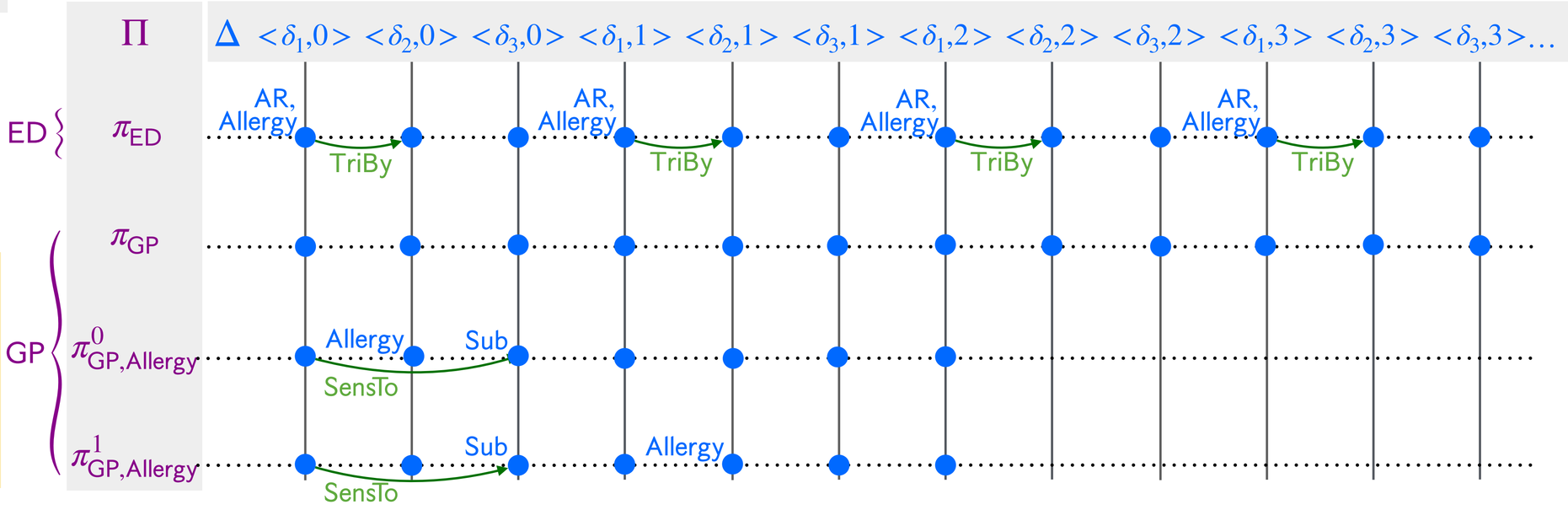
Π	Δ	$f(\pi,\delta,k), g(\pi,\delta,k)$ $k=0$	$k=1$	$k=2$	$k=3$	...
$\pi^0_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_3, 0$	$\pi_5, 0$	$\pi_4, 0$	$\pi_5, 1$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
$\pi^1_{\mathsf{GP,Allergy}}$	δ_1	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	
	δ_2	$\pi_5, 0$	$\pi_3, 0$	$\pi_5, 1$	$\pi_4, 0$	
	δ_3	$\pi_3, 0$	$\pi_4, 0$	$\pi_5, 0$	$\pi_3, 1$	





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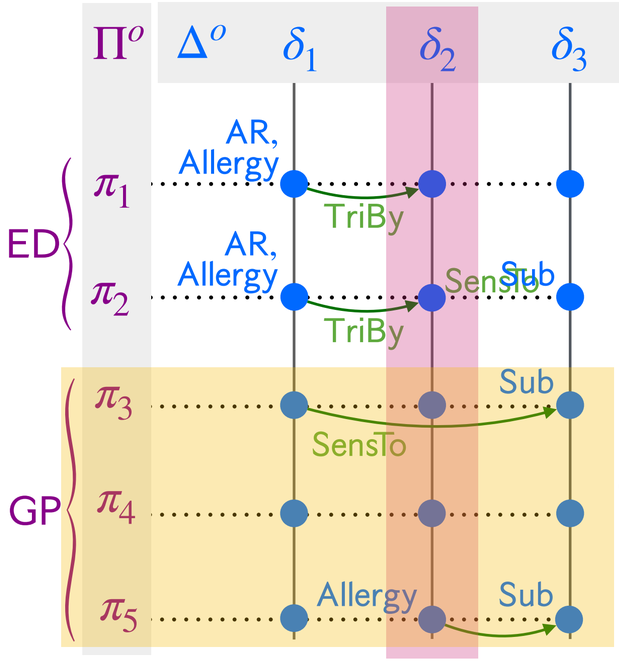

