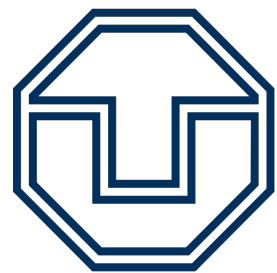


From Horn-*SRIQ* to Datalog:  
A Data-Independent Transformation that  
Preserves Assertion Entailment

David Carral, Larry González, and Patrick Koopmann



**TECHNISCHE  
UNIVERSITÄT  
DRESDEN**

Poster: KRR5901

# Introduction

# The DL Horn-*SRIQ*: Syntax

$$C_1 \sqcap \dots \sqcap C_n \sqsubseteq D$$

$$\exists R. C \sqsubseteq D$$

$$C \sqsubseteq \exists R. D$$

$$C \sqsubseteq \leq 1R. D$$

$$R_1 \circ \dots \circ R_n \sqsubseteq S$$

$$R^- \sqsubseteq S$$

$$C(a)$$

$$R(a, b)$$

# The DL Horn-*SRIQ*: Syntax

- $C_1 \sqcap \dots \sqcap C_n \sqsubseteq D \rightarrow$  EnglishSpeaker  $\sqcap$  FrenchSpeaker  $\sqsubseteq$  Bilingual,  
Vehicle  $\sqsubseteq$  Car, Vertebrate  $\sqcap$  Invertebrate  $\sqsubseteq \perp$
- $\exists R . C \sqsubseteq D \rightarrow$   $\exists$ Attends . Course  $\sqsubseteq$  Student
- $C \sqsubseteq \exists R . D \rightarrow$  Director  $\sqsubseteq \exists$ Directs . Movie
- $C \sqsubseteq \leq 1R . D \rightarrow$  PhDStudent  $\sqsubseteq \leq 1$ HasThesisSupervisor . Faculty
- $R_1 \circ \dots \circ R_n \sqsubseteq S \rightarrow$  HasAncestor  $\circ$  HasAncestor  $\sqsubseteq$  HasAncestor, HasMother  $\sqsubseteq$  HasParent,  
HasParent  $\circ$  HasSister  $\sqsubseteq$  HasAunt
- $R^- \sqsubseteq S \rightarrow$  HasChild<sup>-</sup>  $\sqsubseteq$  HasParent
- $C(a) \rightarrow$  Person(david)
- $R(a, b) \rightarrow$  HasFriend(stan, kyle)

# The DL Horn-*SRIQ*: Semantics

$$\begin{aligned}C_1 \sqcap \dots \sqcap C_n \sqsubseteq D &\mapsto \forall x. C_1(x) \wedge \dots \wedge C_n(x) \rightarrow D(x) \\ \exists R. C \sqsubseteq D &\mapsto \forall x, y. R(x, y) \wedge C(y) \rightarrow D(x) \\ C \sqsubseteq \exists R. D &\mapsto \forall x. C(x) \rightarrow \exists y. R(x, y) \wedge D(y) \\ C \sqsubseteq \leq 1R. D &\mapsto \forall x, y, z. C(x) \wedge R(x, y) \wedge D(y) \wedge R(x, z) \wedge D(z) \rightarrow y \approx z \\ R_1 \circ \dots \circ R_n \sqsubseteq S &\mapsto \forall x_0, \dots, x_n. R_1(x_0, x_1) \wedge \dots \wedge R_n(x_{n-1}, x_n) \rightarrow R(x_0, x_n) \\ R^- \sqsubseteq S &\mapsto \forall x, y. R(x, y) \rightarrow S(y, x) \\ A(a) &\mapsto A(a) \\ R(a, b) &\mapsto R(a, b)\end{aligned}$$

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# The DL Horn-*SRIQ*: Semantics

## Terminological axioms

$C_1 \sqcap \dots \sqcap C_n \sqsubseteq D$	$\mapsto$	$C_1(x) \wedge \dots \wedge C_n(x) \rightarrow D(x)$
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## Assertions / Facts



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**Assertions / Facts**

Ontology

**Set of terminological axioms / TBox**

$$\mathcal{O} = \langle \mathcal{T}, \mathcal{F} \rangle$$

**Set of facts / ABox**

# Datalog

$\text{Features}(x, y) \rightarrow \text{Actor}(y)$

$\text{ActsIn}(x, y) \rightarrow \text{Features}(y, x)$

$\text{HasID}(x, y) \wedge \text{HasID}(x, z) \rightarrow y \approx z$

$\text{Directs}(x, y) \wedge \text{Features}(y, z) \rightarrow \text{DirectsActor}(x, z)$

$\text{Reviews}(x, y) \wedge \text{IsAuthorOf}(z, y) \wedge \text{CollaboratesWith}(x, y) \rightarrow \text{IllegalReviewer}(x, y)$

$\text{P}(x, y, z) \wedge \text{S}(y, w, v) \wedge \text{V}(y, v) \rightarrow \text{D}(x)$

# Datalog

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$\text{HasFriend}(\text{stan}, \text{kyle})$

$R(a, b, c)$

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R(a, b, c)

## Rules

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Program

$$\mathcal{P} = \langle \mathcal{R}, \mathcal{F} \rangle$$

Set of rules

Set of facts

# Datalog

## Assertions / Facts

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**Remark:** Existential quantification is not allowed in Datalog.

# Datalog

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$\text{Director} \sqsubseteq \exists \text{Directs} . \text{Movie}$

$\text{Director}(x) \rightarrow \exists y . \text{Directs}(x, y) \wedge \text{Movie}(y)$

