

#### **Knowledge Representation and Reasoning**

Sebastian Rudolph

Based on slides of Bernardo Cuenca Grau, Ian Horrocks, and Przemysław Wałęga (University of Oxford)





#### Lecture Information

Lecturer: Sebastian Rudolph

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Tutorials: Jonas Karge

jonas.karge@tu-dresden.de

• Mondays, 14:50 – 16:20, APB0005

Mondays, 16:40 – 18:10, APB0005

• Dates, slides, and other info on course webpage:

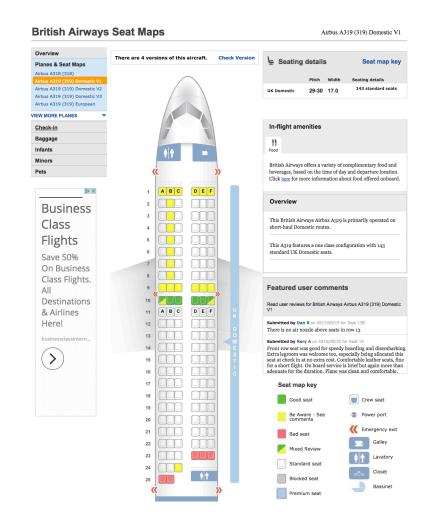
https://iccl.inf.tu-dresden.de/web/Foundations\_of\_Logic\_Programming\_(WS2025)

Oral examintation



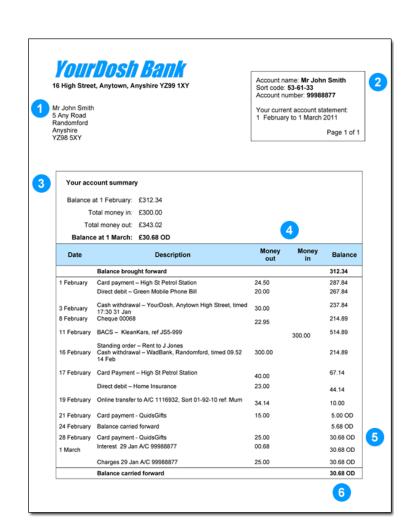


· seats on an aeroplane





- · seats on an aeroplane
- account transactions





- seats on an aeroplane
- account transactions
- tall buildings

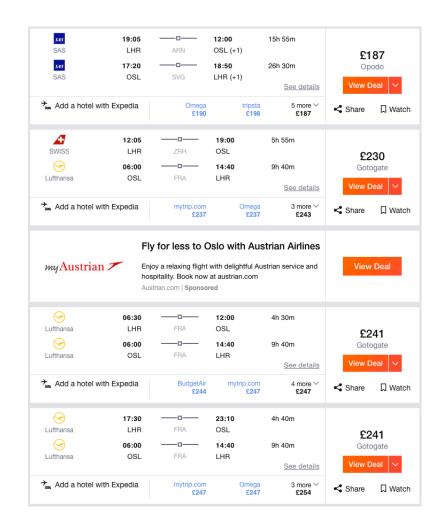
Name +	City ¢	State +	Height ¢	Feet +	Floors ¢	Completed ¢
Federation Tower: East Tower	Moscow	Russia	373.7	1,226	95	2016
OKO: South Tower	Moscow	Russia	354.1	1,162	85	2015
Mercury City Tower	Moscow	Russia	338.8	1,112	75	2013
The Shard <sup>[1]</sup>	London	United Kingdom	309.7	1,017	87	2012
Eurasia <sup>[2]</sup>	Moscow	Russia	308.9	1,014	72	2014
CoC: Moscow Tower <sup>[3]</sup>	Moscow	Russia	301.6	990	76	2010
Skyland İstanbul 1 <sup>[4][5]</sup>	Istanbul	C Turkey	293.1	932	65	2017
Skyland İstanbul 2 <sup>[4][5]</sup>	Istanbul	C Turkey	293.1	932	65	2017
Metropol İstanbul <sup>[6]</sup>	Istanbul	Turkey     ■ Turkey	280	919	66	2017
Emaar Square	Istanbul	<b>C</b> Turkey	280	920	62	2018
Naberezhnaya Tower C	Moscow	Russia	268.4	881	59	2007
Triumph Palace <sup>[7]</sup>	Moscow	Russia	264.1	867	57	2005
Commerzbank Tower <sup>[8][9]</sup>	Frankfurt	Germany	258.7	848	56	1997
CoC: Saint Petersburg Tower <sup>[10]</sup>	Moscow	Russia	256.9	843	65	2010
Messeturm	Frankfurt	Germany	256.5	842	55	1990
Nurol Life <sup>[11][12]</sup>	Istanbul	C Turkey	252	827	60	2017
Torre de Cristal	Madrid	Spain	249	817	45	2008
Torre Cepsa	Madrid	Spain	248.3	815	45	2008
Evolution Tower <sup>[13]</sup>	Moscow	Russia	245.9	807	53	2014
OKO: North Tower <sup>[14]</sup>	Moscow	Russia	245	804	49	2014
Federation: West Tower	Moscow	Russia	243.2	798	62	2007
Main building of Moscow State University	Moscow	Russia	240	787	36	1953
Imperia Tower	Moscow	Russia	238.7	783	60	2011
Palace of Culture and Science	Warsaw	Poland	237	777	43	1955
Torre PwC	Madrid	Spain	236	774	52	2008
1 Canada Square	London	United Kingdom	235	771	50	1991
Istanbul Sapphire <sup>[15][16]</sup>	Istanbul	Turkey     ■ Turkey	234.9	770	54	2010
Tour First	Paris <sup>B</sup>	France	231	758	56	2011
Unicredit Tower	Milan	<b>■</b> Italy	231	758	35	2011
Heron Tower	London	United Kingdom	230	755	46	2011



- seats on an aeroplane
- account transactions
- tall buildings

#### and to answer questions

seats available on flight?

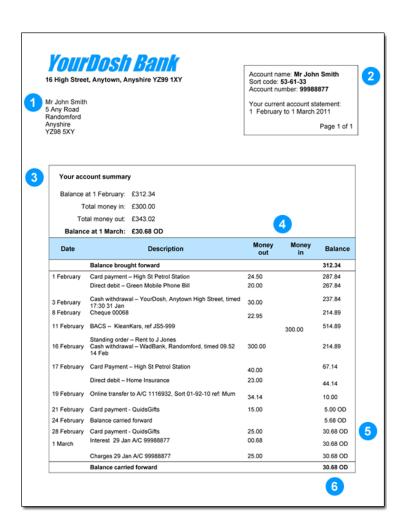




- seats on an aeroplane
- account transactions
- tall buildings

#### and to answer questions

- seats available on flight?
- can afford to pay rent?





- seats on an aeroplane
- account transactions
- tall buildings

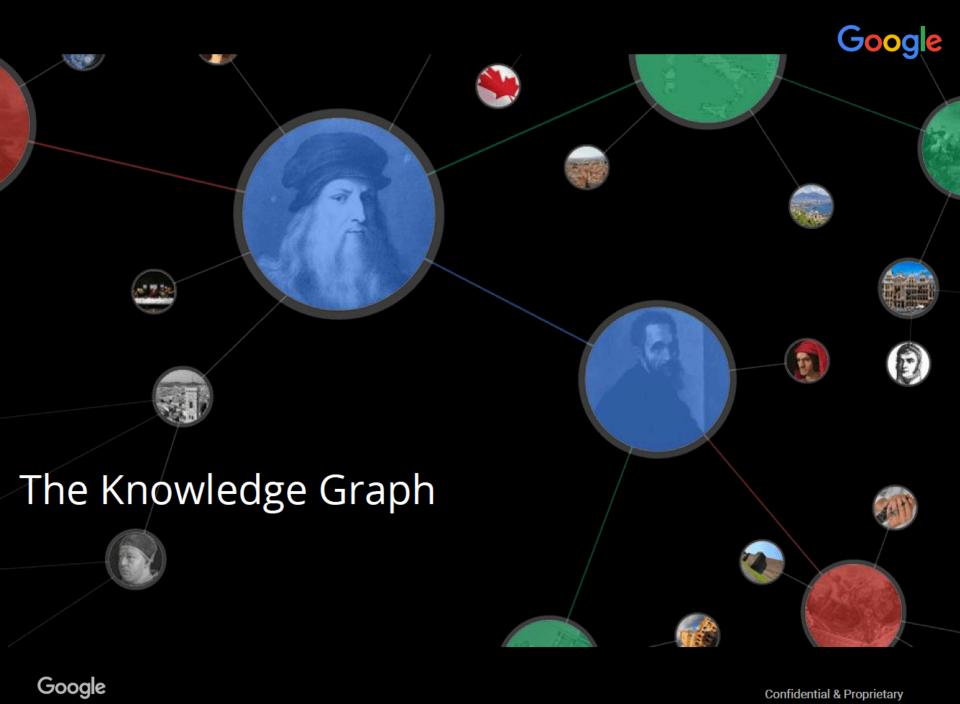
#### and to answer questions

- seats available on flight?
- can afford to pay rent?
- tallest building in Europe?

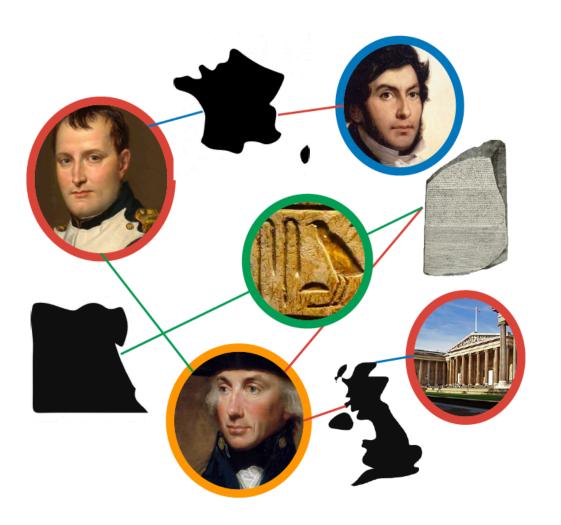




What kind of representation?



### The Knowledge Graph



The Knowledge Graph is a comprehensive collection of real-world entities (people, places, things, and concepts) along with relationships and factual attributes that describe them.