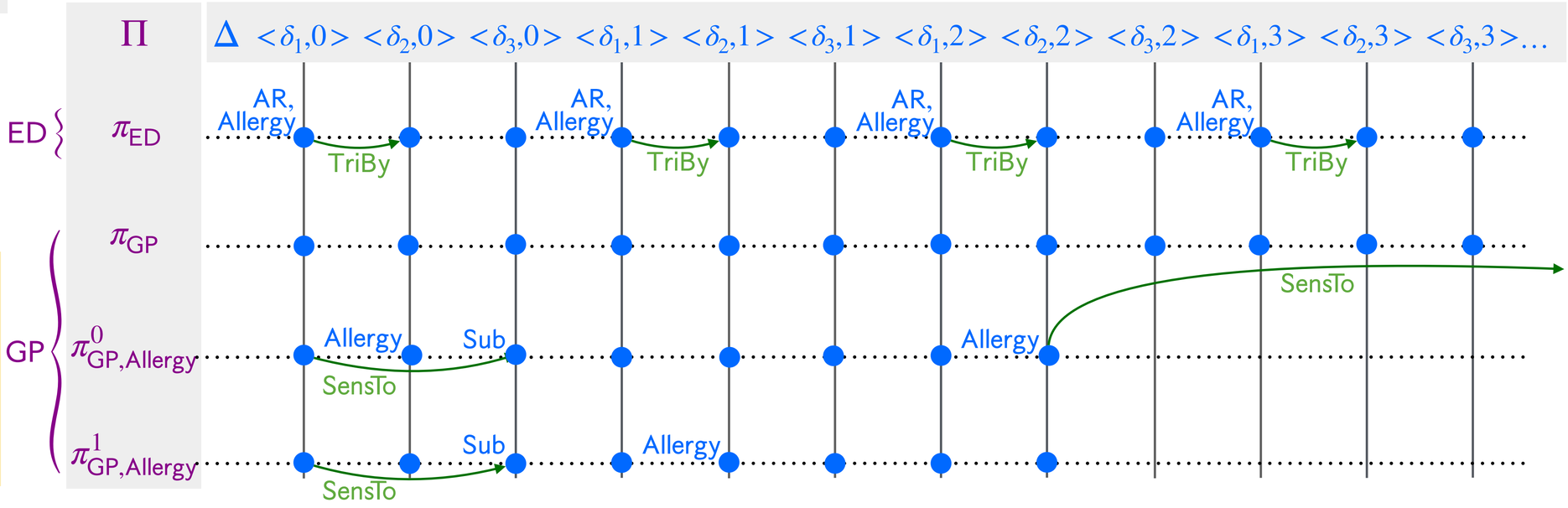

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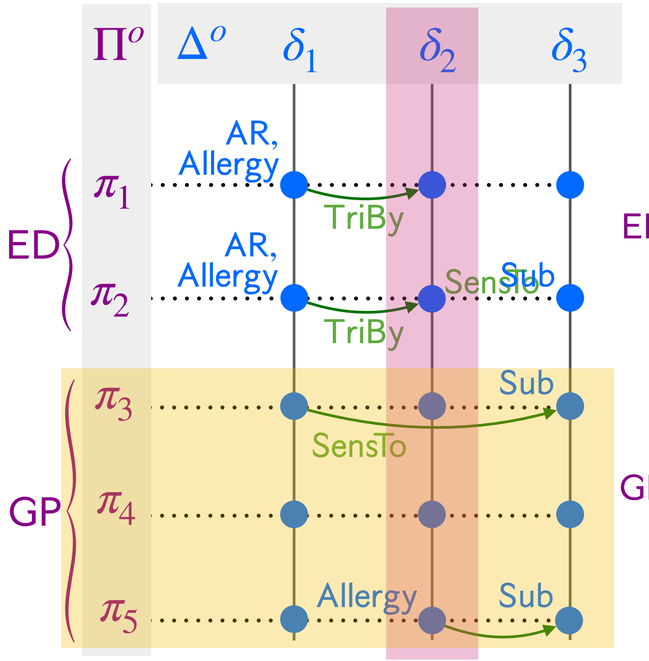