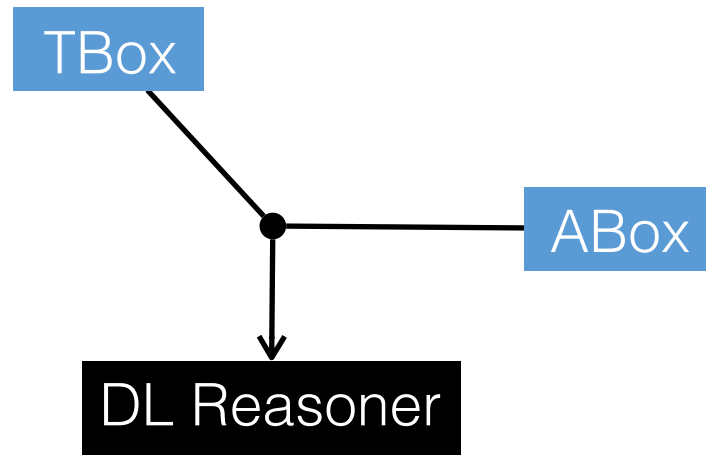


# Solving Assertion Retrieval with Datalog Rewritings

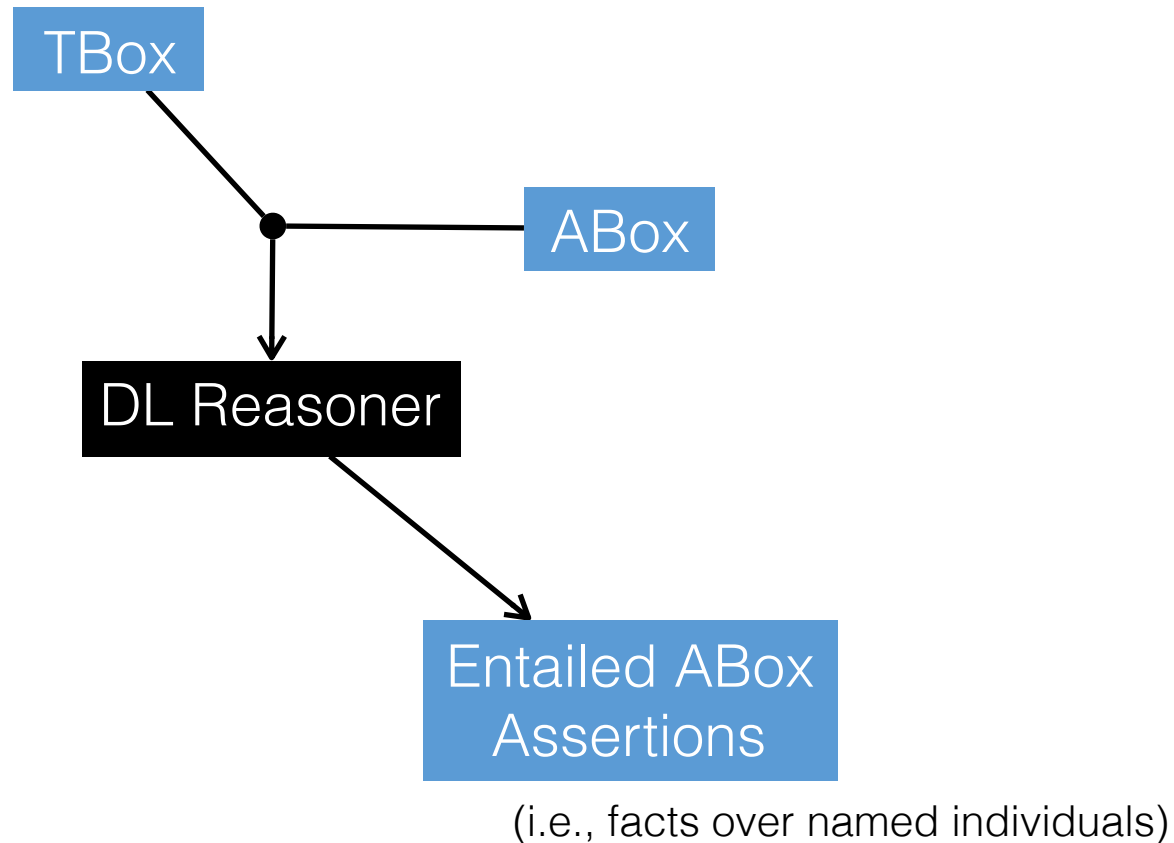
TBox

ABox

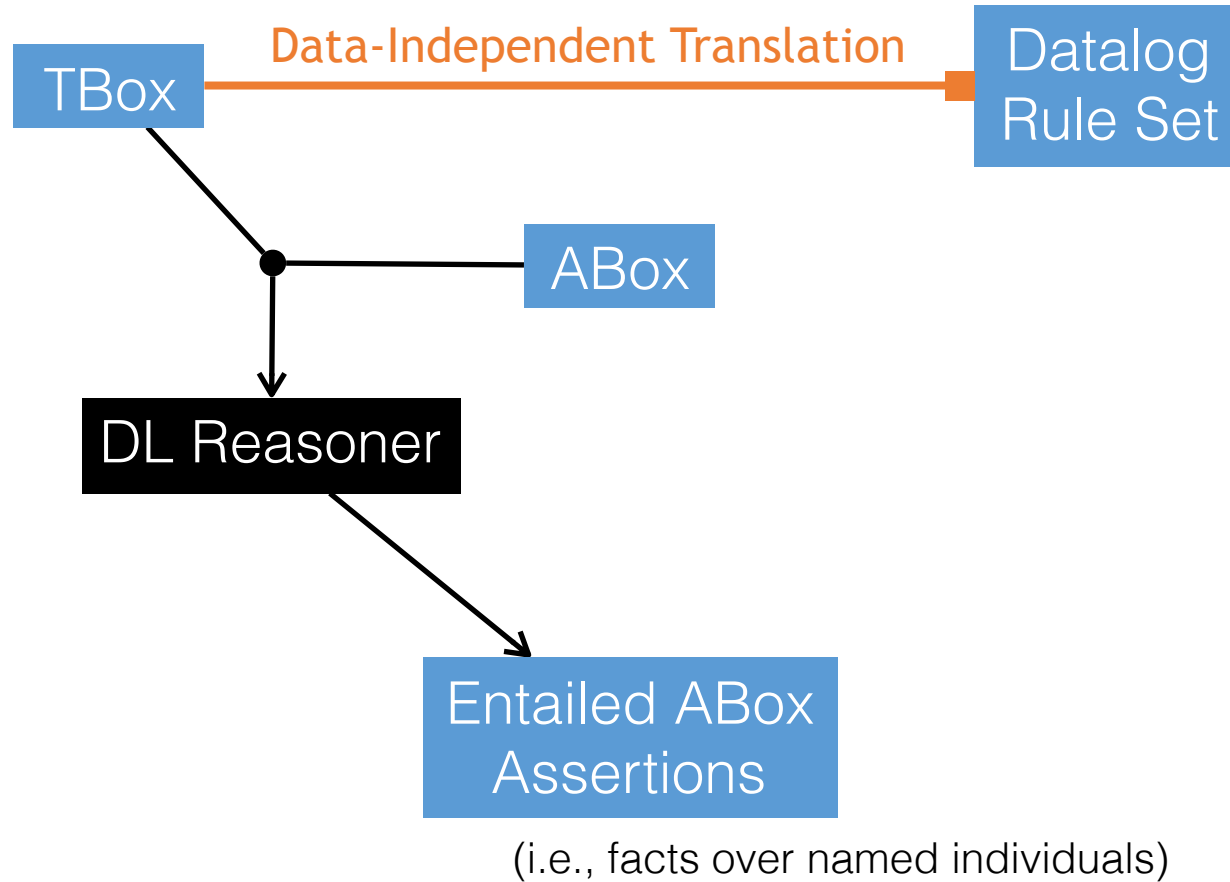
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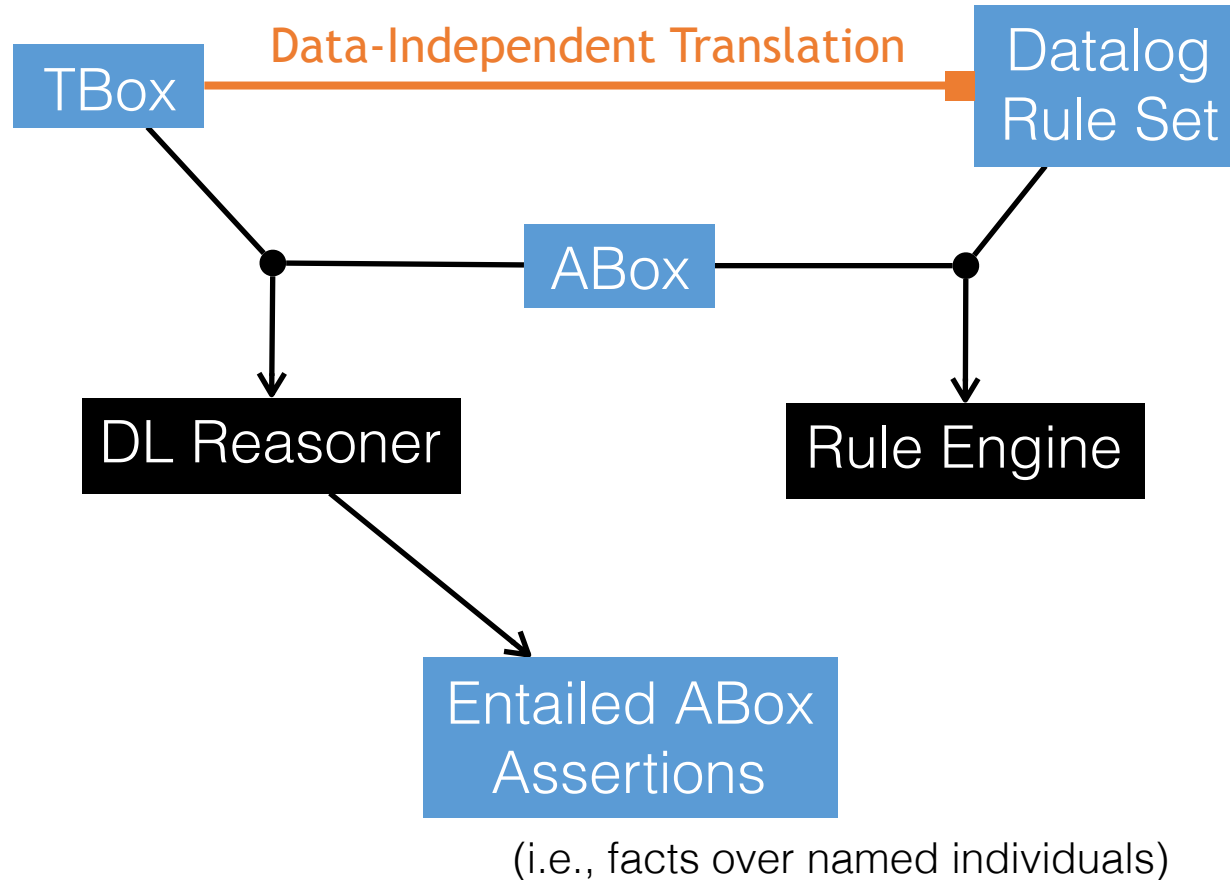
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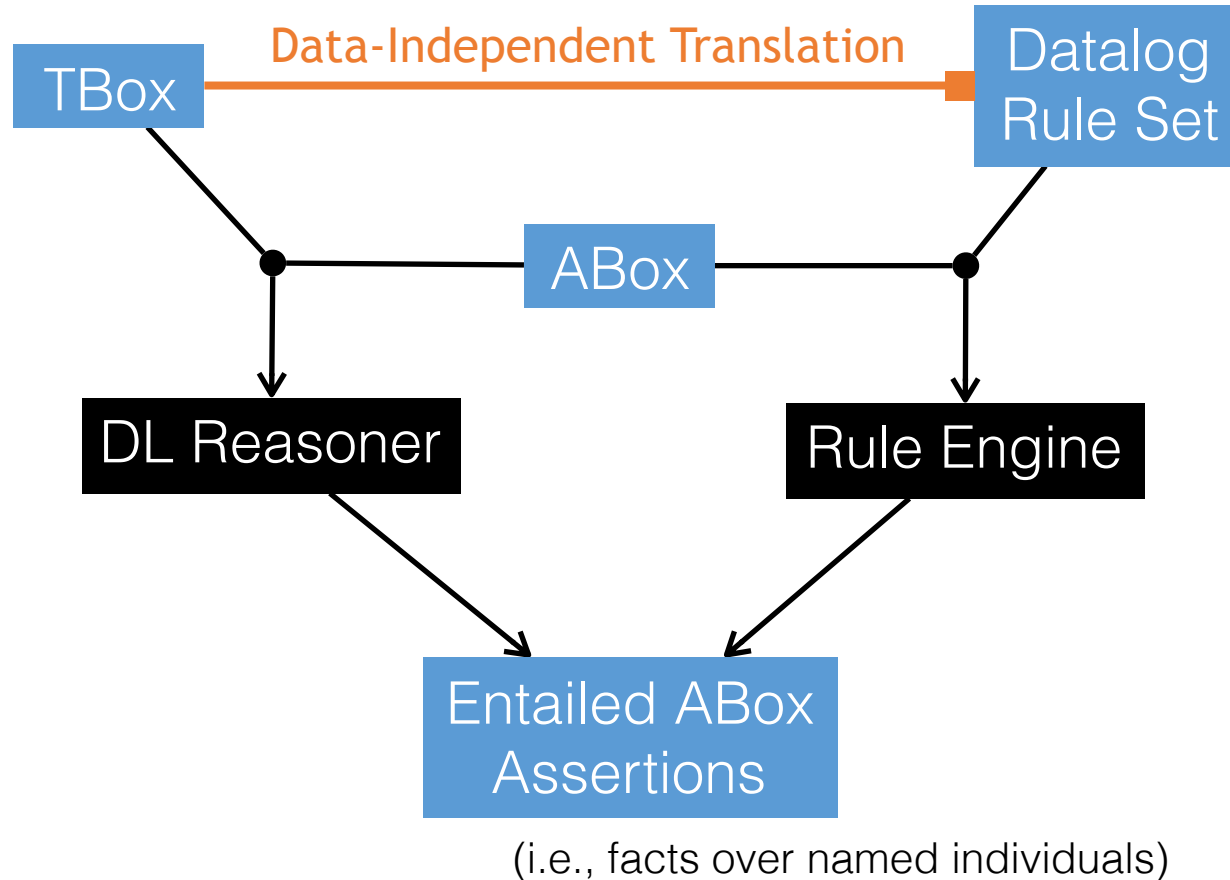
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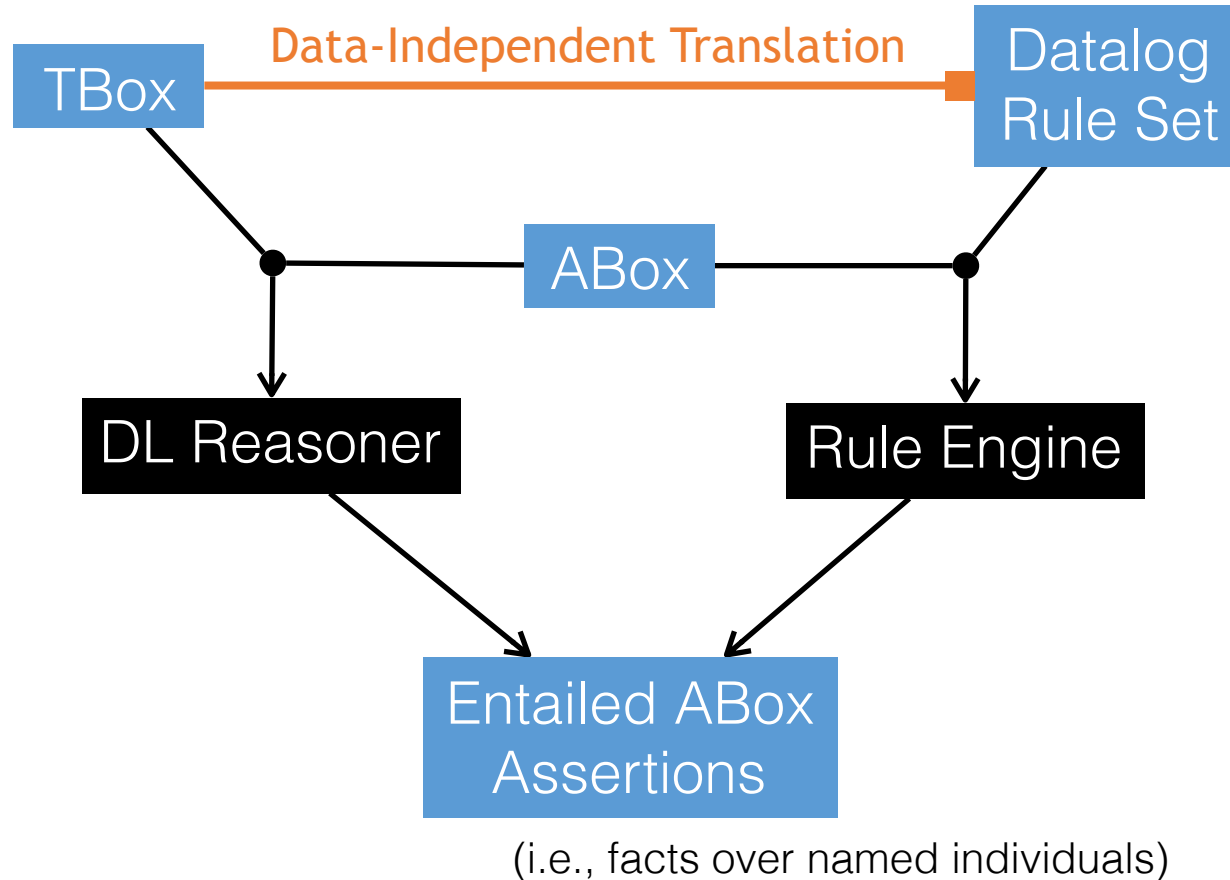
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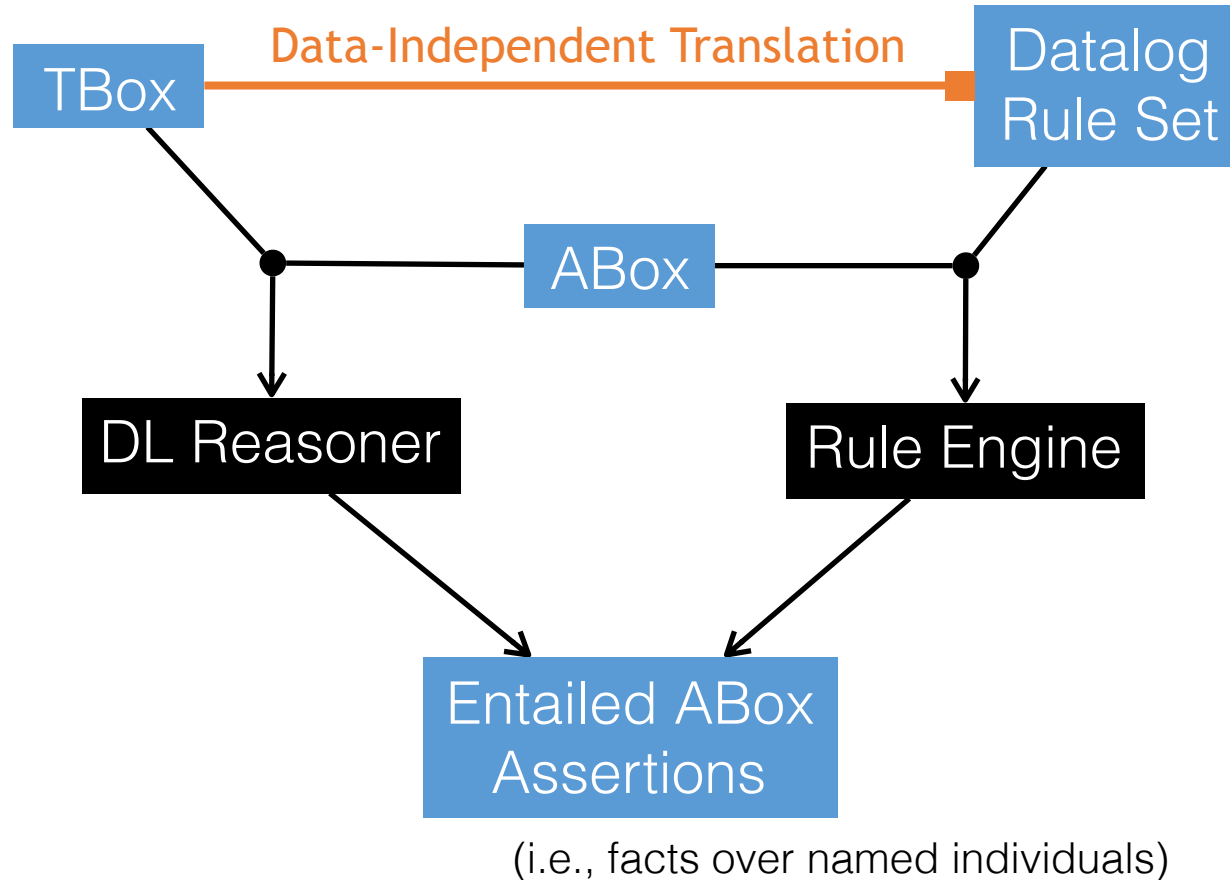
# Solving Assertion Retrieval with Datalog Rewritings