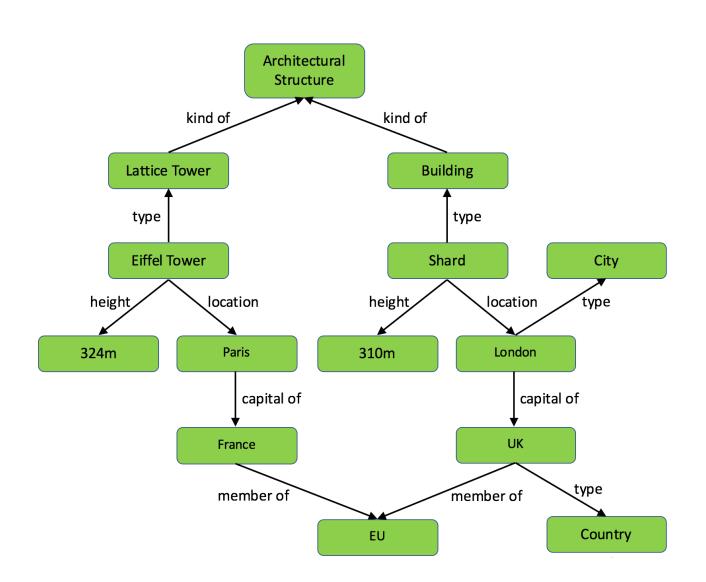


#### Architectural Structure **Lattice Tower** Building **Eiffel Tower Shard** City 324m **Paris** 310m London UK France

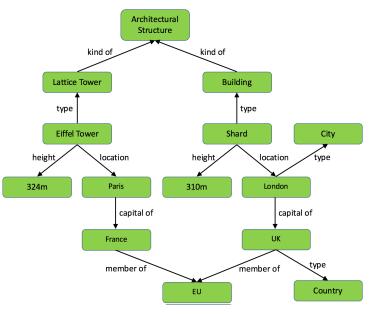
EU

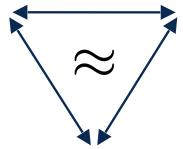
Country











Architectural Structure					
name	location	height	kind		
Shard	London	310m	Building		
Eiffel Tower	Paris	324m	Lattice Tower		

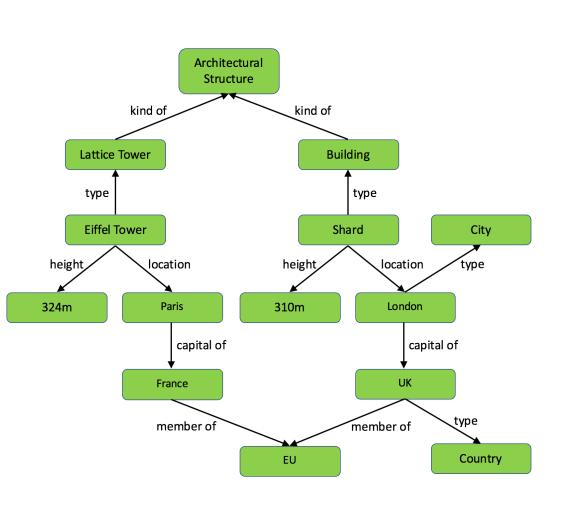
City				
name	capital of			
London	UK			
Paris	France			

member of				
country	organisation			
France	EU			
UK	EU			

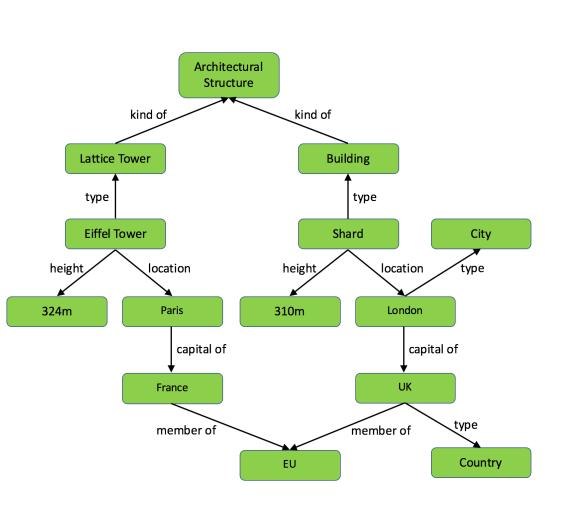
Building(Shard)
City(London)
location(Shard,London)
height(Shard,310m)
capitalOf(London,UK)

. . .





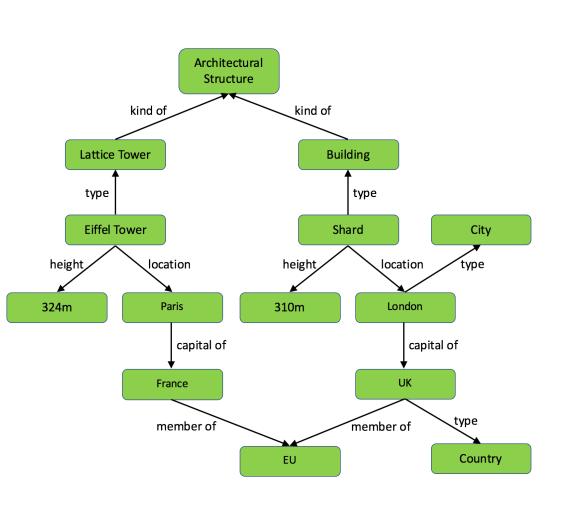


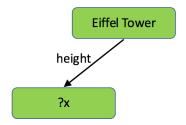


What is the height of the Eiffel Tower?

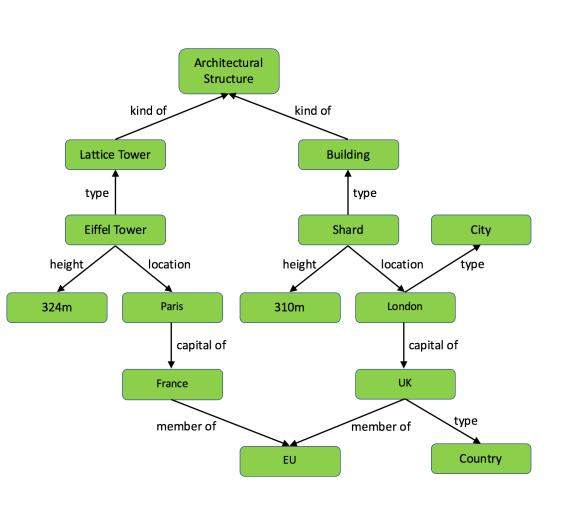
SELECT ?x
WHERE { EiffelTower height ?x. }

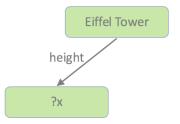




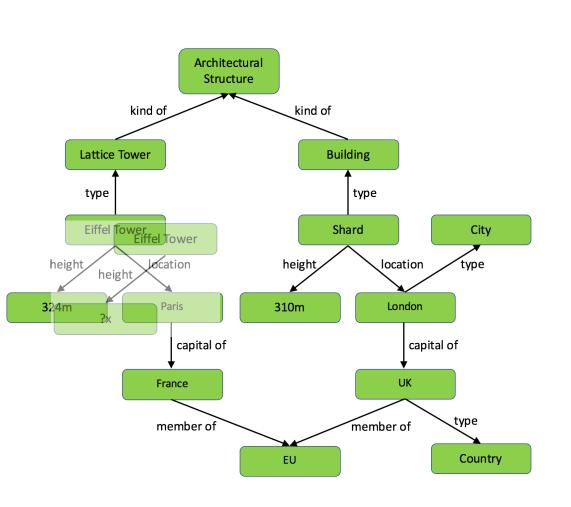




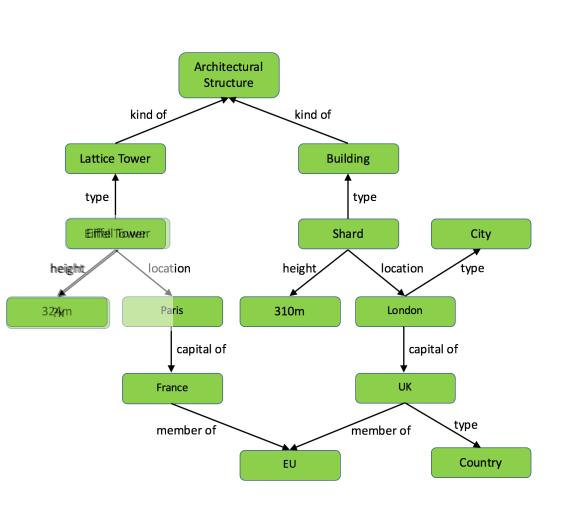








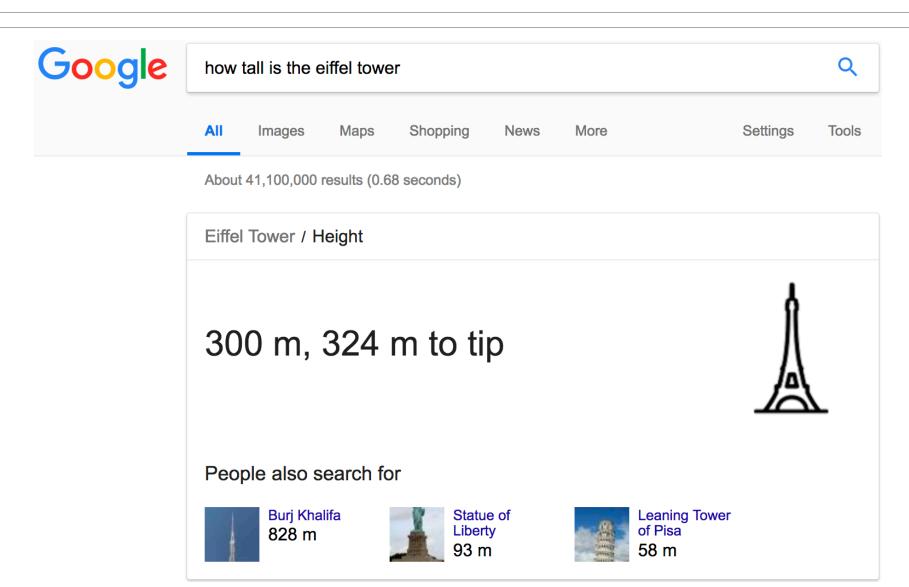




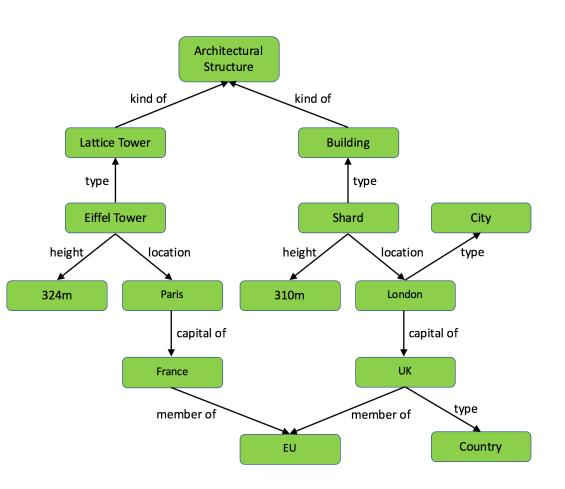
What is the height of the Eiffel Tower?