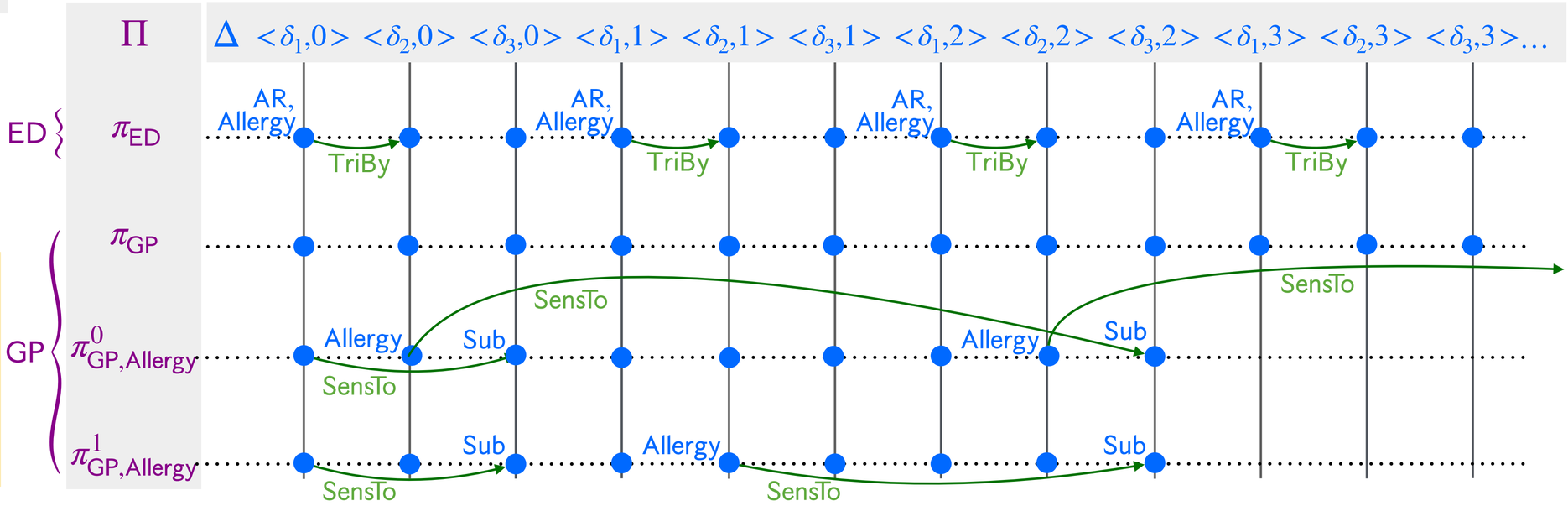

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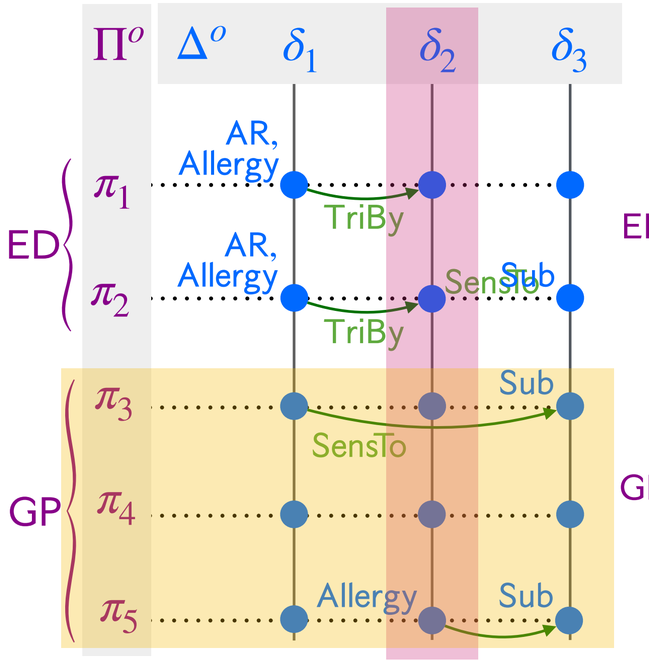