## Motivation

- \* Research expressivity
- \* Performance improvements

# Solving Assertion Retrieval with Datalog Rewritings



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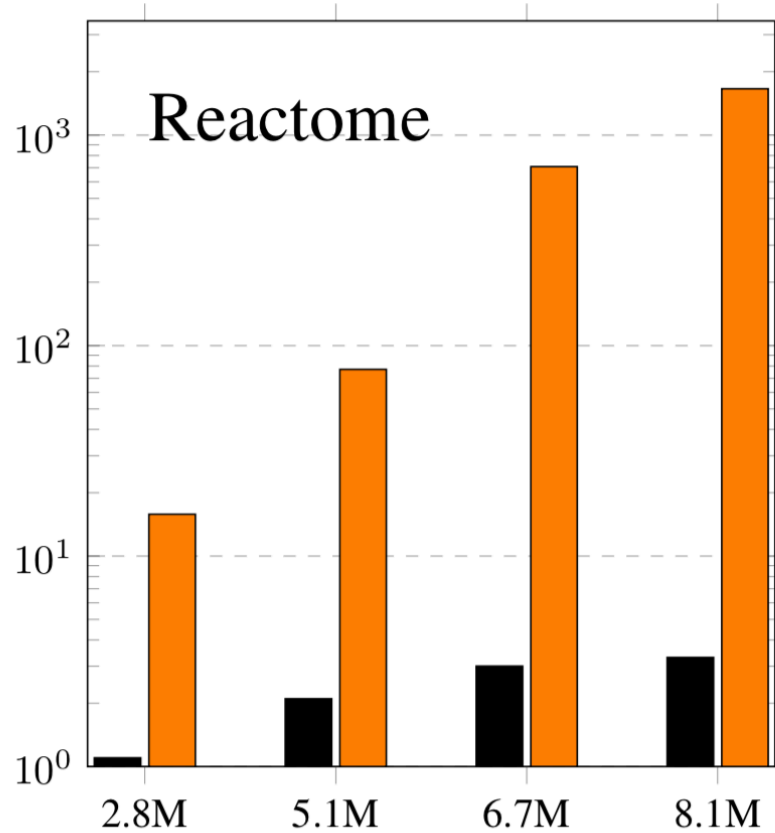
## Challenges

- \* Correctness and complexity
- \* Implement translation and evaluate performance

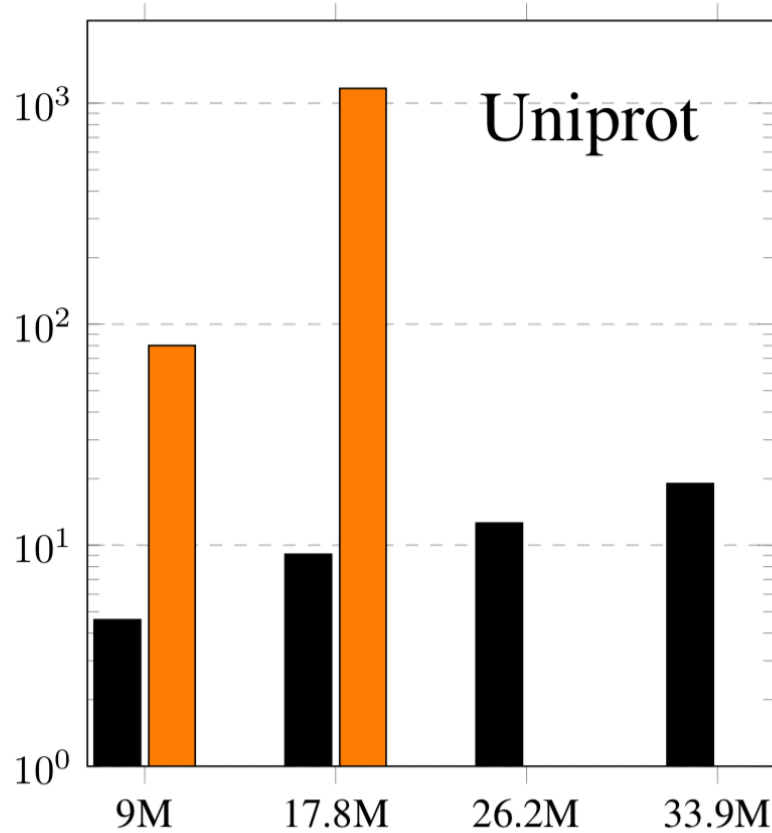


# Evaluation

# Reasoning with Rewritings



TBox size: 485  
Rewriting size: 549



TBox size: 304  
Rewriting size: 367

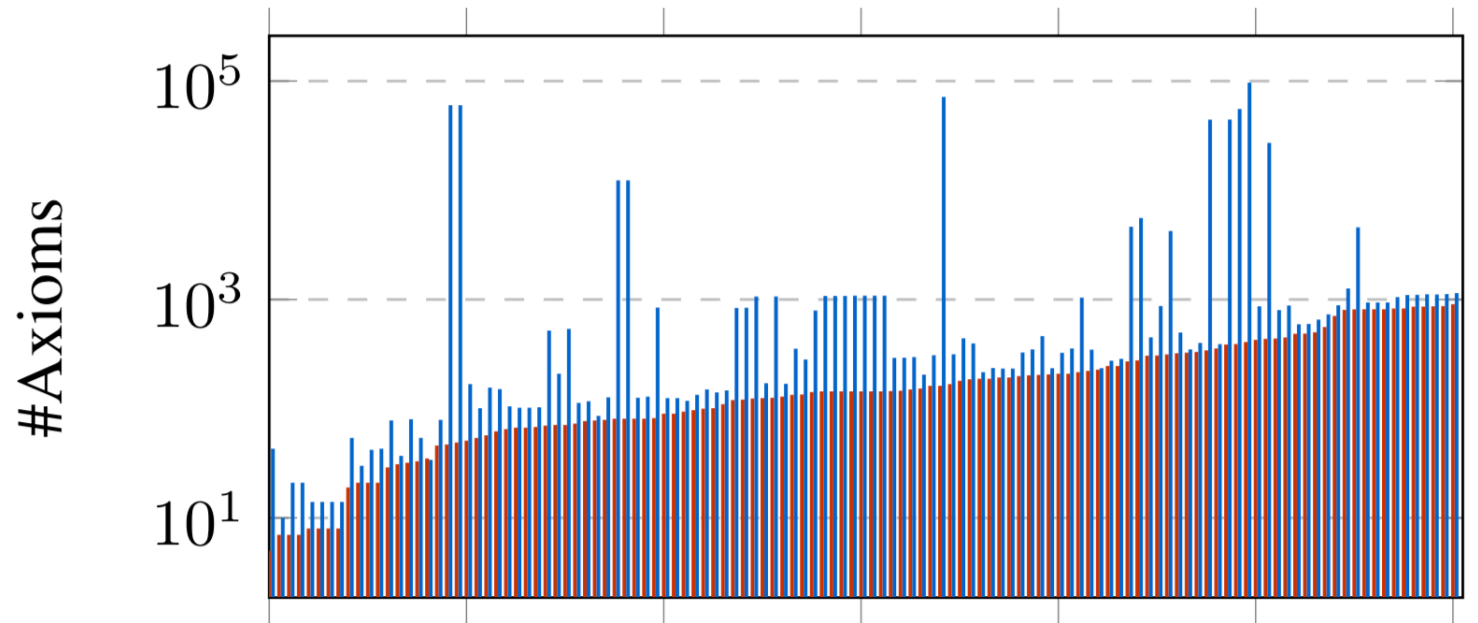


# Size of Rewritings

- MOWLCorpus: TBoxes with less 1000 axioms and containing role chain axioms
- 187 TBoxes
- 121 computed rewritings w/o OOM errors

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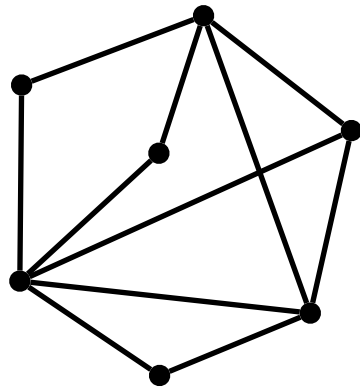