324m

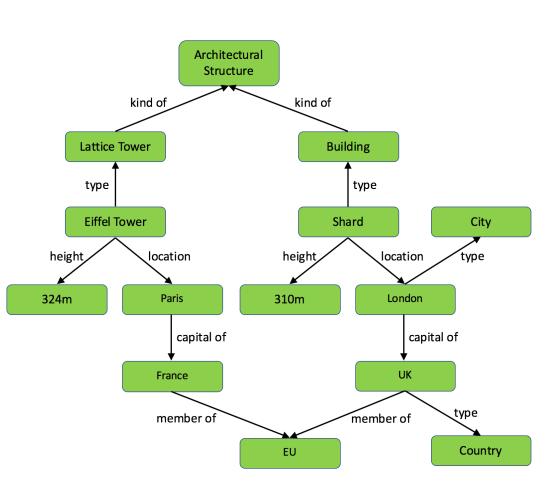






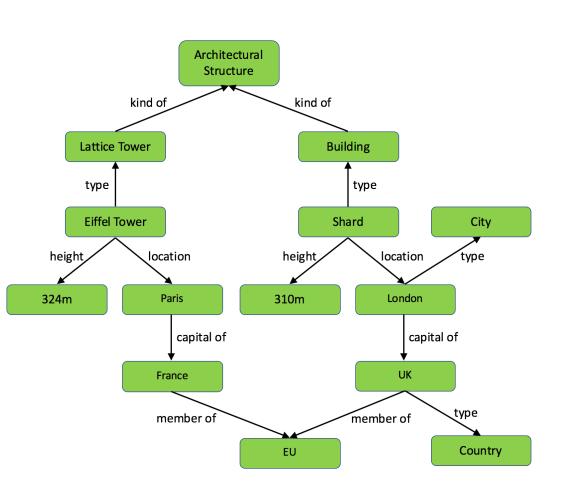






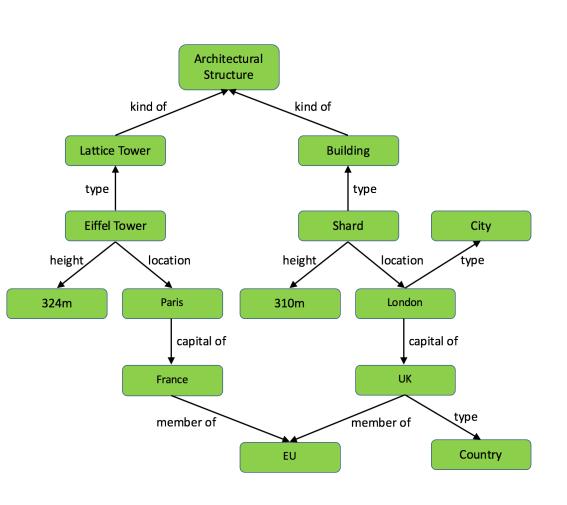
 What is the difference between a Building, a Lattice Tower and an Architectural Structure?





- What is the difference between a Building, a Lattice Tower and an Architectural Structure?
- Is the Eiffel Tower a Building; is it an Architectural Structure?

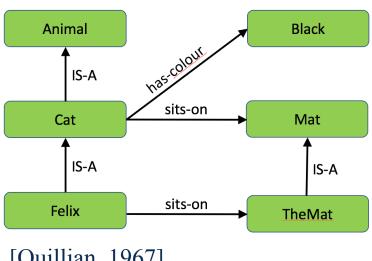




- What is the difference between a Building, a Lattice Tower and an Architectural Structure?
- Is the Eiffel Tower a Building; is it an Architectural Structure?
- Special meaning of, e.g., type and kind of edges?

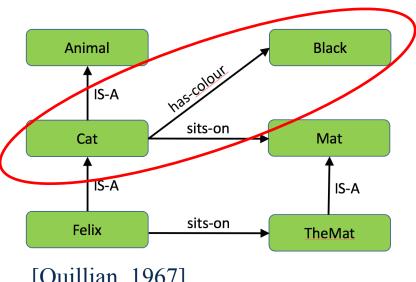






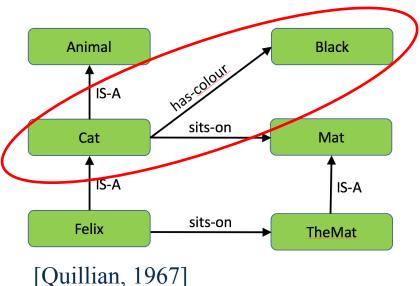
[Quillian, 1967]





[Quillian, 1967]

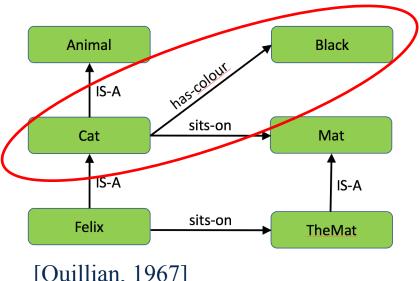






[Quillian, 1967]

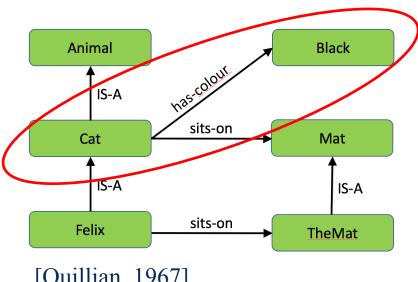






[Quillian, 1967]

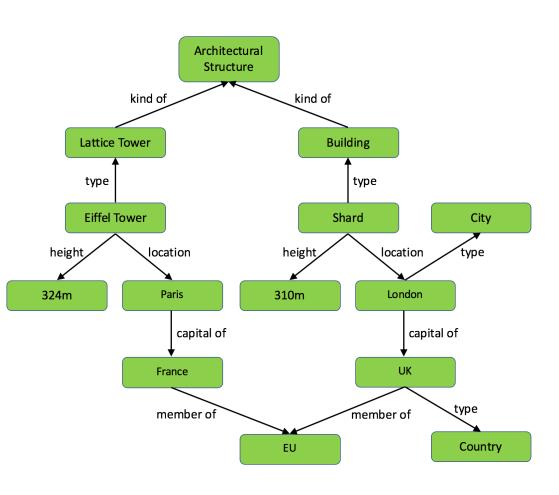






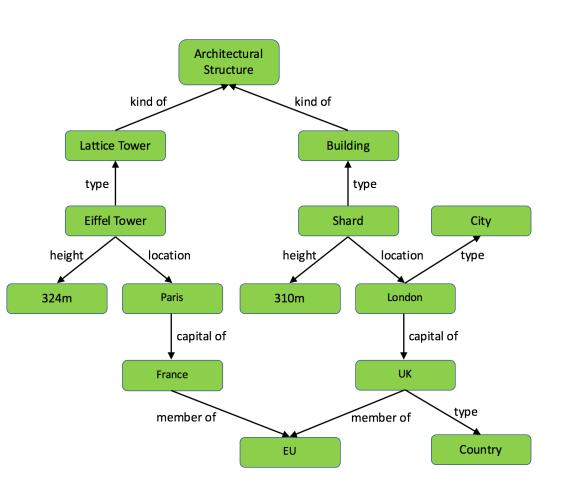
[Quillian, 1967]



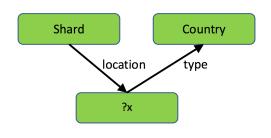


 What country is the Shard located in?

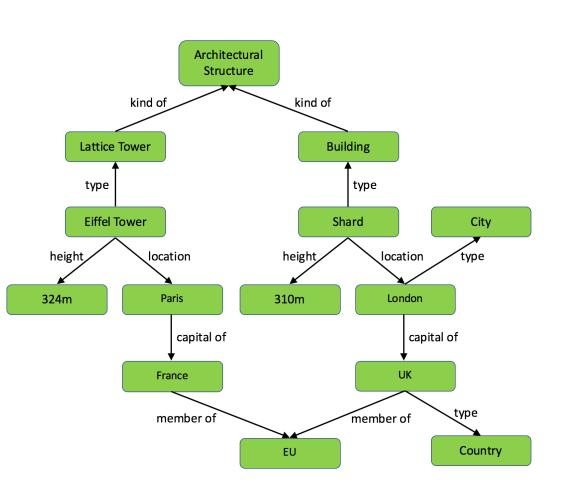


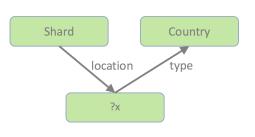


 What country is the Shard located in?