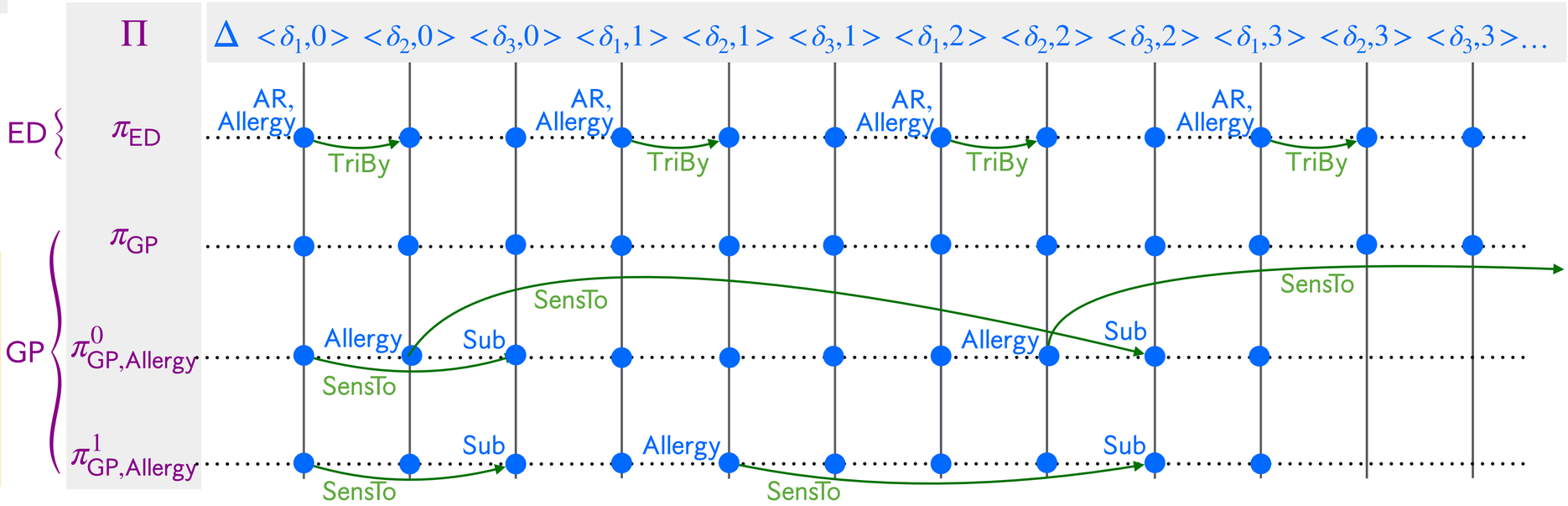

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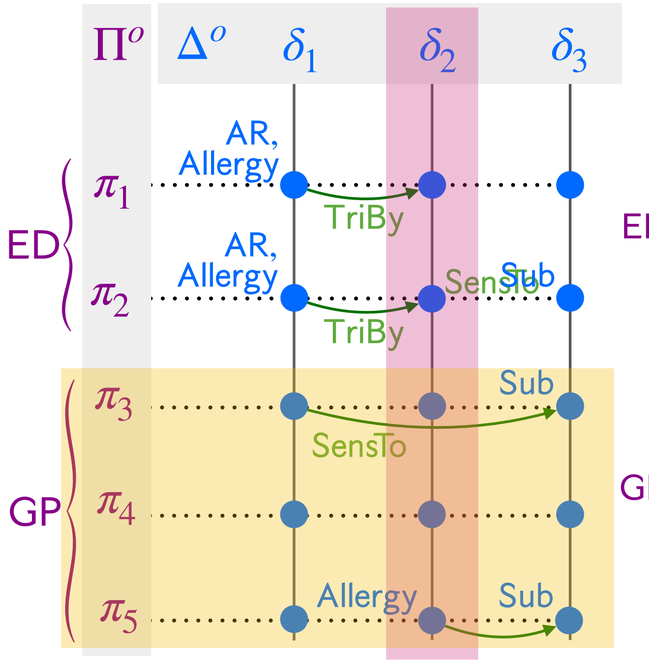