From Horn-*ALCHIQ* to Datalog

# From Horn-*ALCHIQ* to Datalog

$$R_1 \circ \dots \circ R_n \sqsubseteq S \rightarrow R \sqsubseteq S$$

# Forest Model Property



$$C_1 \sqcap \dots \sqcap C_n \sqsubseteq D$$

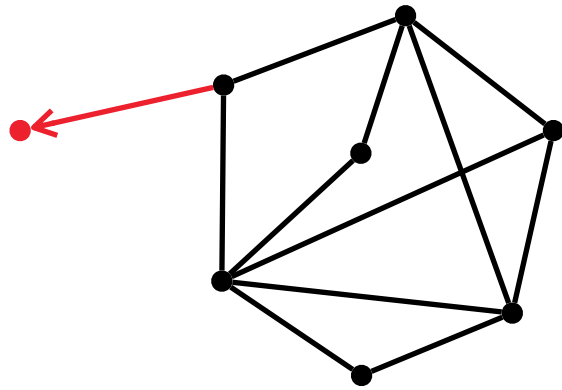
$$C \sqsubseteq \exists R. D$$

$$\exists R. C \sqsubseteq D$$

$$C \sqsubseteq \leq 1 R. D$$

$$R \sqsubseteq S$$

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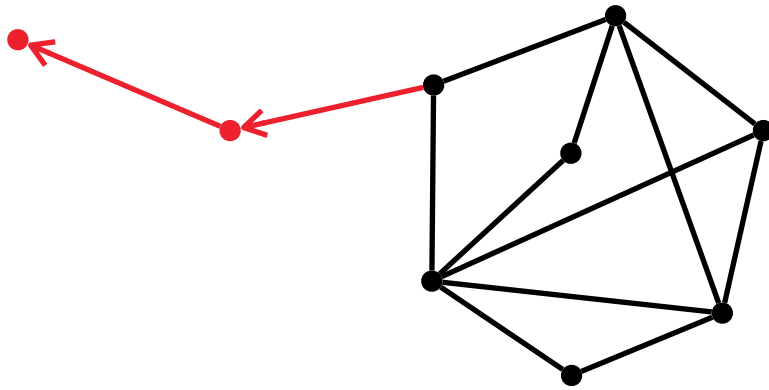
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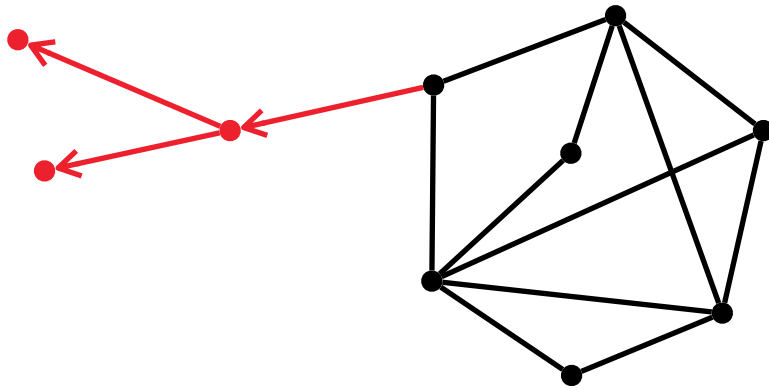
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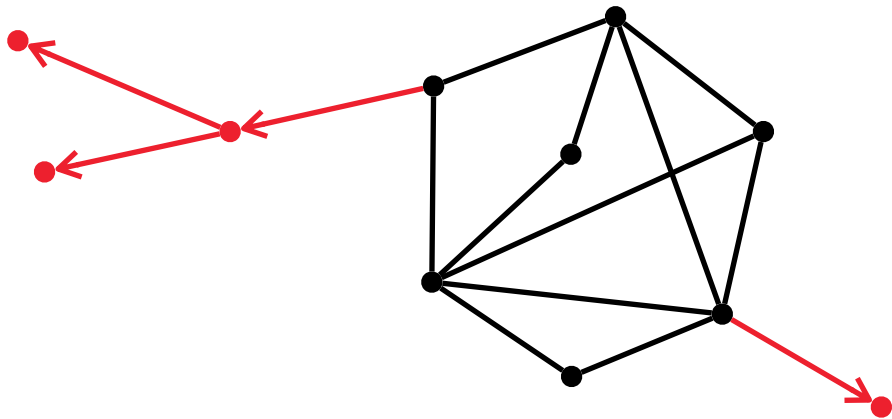
