Exercise 8.1. Describe a polynomial-time ATM solving EXACT INDEPENDENT SET:

Input: Given a graph $G$ and some number $k$.
Question: Does there exists a maximal independent set in $G$ of size exactly $k$?

Find a level of the polynomial hierarchy where this problem is contained in.

Exercise 8.2. Consider the Japanese game go-moku that is played by two players X and O on a 19×19 board. Players alternately place markers on the board, and the first one to have five of its markers in a row, column, or diagonal wins.

Consider the generalised version of go-moku on an $n \times n$ board. Say that a position of go-moku is a placement of markers on such a board as it could occur during the game. Define

$$GM = \{ \langle B \rangle \mid \text{B is a position of go-moku where X has a winning strategy} \}.$$ 

Describe a polynomial-time ATM solving $GM$ and informally argue why this problem is not in any level of the polynomial hierarchy.

Exercise 8.3. Show that if $P = NP$, then $P = PH$.

Exercise 8.4. Show $NP^{SAT} \subseteq \Sigma_2P$.

Exercise 8.5. Show the following result: If there is any $k$ such that $\Sigma_k^P = \Sigma_{k+1}^P$ then $\Sigma_j^P = \Pi_j^P = \Sigma_k^P$ for all $j > k$, and therefore $PH = \Sigma_k^P$.

Exercise 8.6. Show that $PH \subseteq PSPACE$. 