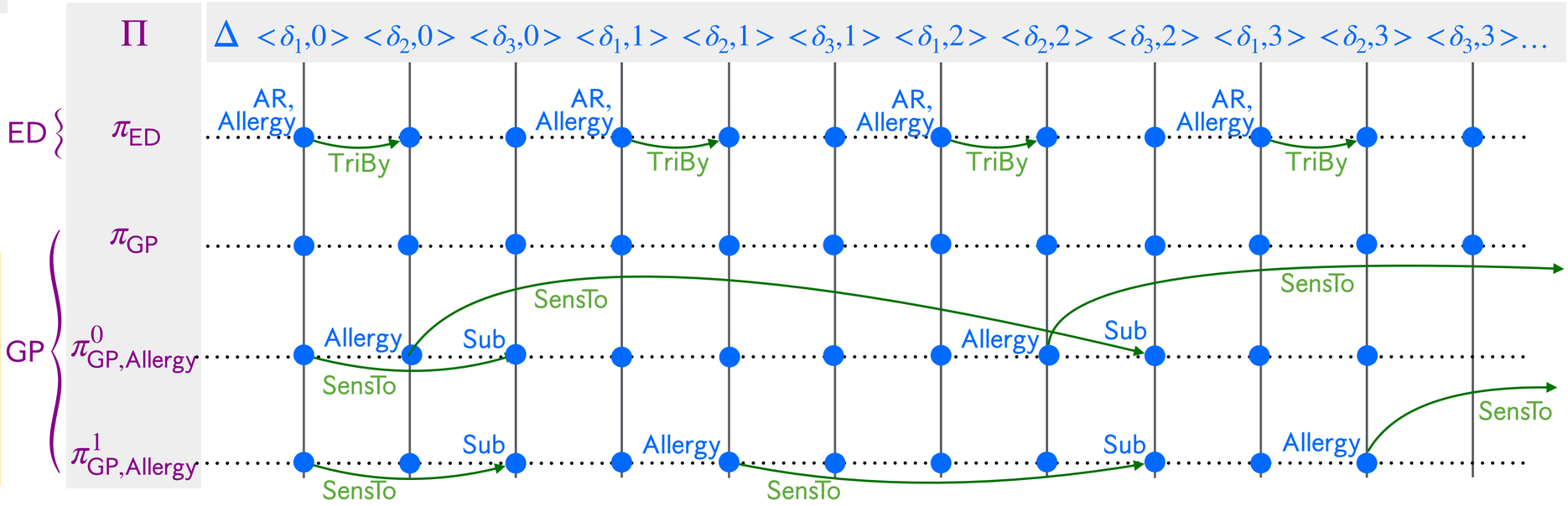

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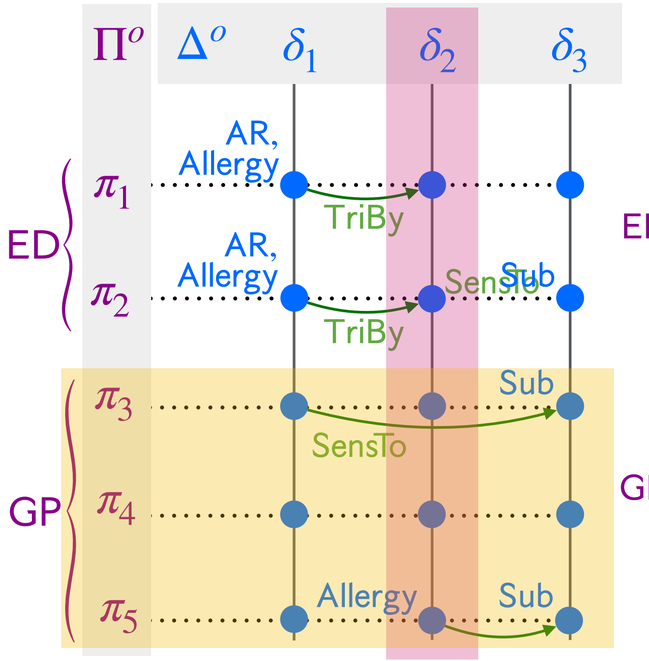