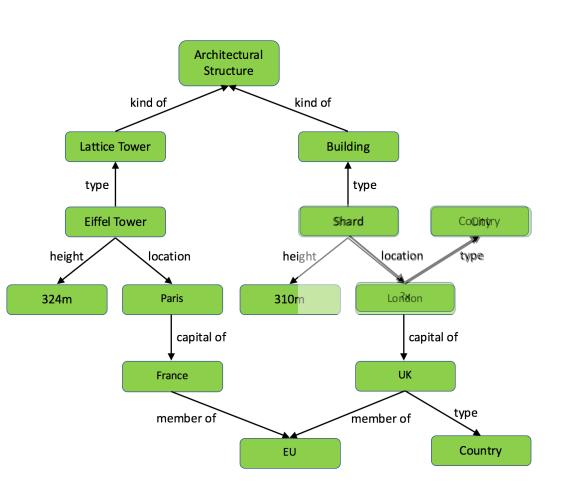




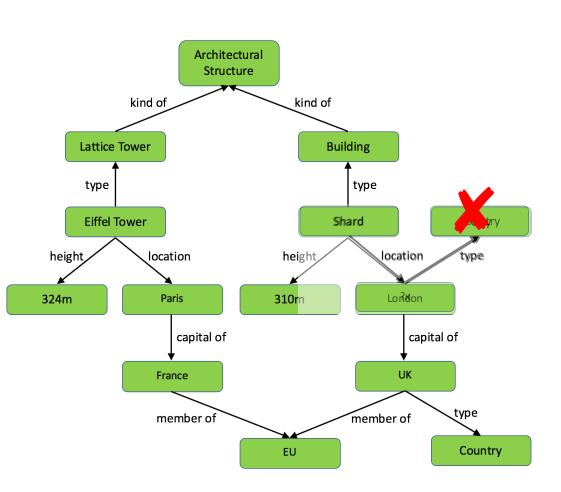




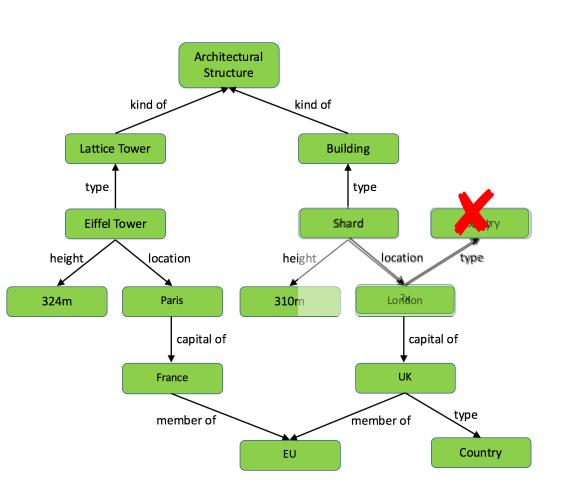








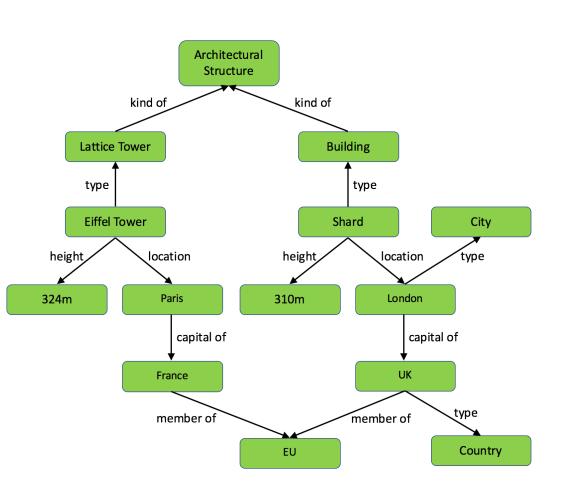




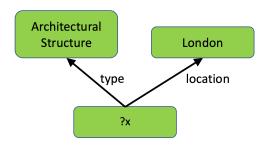
 What country is the Shard located in?

• ???

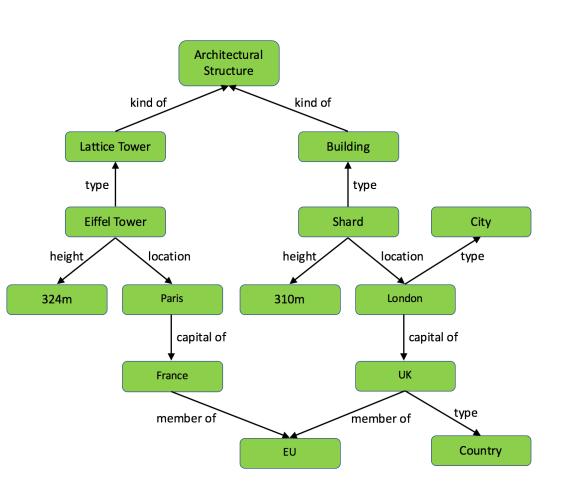




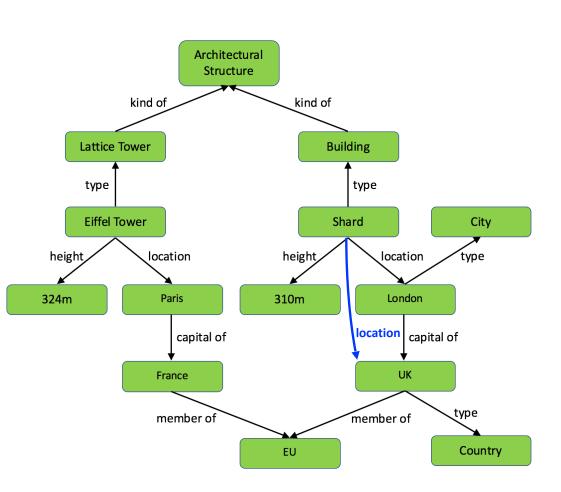
 What Architectural Structures are located in London?



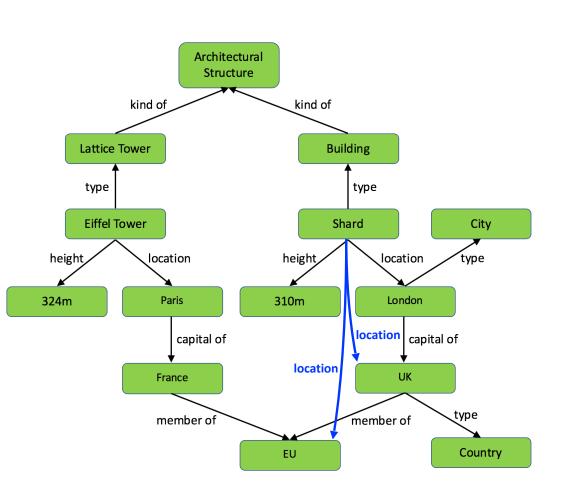




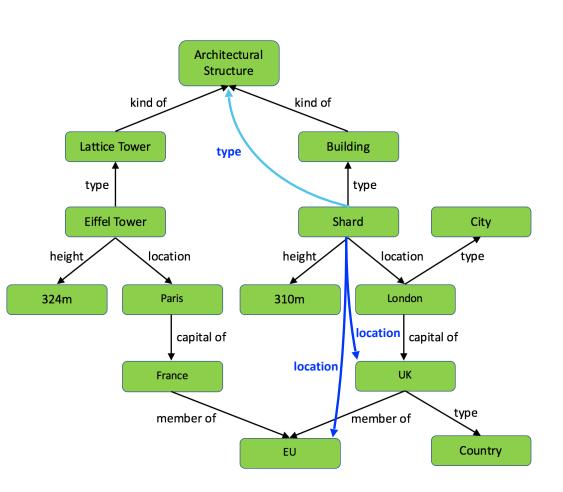




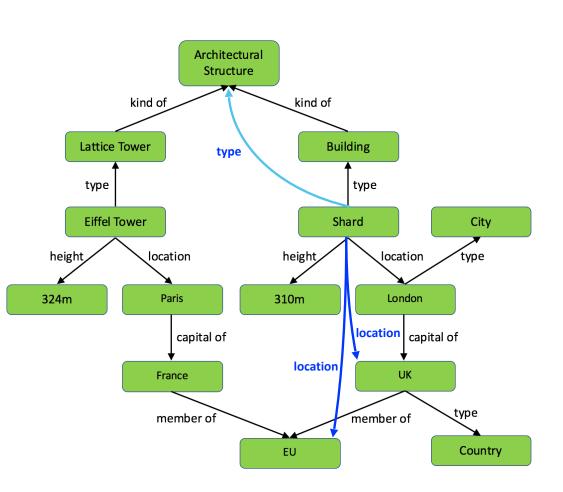




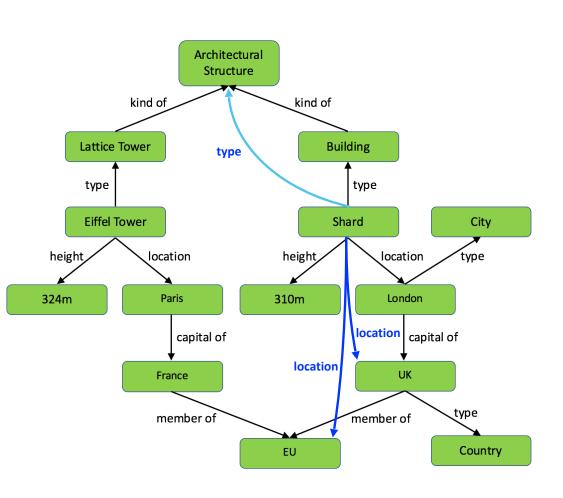






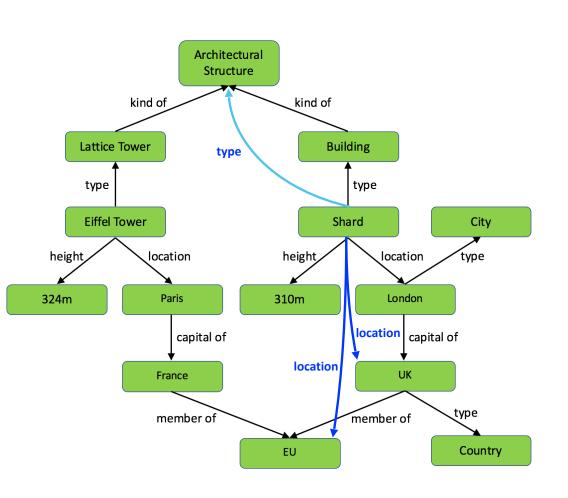






- Every Building located in London is also located in
  - UK
  - EU
  - England
  - Northern Hemisphere
  - ..