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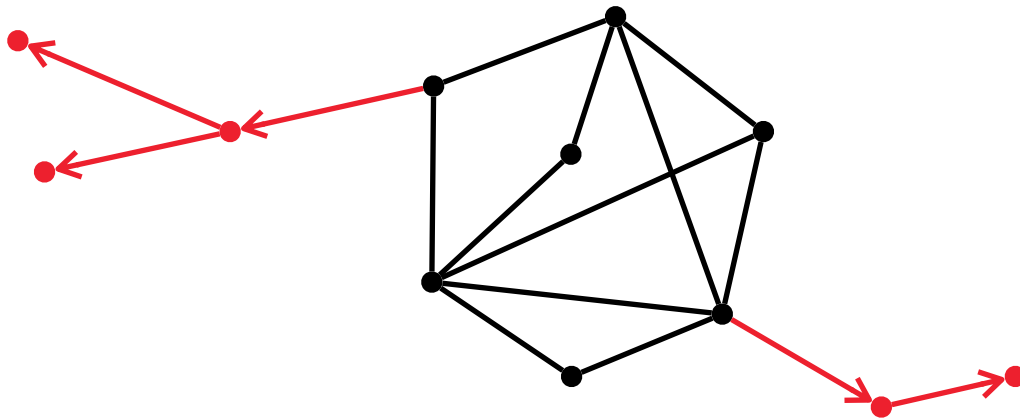
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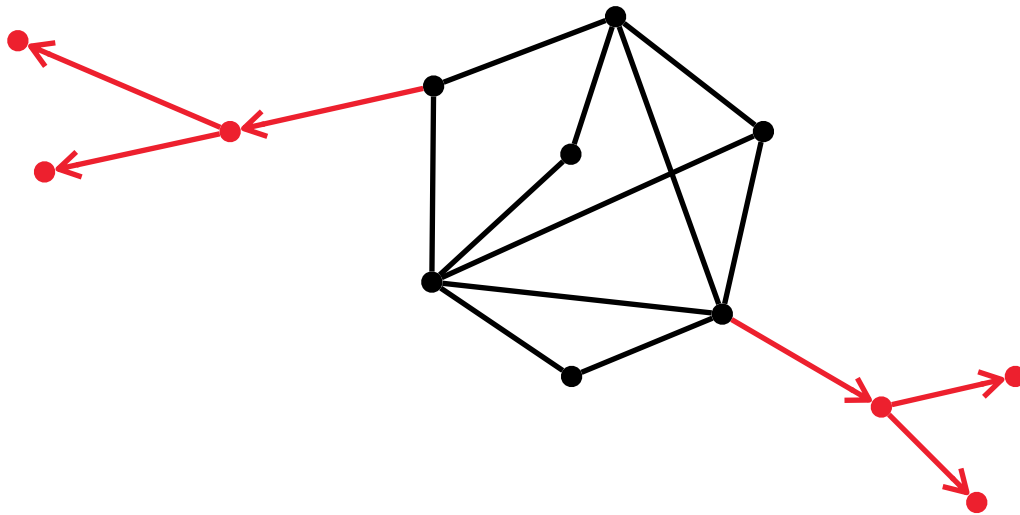
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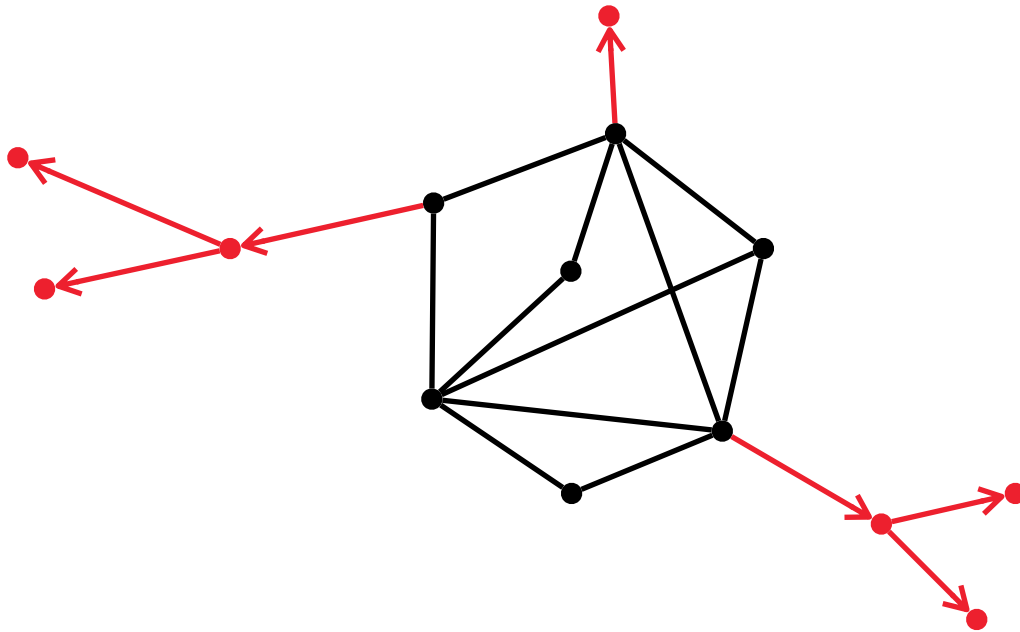
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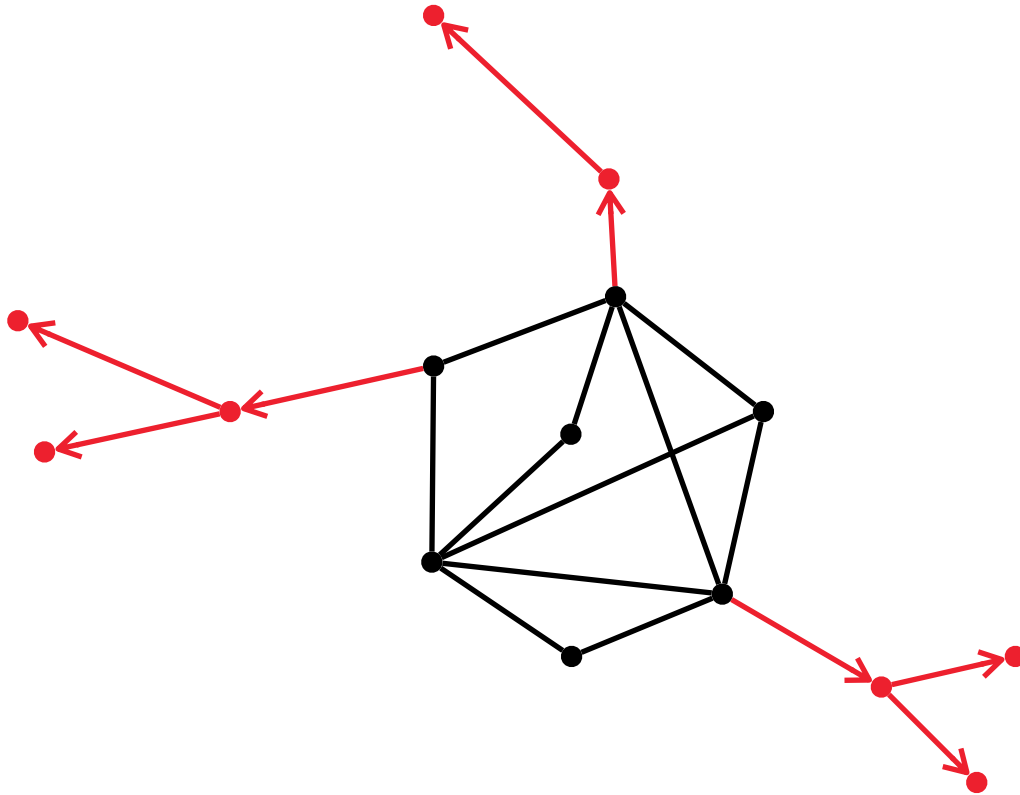
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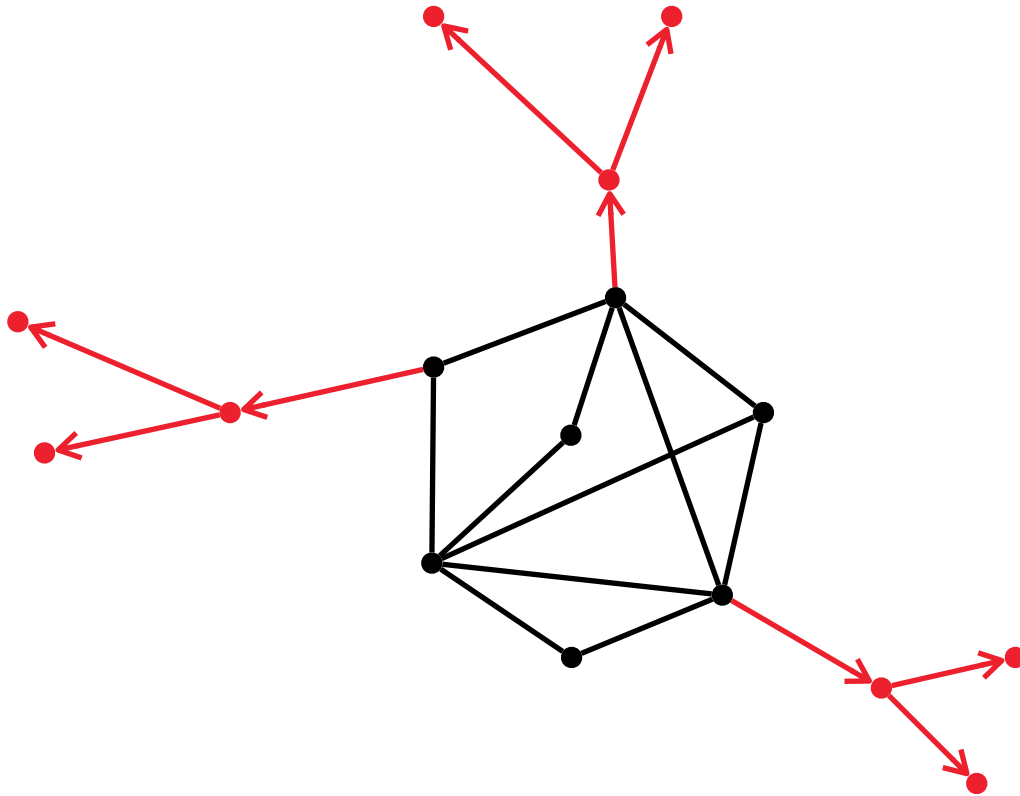
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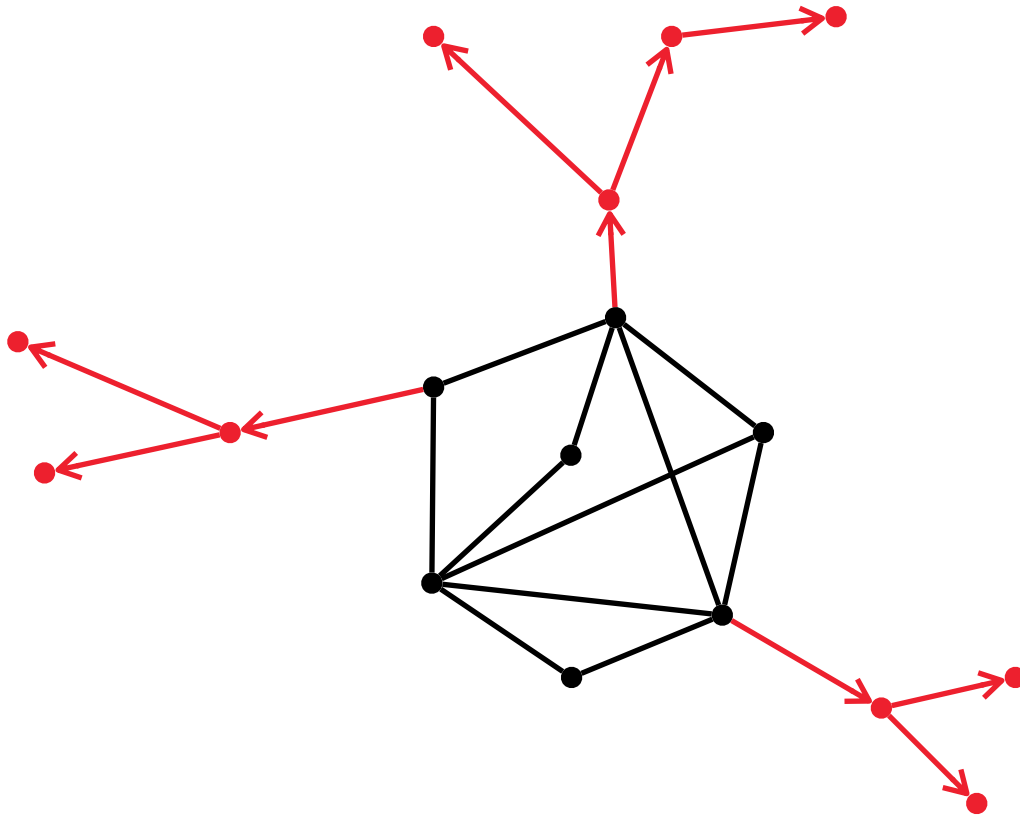