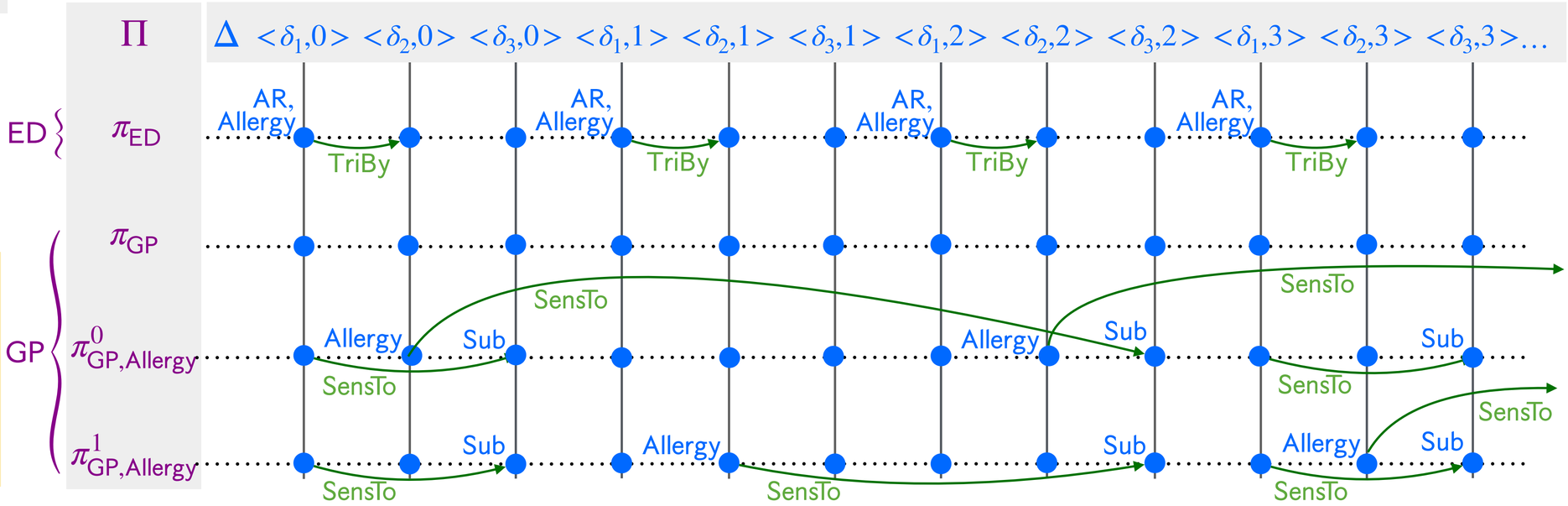

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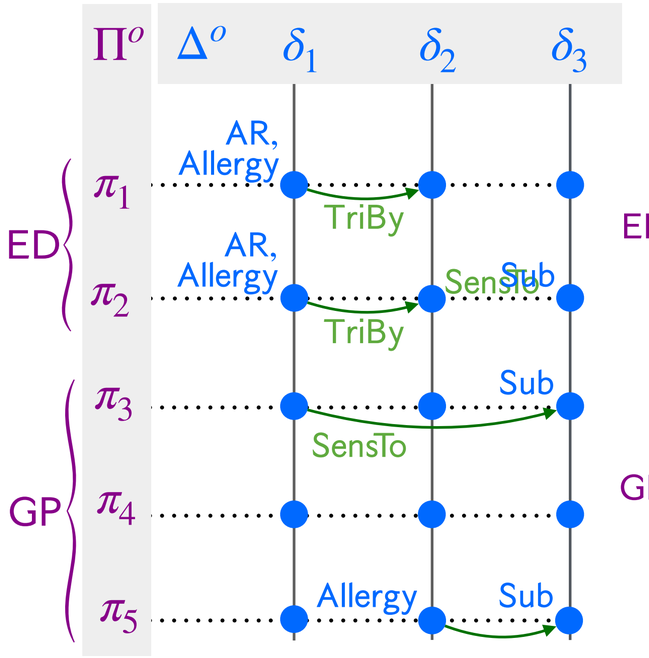

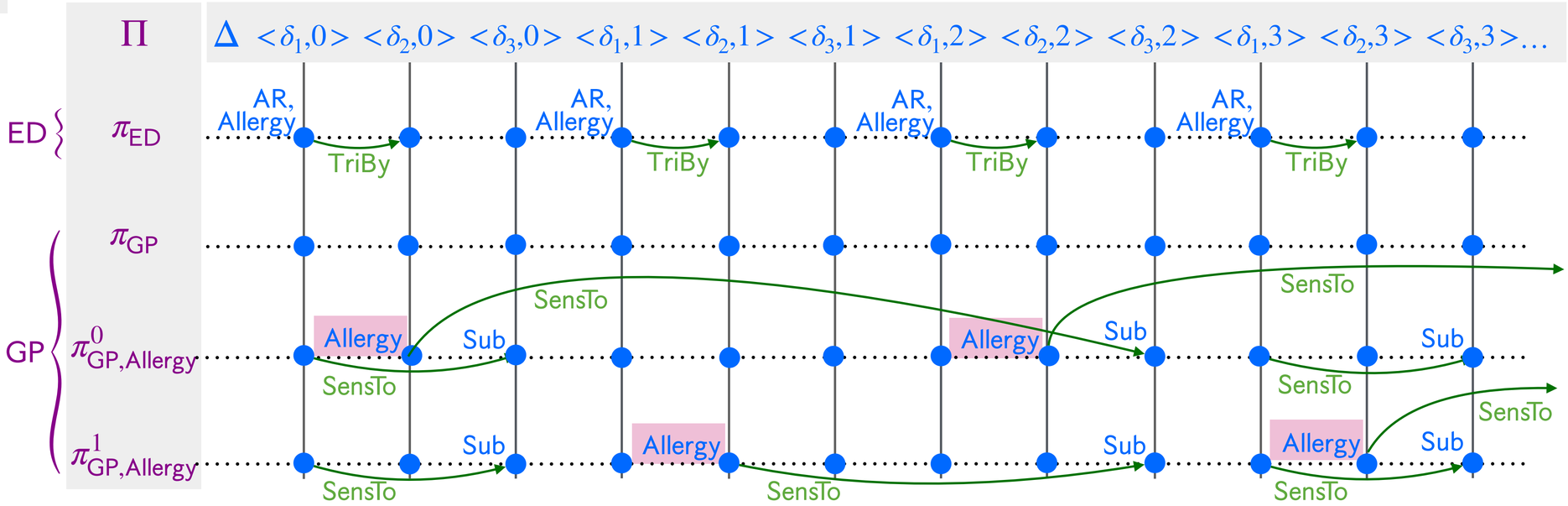

Translation to $\mathcal{SHIQ}$ and complexity

Translation from $\mathbb{S}_{\mathcal{SHIQ}}$ to $\mathcal{SHIQ}$

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δ carries concept C in the *k*th precisification → (encoded by) δ carries the *k*th copy of C.

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Corollary 9 . *Satisfiability and statement entailment in $\mathbb{S}_{\mathcal{SHIQ}}$ are EXPTIME-complete*

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- Implementation of the translation and integration with existing reasoners
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- Towards conceptual modelling with standpoints for knowledge integration challenges

The end.