- Every Building located in London is also located in
  - UK
  - EU
  - England
  - Northern Hemisphere
  - •
- Need to add a very large number of edges



A city that is the capital of a country is a (geographical) part of that country<sup>†</sup>

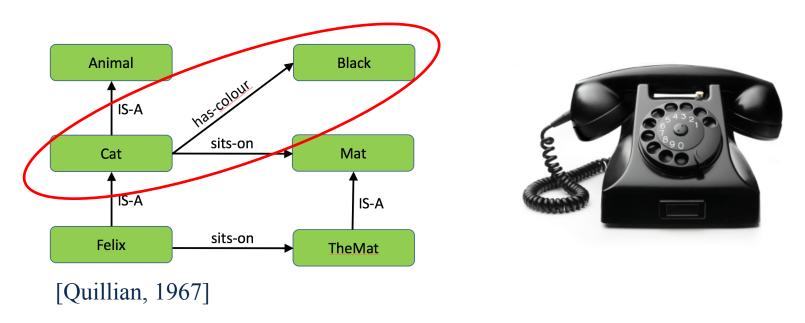
A thing that is located in a city that is a (geographical) part of a country is also located in that country

A thing that is located in a country that is a member of a supranational union is also located in that supranational union

<sup>†</sup> Part-whole relationships are complicated! They are the subject of a whole field of study in logic and philosophy: mereology

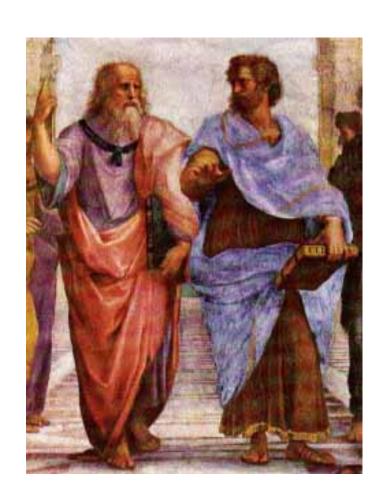


• Semantics: the study of meaning

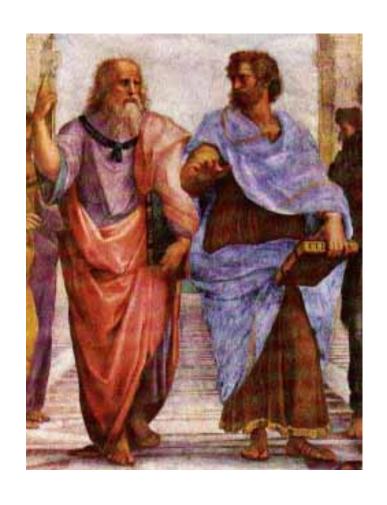


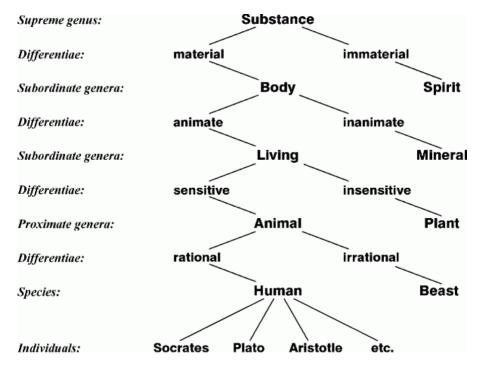
(Precise) semantics needed in order to define what (correct) query answers should be



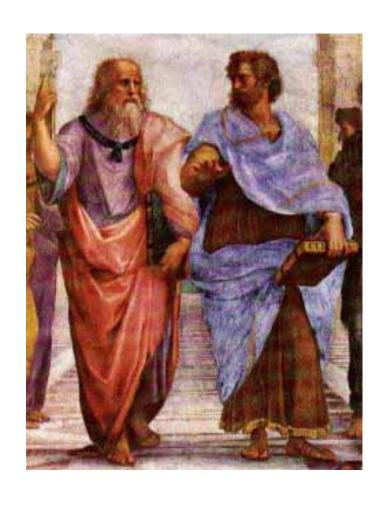


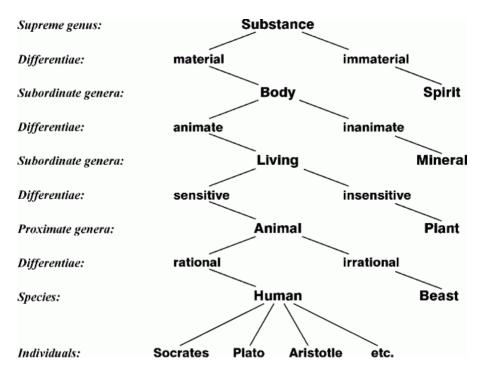












"All men are mortal, all Greeks are men, therefore all Greeks are mortal" (syllogism)



Modern KR languages are often based on logic Typically (subsets of) First Order Predicate Calculus



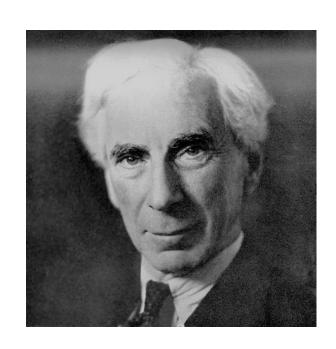
## Modern KR languages are often based on logic Typically (subsets of) First Order Predicate Calculus