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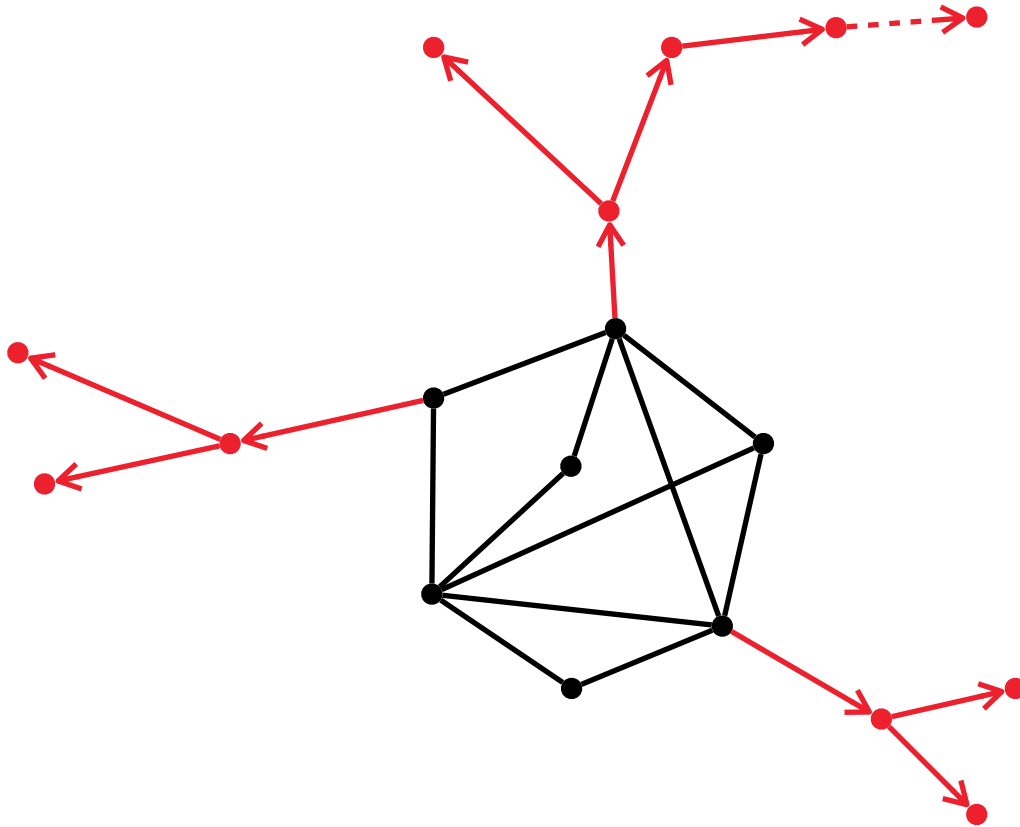
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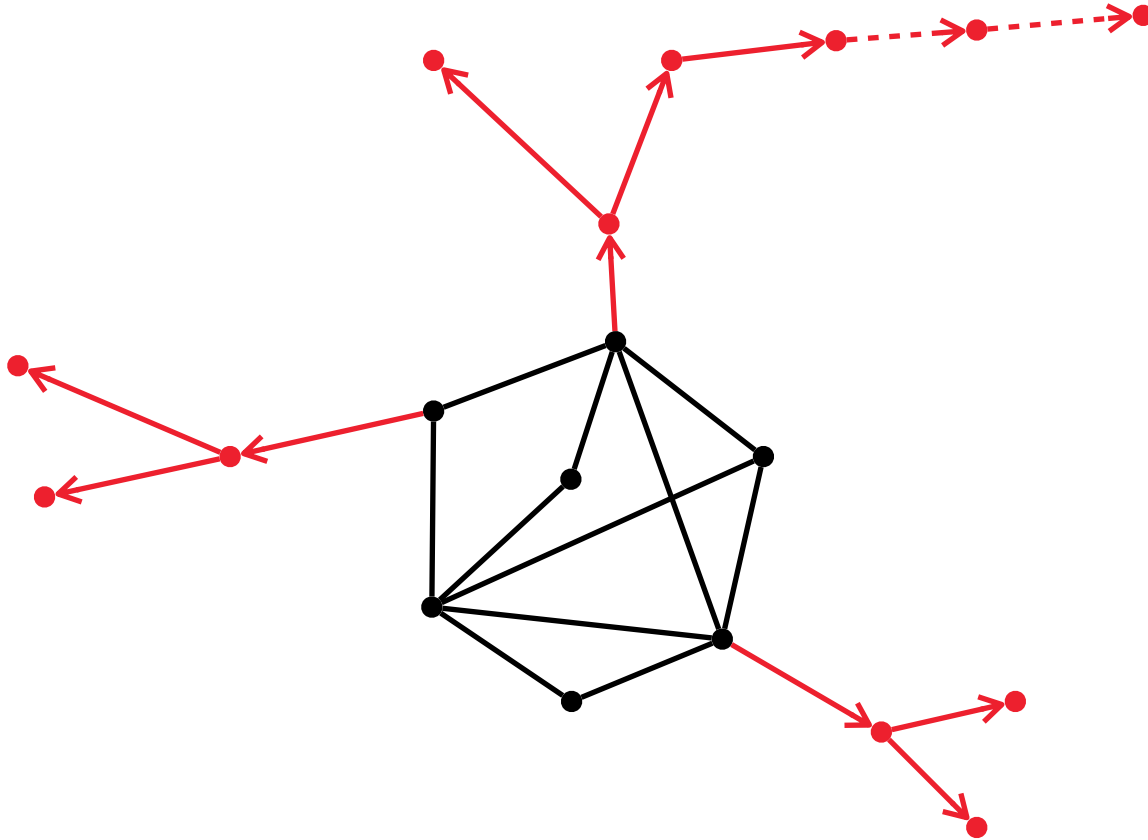
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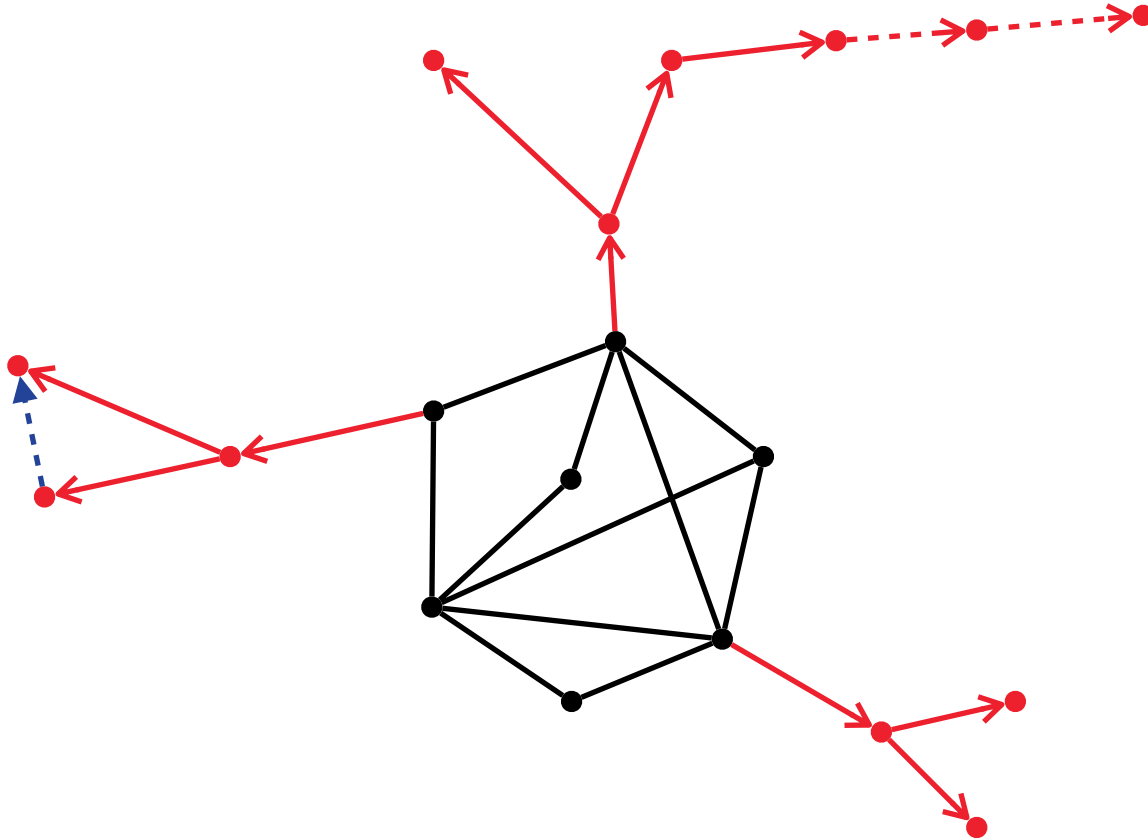
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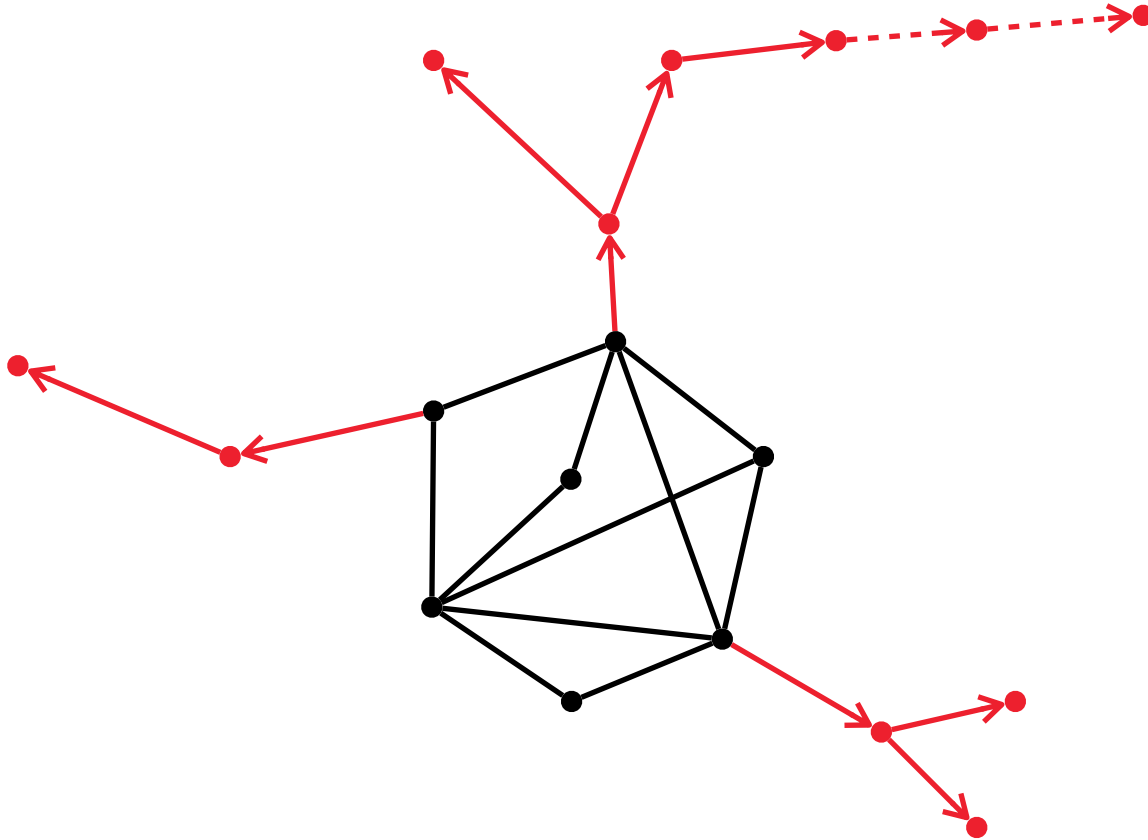
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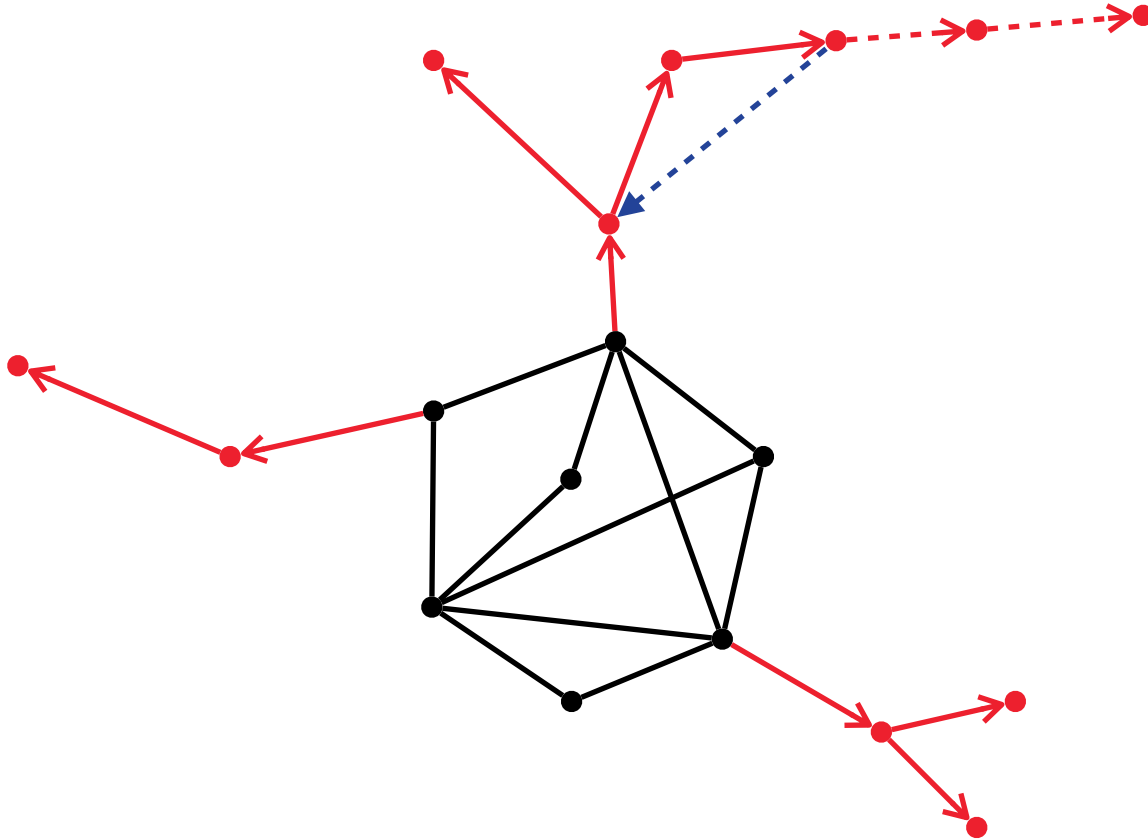
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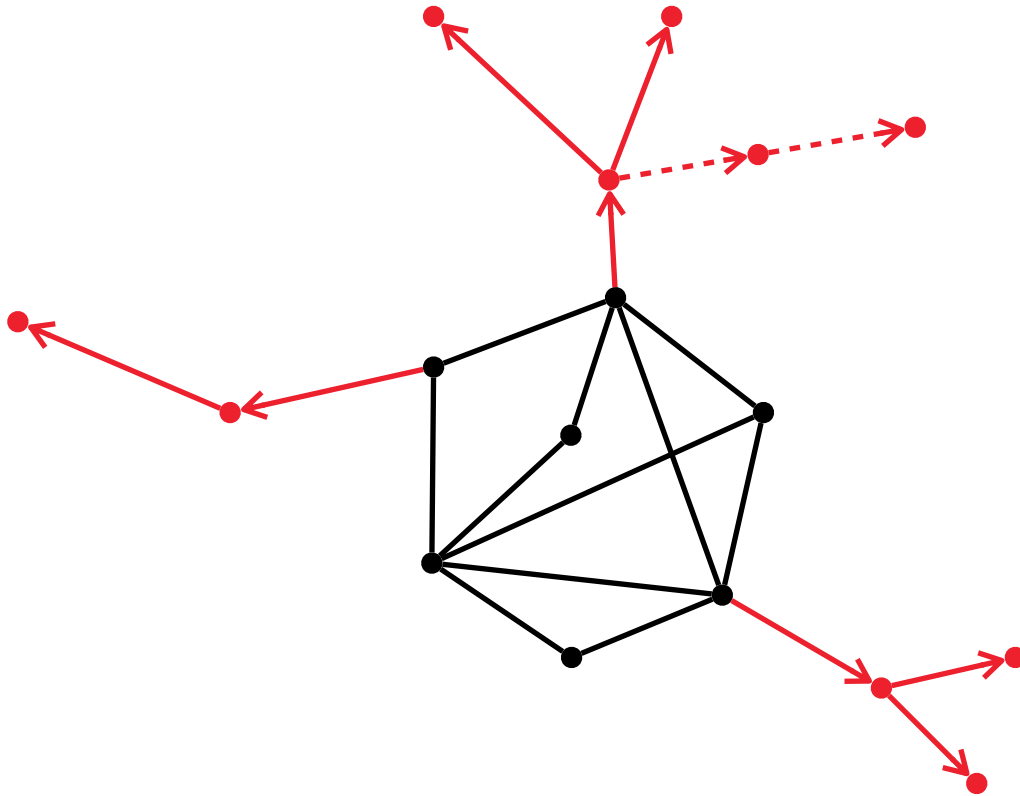
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$$C \sqsubseteq \leq 1 R. D$$