**Gottlob Frege** 

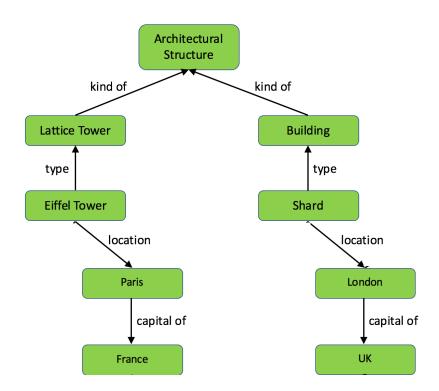


**Charles Sanders Peirce** 

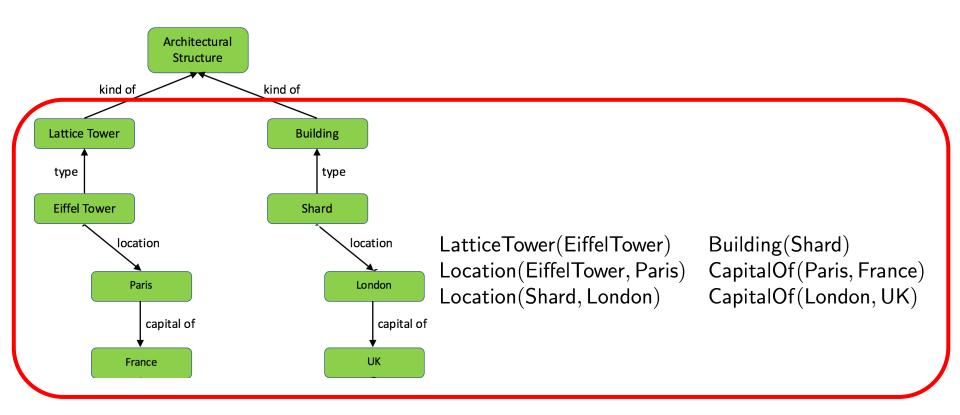


**Bertrand Russell** 



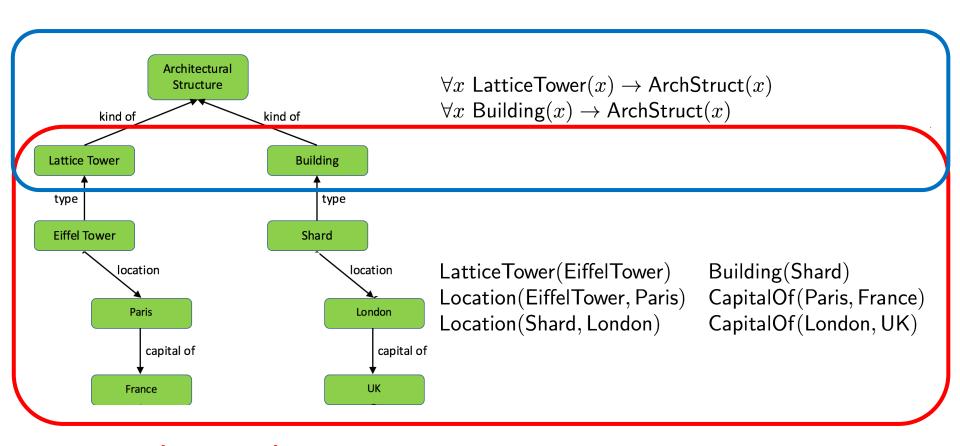




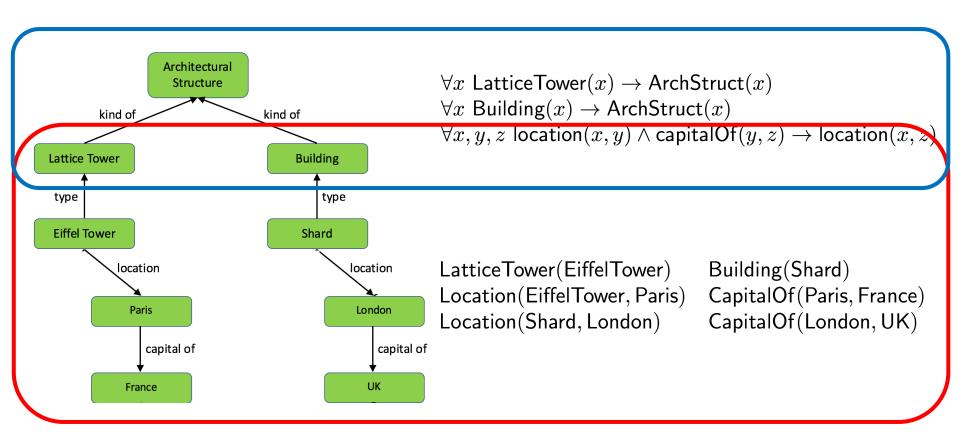


facts / data / (RDF) graph

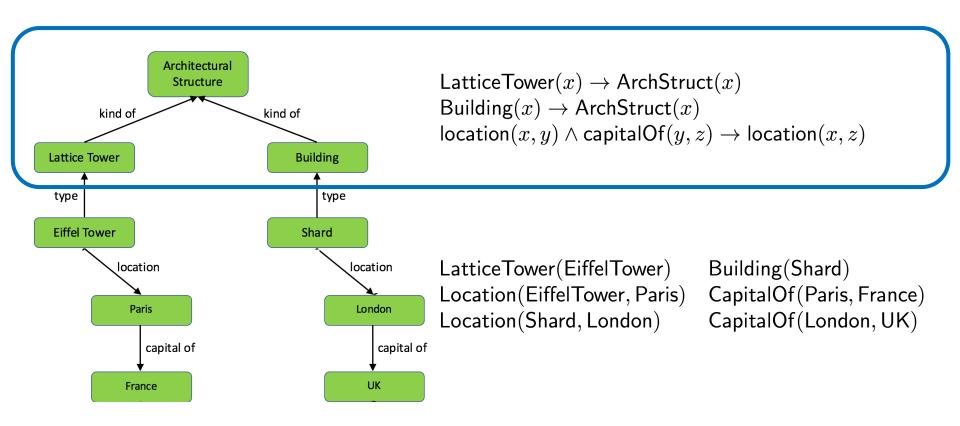




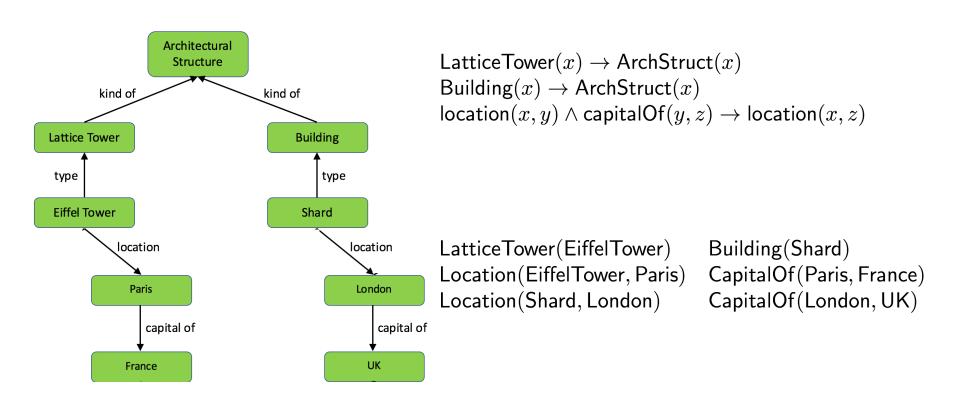




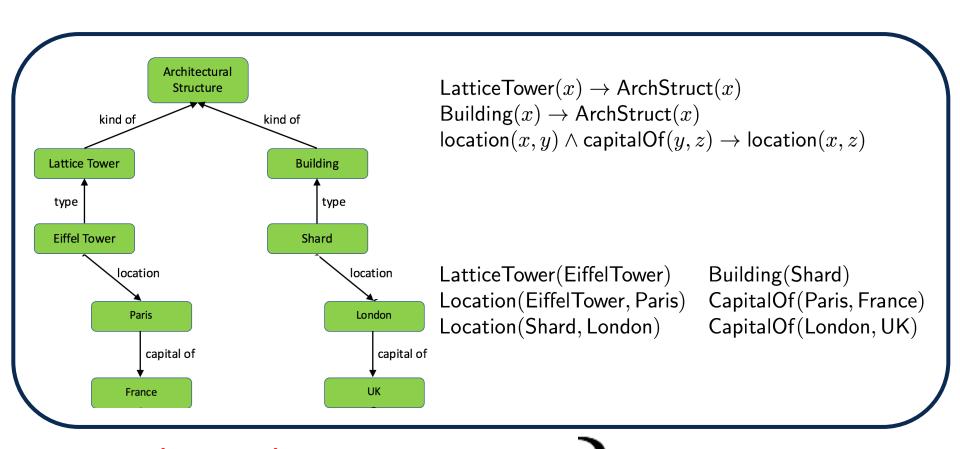






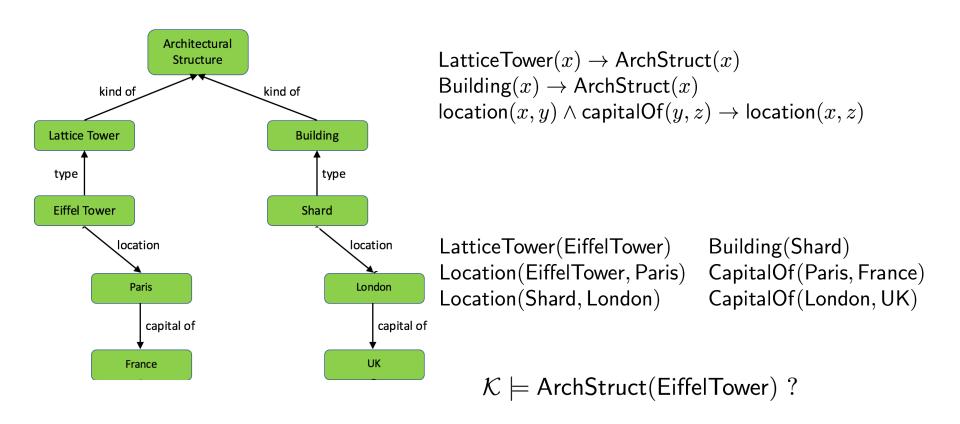




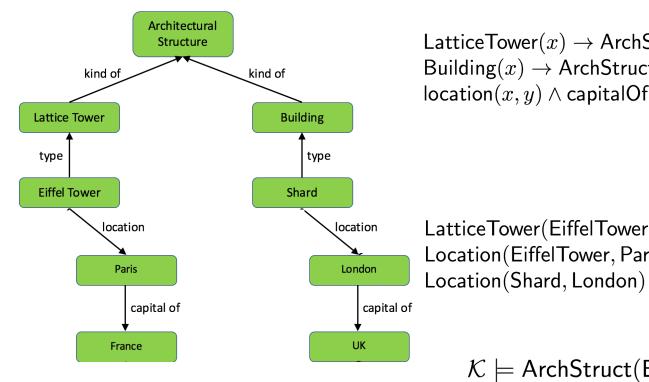


knowledge base





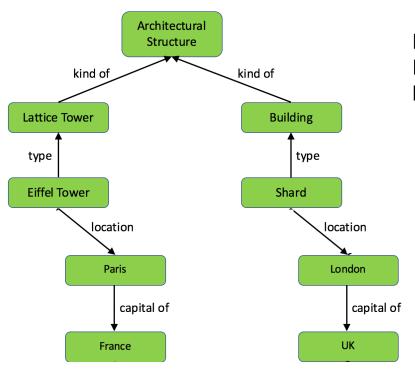




LatticeTower(EiffelTower) Building(Shard) Location(EiffelTower, Paris) CapitalOf(Paris, France) Location(Shard, London) CapitalOf(London, UK)

 $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{EiffelTower})$ ? True





 $\mathsf{LatticeTower}(x) \to \mathsf{ArchStruct}(x)$  $\mathsf{Building}(x) \to \mathsf{ArchStruct}(x)$  $location(x, y) \land capitalOf(y, z) \rightarrow location(x, z)$ 

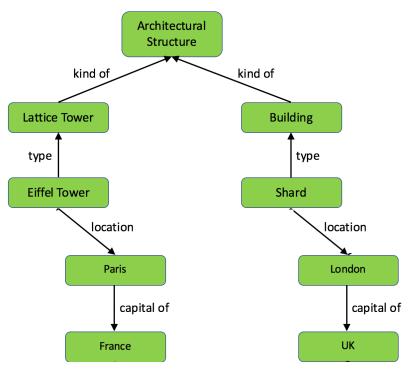
LatticeTower(EiffelTower) Location(EiffelTower, Paris) Location(Shard, London) CapitalOf(London, UK)

Building(Shard) CapitalOf(Paris, France)

 $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{EiffelTower}) ?$ True

 $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{Shard})$ ?





 $\begin{aligned} \mathsf{LatticeTower}(x) &\to \mathsf{ArchStruct}(x) \\ \mathsf{Building}(x) &\to \mathsf{ArchStruct}(x) \\ \mathsf{location}(x,y) \land \mathsf{capitalOf}(y,z) &\to \mathsf{location}(x,z) \end{aligned}$ 

LatticeTower(EiffelTower)
Location(EiffelTower, Paris)
Location(Shard, London)

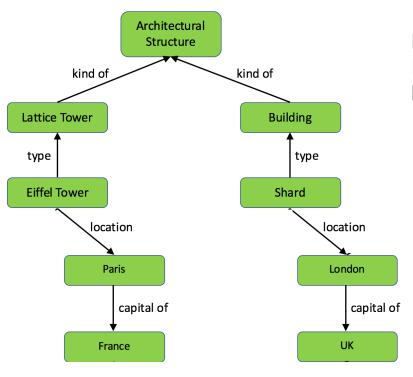
Building(Shard)

(CapitalOf(Paris, France)

(CapitalOf(London, UK)

 $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{EiffelTower}) ?$  True  $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{Shard}) ?$  True





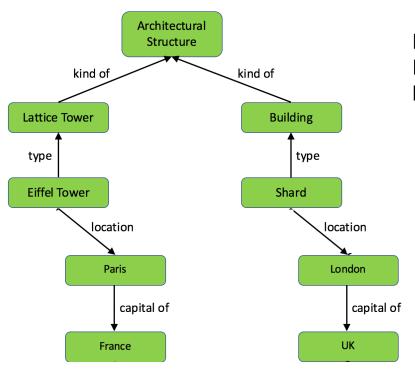
LatticeTower(EiffelTower) Location(EiffelTower, Paris) CapitalOf(Paris, France) Location(Shard, London) CapitalOf(London, UK)

Building(Shard)

 $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{EiffelTower}) ?$ True  $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{Shard})$ ? True

 $\mathcal{K} \models location(EiffelTower, France)$ ?

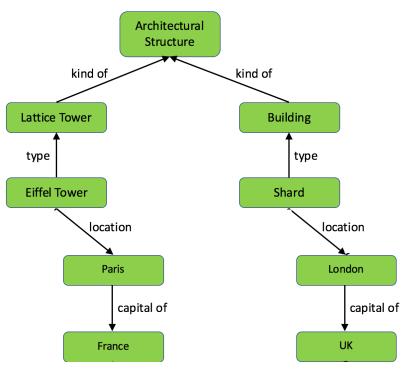




LatticeTower(EiffelTower) Building(Shard) Location(EiffelTower, Paris) CapitalOf(Paris, France) Location(Shard, London) CapitalOf(London, UK)

 $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{EiffelTower}) ?$ True True  $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{Shard})$ ?  $\mathcal{K} \models location(EiffelTower, France) ? True$ 





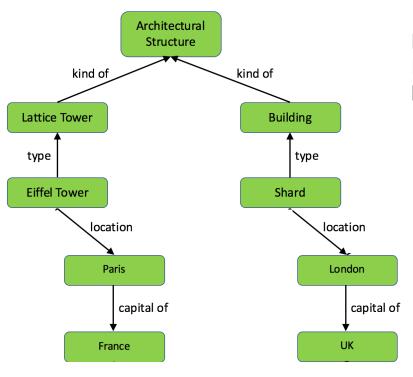
LatticeTower(EiffelTower) Location(EiffelTower, Paris) CapitalOf(Paris, France) Location(Shard, London) CapitalOf(London, UK)

Building(Shard)

 $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{EiffelTower})$ ? True  $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{Shard})$ ? True  $\mathcal{K} \models location(EiffelTower, France) ? True$ 

 $\mathcal{K} \models location(Shard, UK)$ ?