$$R \sqsubseteq S$$

# Forest Model Property



$$C_1 \sqcap \dots \sqcap C_n \sqsubseteq D$$

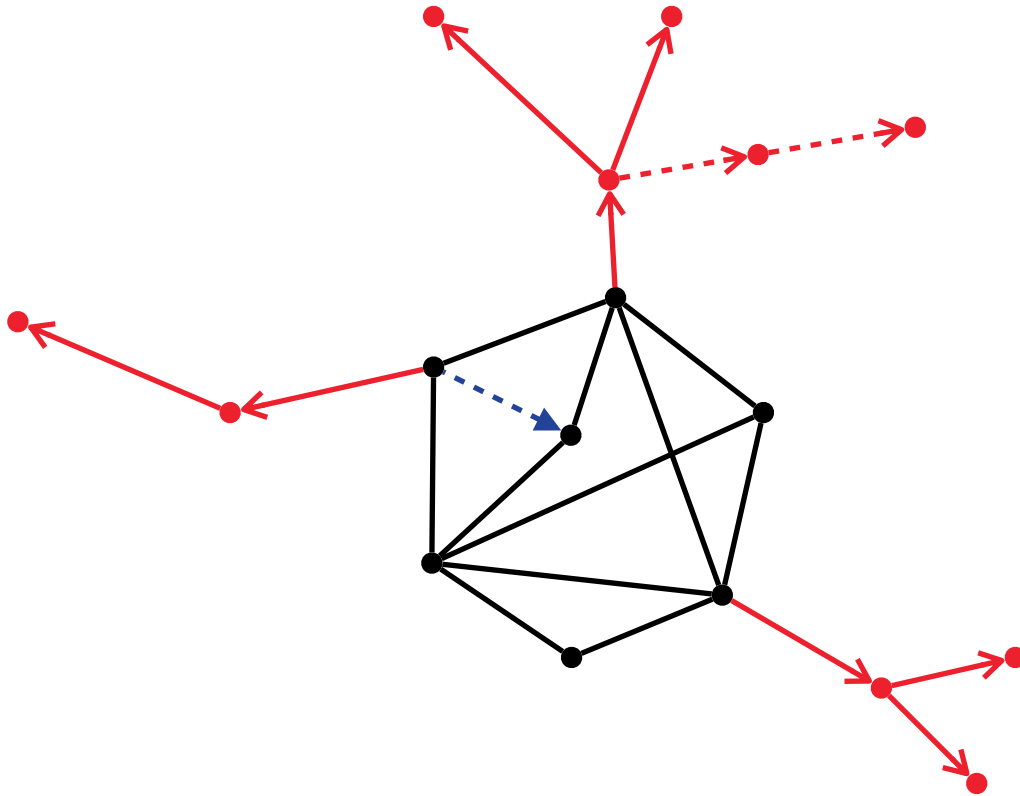
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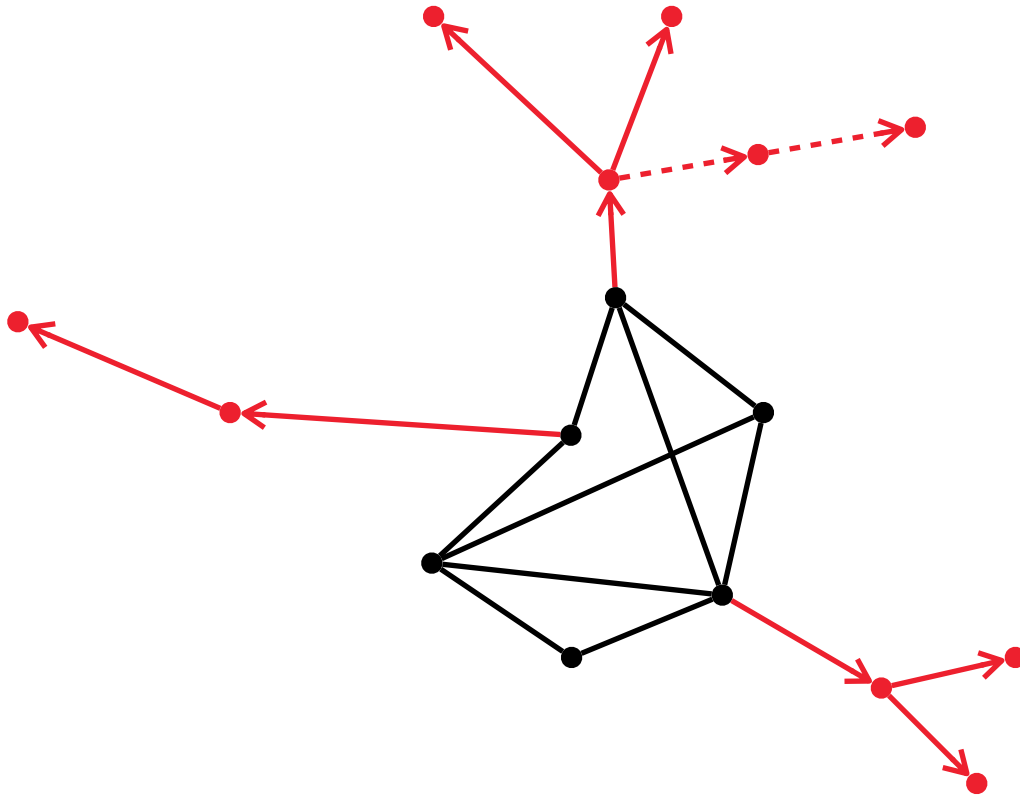
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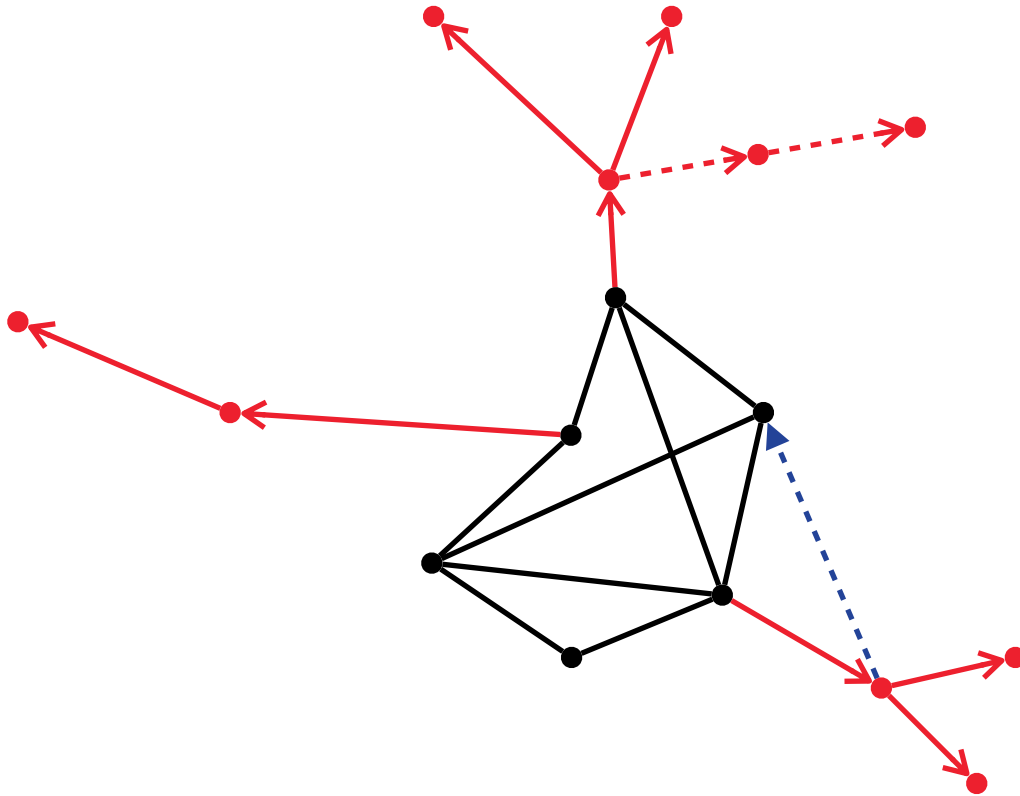
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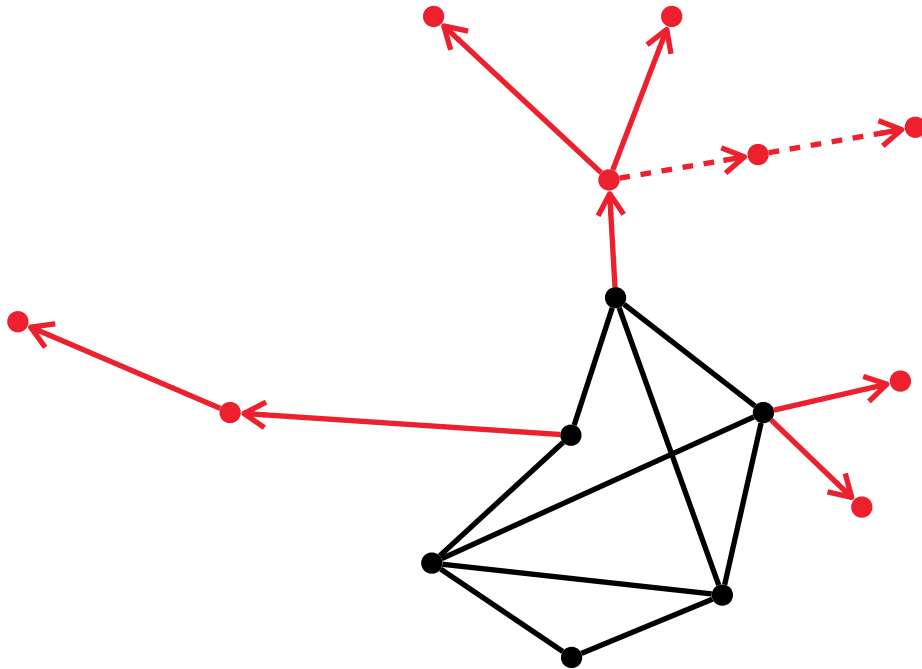
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Successor-to-predecessor

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Successor-to-predecessor

$$C \sqsubseteq \exists R . D$$

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$$\bullet a : C$$

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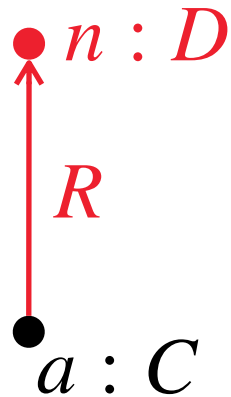
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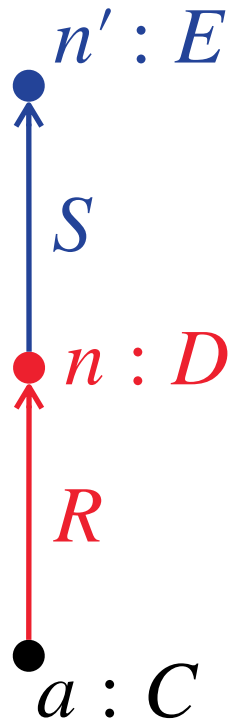
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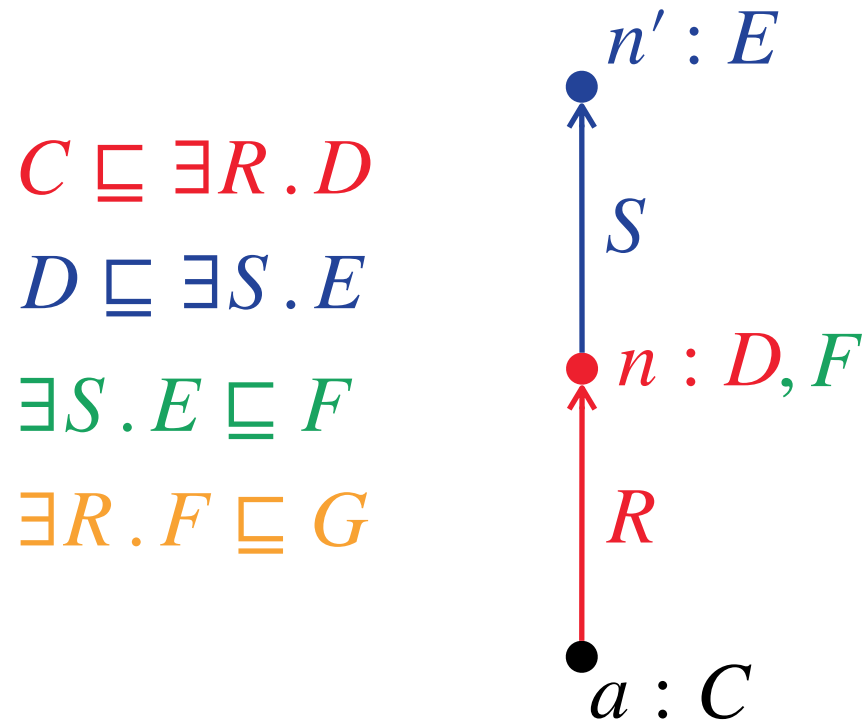
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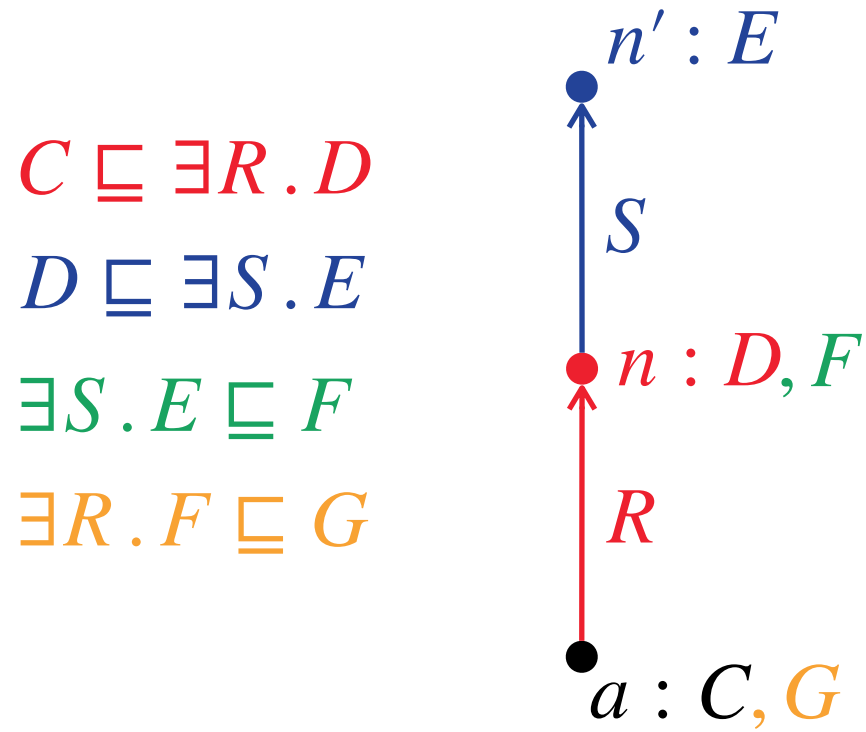
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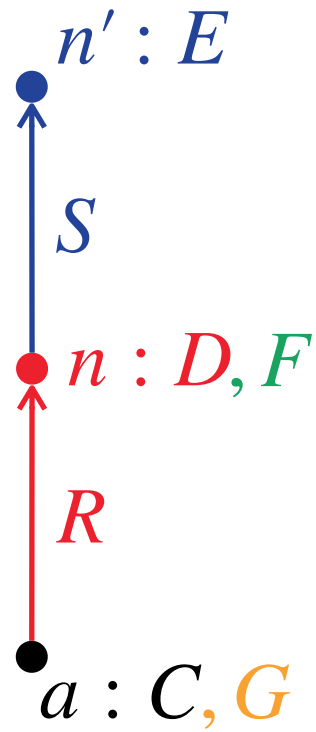
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Folding