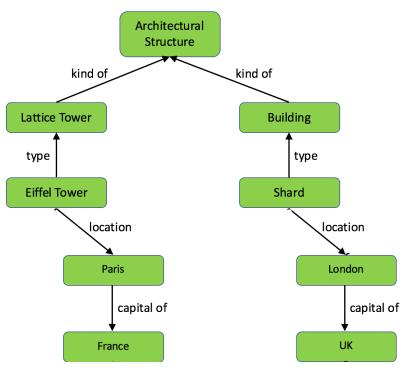


LatticeTower $(x) \to \mathsf{ArchStruct}(x)$ Building $(x) \to \mathsf{ArchStruct}(x)$ location $(x,y) \land \mathsf{capitalOf}(y,z) \to \mathsf{location}(x,z)$ 

 $\begin{array}{ll} {\sf LatticeTower}({\sf EiffelTower}) & {\sf Building}({\sf Shard}) \\ {\sf Location}({\sf EiffelTower}, {\sf Paris}) & {\sf CapitalOf}({\sf Paris}, {\sf France}) \\ {\sf Location}({\sf Shard}, {\sf London}) & {\sf CapitalOf}({\sf London}, {\sf UK}) \end{array}$ 

 $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{EiffelTower}) ?$  True  $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{Shard}) ?$  True  $\mathcal{K} \models \mathsf{location}(\mathsf{EiffelTower}, \mathsf{France}) ?$  True  $\mathcal{K} \models \mathsf{location}(\mathsf{Shard}, \mathsf{UK}) ?$  True





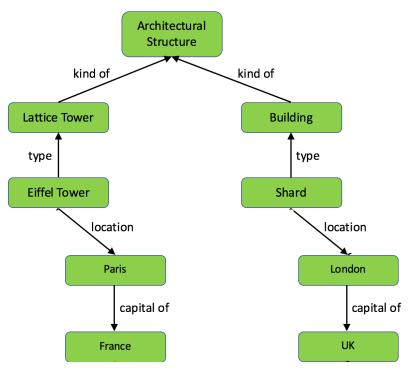
LatticeTower(EiffelTower) Location(EiffelTower, Paris) Location(Shard, London)

Building(Shard) CapitalOf(Paris, France) CapitalOf(London, UK)

 $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{EiffelTower}) ?$ True  $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{Shard})$ ? True  $\mathcal{K} \models location(EiffelTower, France) ? True$ True  $\mathcal{K} \models location(Shard, UK)$ ?

 $\mathcal{K} \models location(Shard, France)$ ?





 $\begin{aligned} \mathsf{LatticeTower}(x) &\to \mathsf{ArchStruct}(x) \\ \mathsf{Building}(x) &\to \mathsf{ArchStruct}(x) \\ \mathsf{location}(x,y) \land \mathsf{capitalOf}(y,z) &\to \mathsf{location}(x,z) \end{aligned}$ 

LatticeTower(EiffelTower) Building(Shard, London) Building(Shard, Shard, CapitalOf(Notation(Shard, London) C

Building(Shard)
CapitalOf(Paris, France)
CapitalOf(London, UK)

 $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{EiffelTower}) ?$  True  $\mathcal{K} \models \mathsf{ArchStruct}(\mathsf{Shard}) ?$  True  $\mathcal{K} \models \mathsf{location}(\mathsf{EiffelTower}, \mathsf{France}) ?$  True  $\mathcal{K} \models \mathsf{location}(\mathsf{Shard}, \mathsf{UK}) ?$  True  $\mathcal{K} \models \mathsf{location}(\mathsf{Shard}, \mathsf{France}) ?$  False



## Devise algorithms that compute query answers E.g., using natural deduction rules:

$$\frac{\forall x . P(x) \rightarrow Q(x) \quad \forall x . Q(x) \rightarrow R(x)}{\forall x . P(x) \rightarrow R(x)} \qquad \frac{\forall x . \text{Greek}(x) \rightarrow \text{Man}(x) \quad \forall x . \text{Man}(x) \rightarrow \text{Mortal}(x)}{\forall x . \text{Greek}(x) \rightarrow \text{Mortal}(x)}$$

$$\frac{\forall x . P(x) \rightarrow Q(x) \quad P(a)}{Q(a)} \qquad \frac{\forall x . \text{Building}(x) \rightarrow \text{ArchStruc}(x) \quad \text{Building}(\text{EiffelTower})}{\text{ArchStruc}(\text{EiffelTower})}$$

Can check/prove algorithms are sound and complete w.r.t. semantics



Turing showed that some problems cannot be completely solved using standard computational model

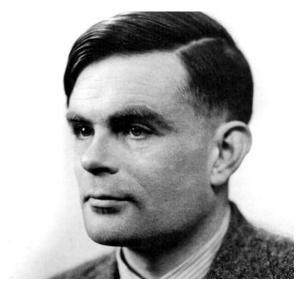
- halting problem
- FOL entailment problem

Even if decidable, reasoning might be of inherently high complexity and so take an infeasibly long time



Turing showed that some problems cannot be completely solved using standard computational model

- halting problem
- FOL entailment problem



Even if decidable, reasoning might be of inherently high complexity and so take an infeasibly long time



# "Scruffy" approach:

Ad-hoc representation Efficient but (at least) incomplete algorithms

- ✓ Can use arbitrarily powerful representation
- √ Favourable scalability properties
- Incomplete answers
  - Degree of incompleteness unknown
  - Incompleteness can easily become unsoundness



# "Neat" approach:

Study KR languages to find appropriate balance of expressive power and computability

Design algorithms that work well in typical cases

Develop highly optimised implementations

- ✓ Precisely defined semantics
- ✓ Formal properties well understood
- √ Sound and complete reasoning
- Limited representation power
- Optimisations may not offer robust scalability



Family of logic-based KR languages

Most are decidable subsets of FO logic

Provide a range of different constructors

- Booleans (and, or, not)
- Restricted forms of quantification (exists, forall)
- Counting (atmost, atleast)
- •

Decidability/complexity and (efficient) algorithms known for many combinations of constructors Highly optimised implementations for various "sweet spot" languages





#### **Complexity of reasoning in Description Logics**

Note: the information here is (always) incomplete and  $\underline{\text{updated}}$  often

Base description logic: Attributive Language with  ${\mathcal C}$ omplements

 $\mathcal{ALC} := \bot \mid A \mid \neg C \mid C \land D \mid C \lor D \mid \exists R.C \mid \forall R.C$ 



Concept constructors:	Role constructors:	trans reg
■ $F$ - functionality $^2$ : ( $\leq 1\ R$ )  ■ $\mathcal{N}$ - (unqualified) number restrictions: ( $\geq n\ R$ ), ( $\leq n\ R$ )  ■ $Q$ - qualified number restrictions: ( $\geq n\ R$ . $C$ ), ( $\leq n\ R$ . $C$ )  ■ $Q$ - nominals: { $a$ } or { $a_1$ ,, $a_n$ } ("one-of" constructor)  ■ $\mu$ - least fixpoint operator: $\mu X.C$ ■ $R \subseteq S$ - role-value-maps  ■ $f = g$ - agreement of functional role chains ("same-as")	✓ I – role inverses: R –  □ ∩ – role intersection³: R∩S □ ∪ – role union: R∪S □ ¬ – role complement: full ⇒ □ o – role chain (composition): RoS □ * – reflexive-transitive closure⁴: R* □ id – concept identity: id(C)  Forbid ⇒ complex roles⁵ in number restrictions⁶	
<b>TBox</b> is internalized in extensions of $ALCIO$ , see [76, Lemma 4.12], [54, p.3]  • Empty TBox  • Acyclic TBox ( $A \equiv C$ , $A$ is a concept name; no cycles)  • General TBox ( $C \subseteq D$ for arbitrary concepts $C$ and $D$ )	Role axioms (RBox):	OWL-Lite OWL-DL OWL 1.1
You have selected the Description Logic:	SHOIN	

Complexity of reasoning problems <sup>7</sup>			
Reasoning problem	Complexity <sup>8</sup>	Comments and references	
Concept satisfiability	NExpTime-complete	<ul> <li>Hardness of even ALCFIO is proved in [76, Corollary 4.13]. In that paper, the result is formulated for ALCQIO, but only number restrictions of the form (≤1R) are used in the proof.</li> <li>A different proof of the NExpTime-hardness for ALCFIO is given in [54] (even with 1 nominal, and role inverses not used in number restrictions).</li> <li>Upper bound for SHOIQ is proved in [77, Corollary 6.31] with numbers coded in unary (for binary coding, the upper bound remains an open problem for all logics in between ALCNIO and SHOIQ.</li> <li>Important: in number restrictions, only simple roles (i.e. which are neither transitive nor have a transitive subroles) are allowed; otherwise we gain undecidability even in SHN, see [46].</li> <li>Remark: recently [47] it was observed that, in many cases, one can use transitive roles in number restrictions – and still have a decidable logic! So the above notion of a simple role could be substantially extended.</li> </ul>	
ABox consistency	NExpTime-complete	By reduction to concept satisfiability problem in presence of nominals shown in [69, Theorem 3.7].	





### List of reasoners

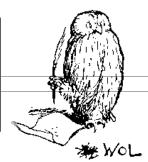
Reasoner	Instution	Download	Publication
BaseVISor	VIStology, Inc.	Download	Core publication
BUNDLE	University of Ferrara	Download	Core publication
CEL	Technische Universität Dresden	Download	Core publication
Chainsaw	The University of Manchester	Download	Core publication
Clipper	Vienna University of Technology	Download	Core publication
DBOWL	University of Malaga	Download	Core publication
DeLorean	Not given	Download	Core publication

DistEL	Wright State University	Download	Core publication
DRAOn	University of Paris 8, IUT of Montreuil	Download	Core publication
DReW	Vienna University of Technology	Download	Core publication
ELepHant	Not given	Download	Core publication
ELK	University of Ulm, Germany	Download	Core publication
ELOG	Not given	Download	Core publication
FaCT++	The University of Manchester	Download	Core publication
fuzzyDL	ISTI – CNR	Download	Core publication

HermiT	University of Oxford	Download	Core publication
jcel	Technische Universität Dresden	Download	Core publication
JFact	The University of Manchester	Download	Core publication
Konclude	University of Ulm, derivo GmbH	Download	Core publication
LIFR	Centre for Research and Technology Hellas (CERTH)	Download	Core publication
Mastro	Sapienza University of Rome	Download	Core publication
MORe	University of Oxford	Download	Core publication
ontop	Free University of Bozen-Bolzano	Download	Core publication

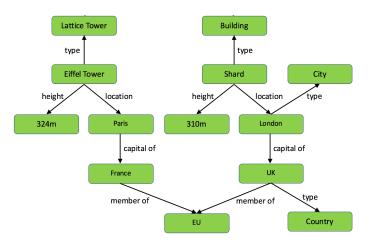






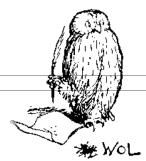
- RDF provides a graphical data model
- OWL provides a DL-based ontology language

 $\begin{aligned} \mathsf{LatticeTower}(x) &\to \mathsf{ArchStruct}(x) \\ \mathsf{Building}(x) &\to \mathsf{ArchStruct}(x) \\ \mathsf{location}(x,y) \land \mathsf{capitalOf}(y,z) &\to \mathsf{location}(x,z) \end{aligned}$ 









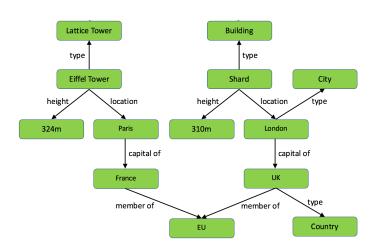
- RDF provides a graphical data model
- OWL provides a DL-based ontology language

 $\mathsf{LatticeTower}(x) \to \mathsf{ArchStruct}(x)$ 

**OWL ontology:** Building $(x) \rightarrow ArchStruct(x)$ 

 $\mathsf{location}(x,y) \land \mathsf{capitalOf}(y,z) \to \mathsf{location}(x,z)$ 

RDF data:









- RDF provides a graphical data model
- OWL provides a DL-based ontology language

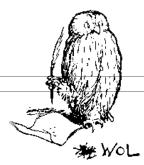
Developed as part of **W3C's Semantic Web** project

"A new form of Web content that is meaningful to computers will unleash a revolution of new possibilities" (!)









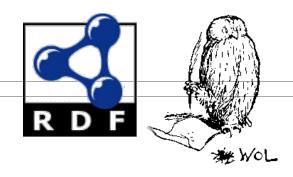
- RDF provides a graphical data model
- OWL provides a DL-based ontology language

Developed as part of W3C's Semantic Web project

Now widely used in science, healthcare and Industry
Often referred to as "semantic technology"







# Based on powerful but still decidable DL (SROIQ) Three "profiles" based on tractable subsets

- QL: based on the DL-Lite description logic
- EL: based on the EL description logic
- RL: based on the DL fragment of Datalog (aka DLP)

# Different algorithmic techniques

- (Hyper-) Tableau for full language
- Query rewriting for QL
- Consequence-based for EL
- Materialisation for RL

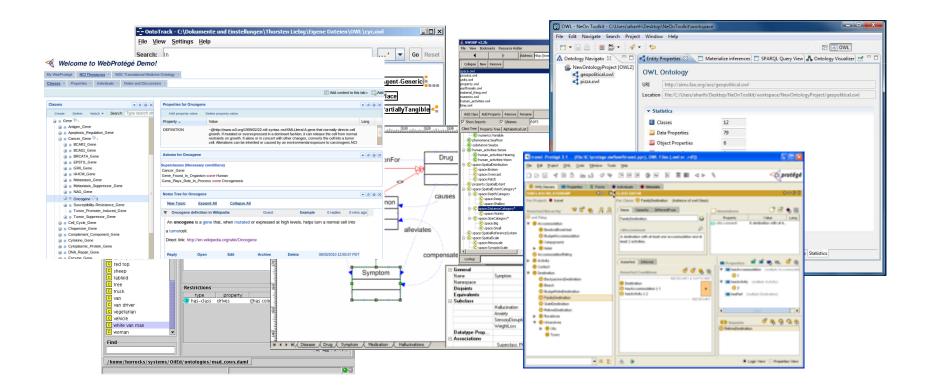
Highly optimised implementations



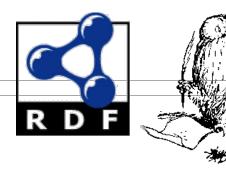




### **Tools:**







### **Reasoners:**

# Hermit FaCT++ ORACLE













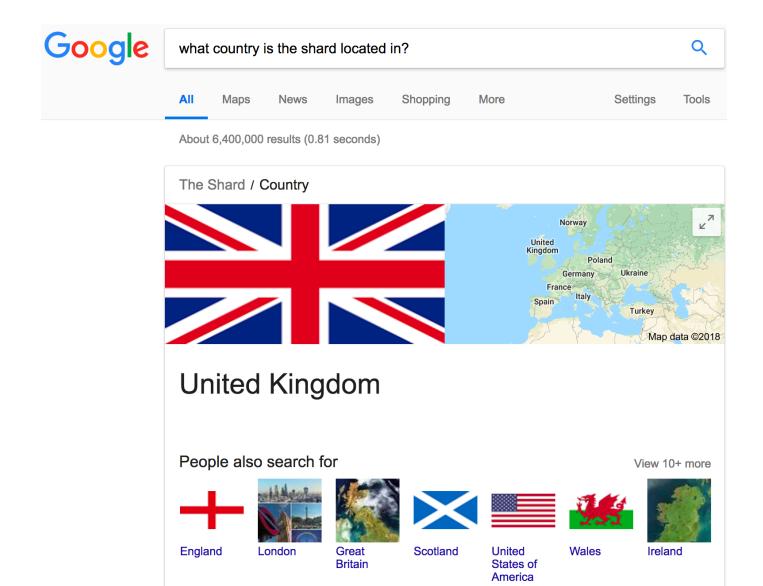




# Applications: Question Answering











### **KR Success Story:**

SNOMED is a **huge** medical ontology

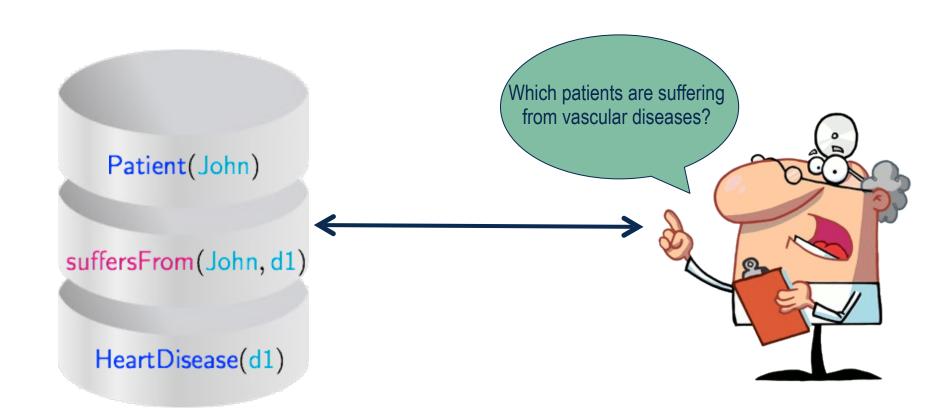
More than 500,000 terms!

Why SNOMED? Let's ask Healthcare experts!

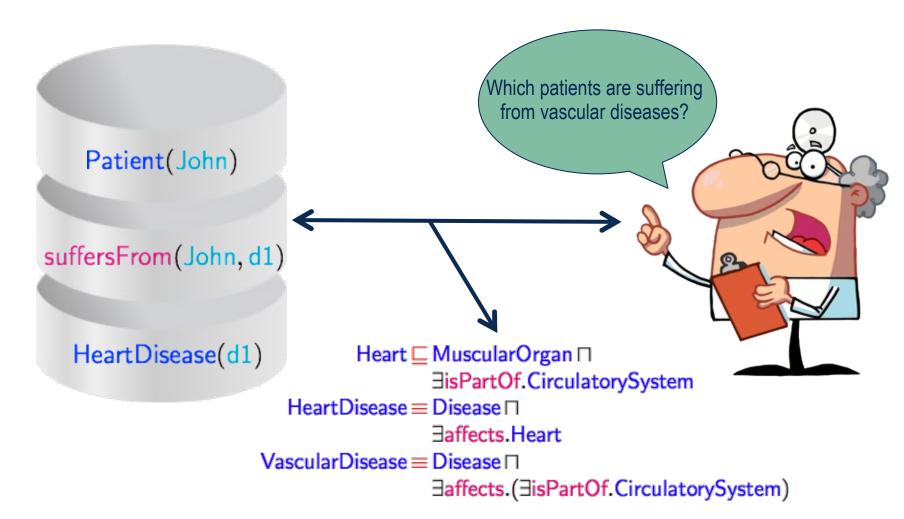
- "We need a clinical ontology that is universal, so any term I use is the same as every other colleague around the country"
- "SNOMED is the glue that binds the clinical community together and is the platform for all clinically relevant information"

Used to annotate patient records in more than 20 countries, including UK, USA, New Zealand, ...











The end?



### **Extensions**

- Arithmetic functions and aggregation
- Reasoning about time
- Data streams

### **Algorithms**

- Consequence-based reasoning
- Hybrid rewriting/materialisation

## **Optimisation and implementation**

- Incremental reasoning
- Query planning
- HPC, including large-scale and distributed architectures

# **Tools and applications**



### Course Structure

Logics for Knowledge Representation

Horn Logics and Datalog

Description Logics – Syntax and Semantics

Description Logics - Reasoning with Data

Nonmonotonic Reasoning

Inconsistency Handling

Argumentation

Uncertainty



## Extra Reading Material

### **Primary Text**

 An Introduction to Description Logic. Franz Baader, Ian Horrocks, Carsten Lutz, Uli Sattler

### Supplementary Texts

- Handbook of Knowledge Representation. Frank van Harmelen, Vladimir Lifschitz and Bruce Porter (Eds). Foundations of Artificial Intelligence, 2008.
- Foundations of Semantic Web Technologies. Chapman & Hall/ CRC Textbooks in Computing. Pascal Hitzler, Markus Kroetsch, and Sebastian Rudolph, 2009.