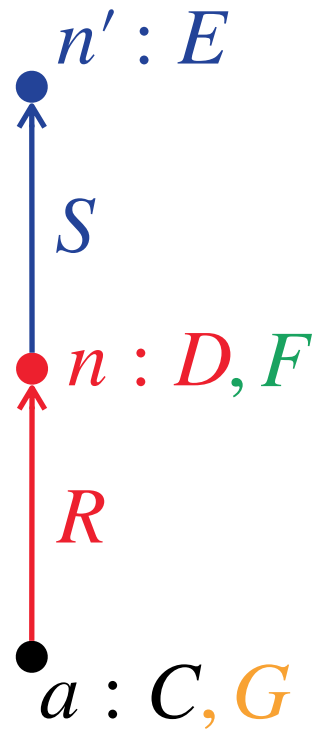
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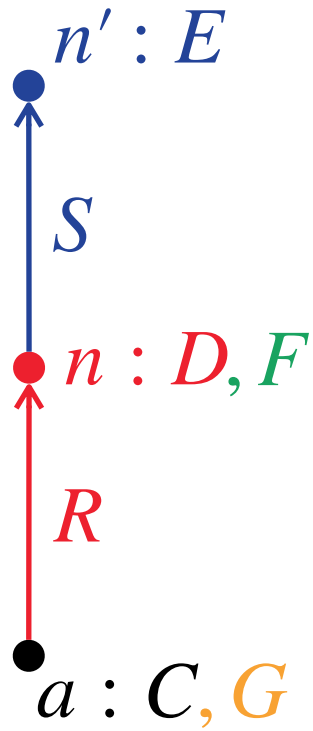
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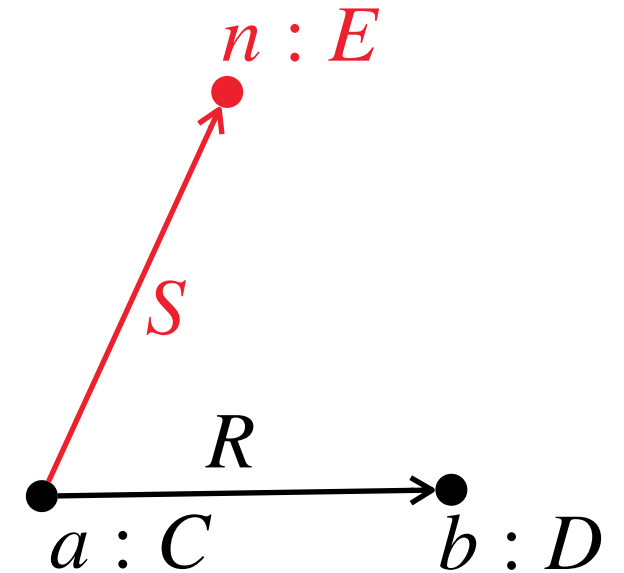


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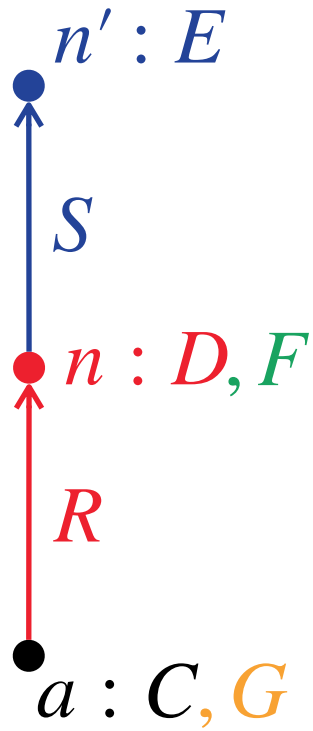
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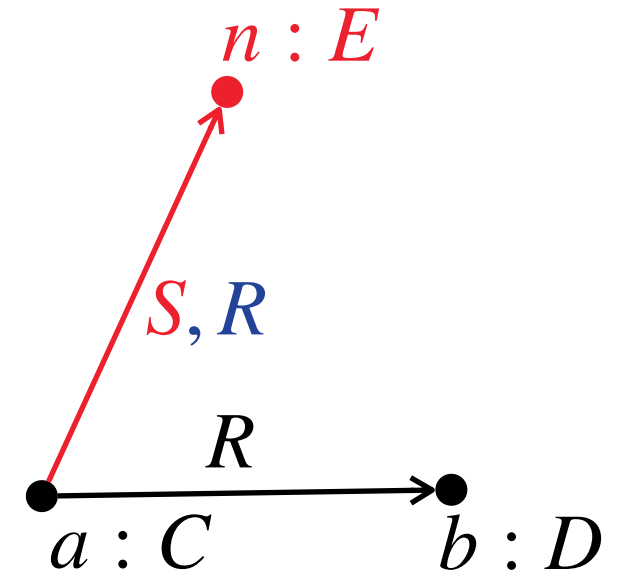


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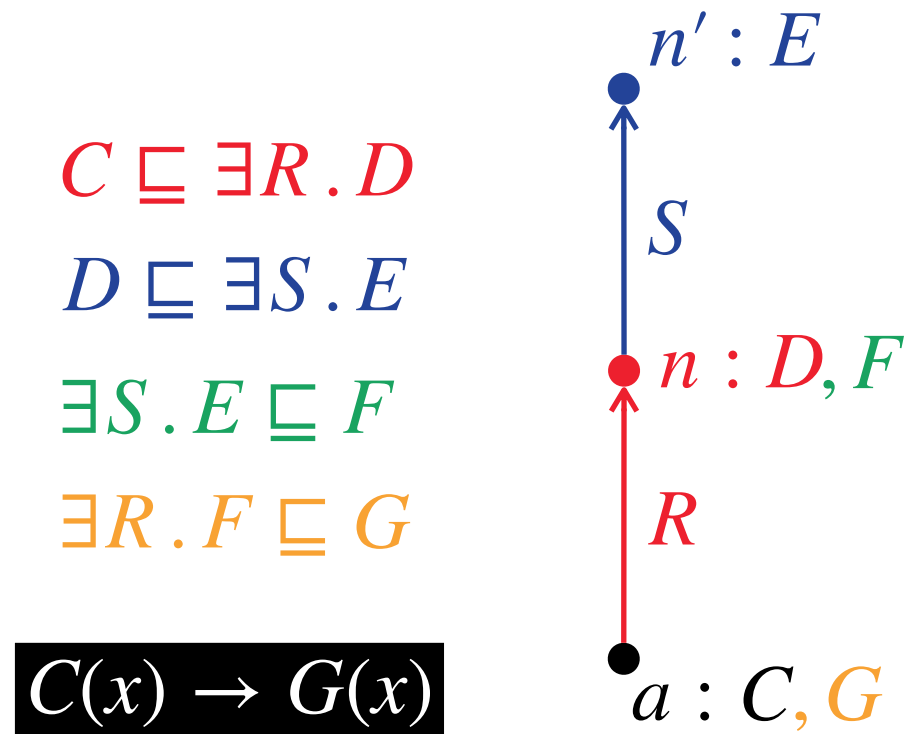
Folding



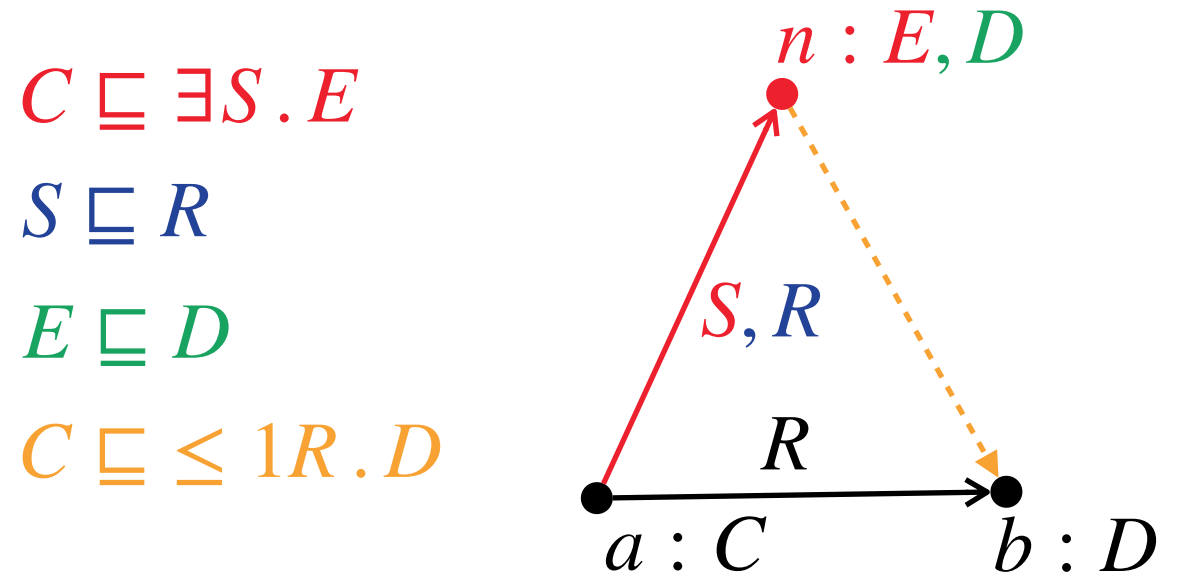


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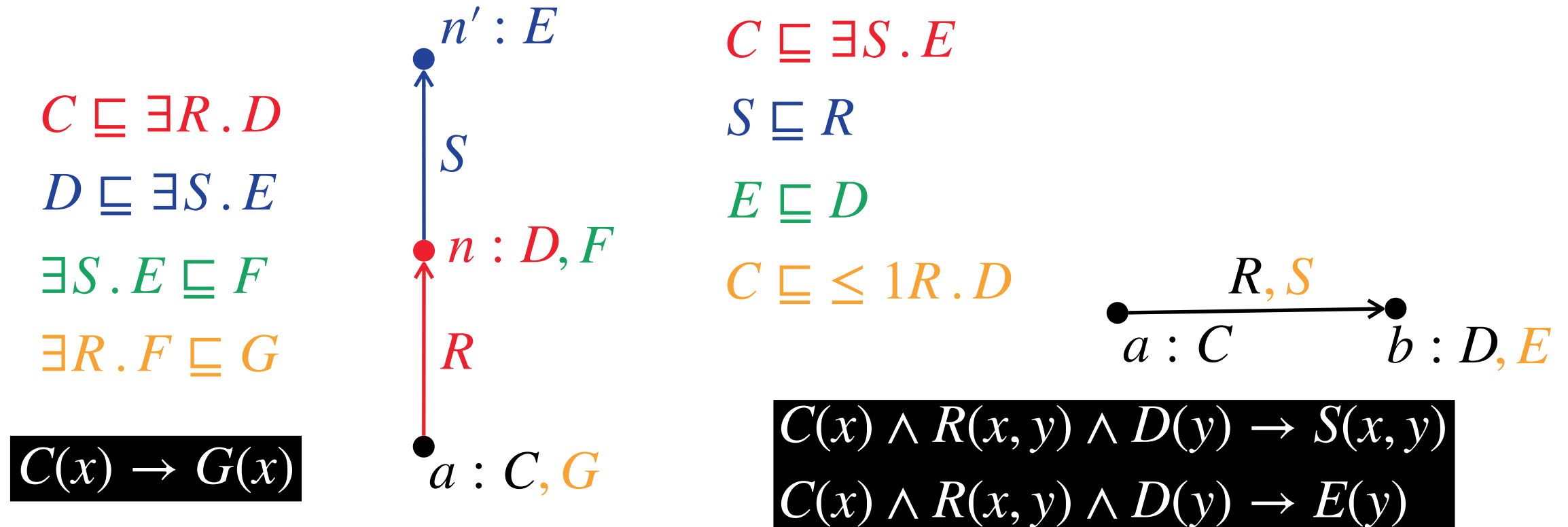
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# Computing IQ Rewritings for Horn-*ALCHIQ*

Consider some Horn-ALCHIQ TBox  $\mathcal{T}$ .

Then, the rule set  $\mathcal{R}_{\mathcal{T}}$  defined as follows is an IQ-preserving rewriting for  $\mathcal{T}$ .

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Consequence-based Reasoning Calculi  
[IJCAI 2017] Yevgeny  
[AAAI 2012] Eiter et al.

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For all  $C \sqsubseteq \leq 1R.D \in \mathcal{T}$ ,

$$C(x) \wedge R(x, y) \wedge D(y) \wedge R(x, z) \wedge D(z) \rightarrow y \approx z \in \mathcal{R}_{\mathcal{T}},$$

$$C(x) \wedge C_1(x) \wedge \dots \wedge C_n(x) \wedge R(x, y) \wedge D(y) \rightarrow E(y) \in \mathcal{R}_{\mathcal{T}} \text{ if } C_1 \sqcap \dots \sqcap C_n \sqsubseteq \exists R.(D \sqcap E) \in \Omega(\mathcal{T}), \text{ and}$$

$$C(x) \wedge C_1(x) \wedge \dots \wedge C_n(x) \wedge R(x, y) \wedge D(y) \rightarrow S(x, y) \in \mathcal{R}_{\mathcal{T}} \text{ if } C_1 \sqcap \dots \sqcap C_n \sqsubseteq \exists (R \sqcap S).D \in \Omega(\mathcal{T})$$

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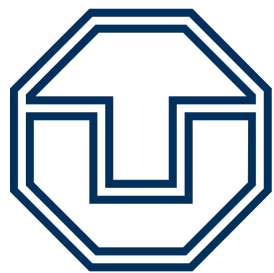
From Horn-*SRIQ* to Datalog

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Check out our poster!

From Horn-*SRIQ* to Datalog:  
A Data-Independent Transformation that  
Preserves Assertion Entailment

David Carral, Larry González, and Patrick Koopmann



**TECHNISCHE  
UNIVERSITÄT  
DRESDEN**

Poster: KRR5901