

Science of Computational Logic

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Problem 1.1

In the lectures the following example from Description Logics was presented:

$$\begin{array}{ll}
 \mathcal{K}_T : & \text{woman} \sqsubseteq \text{person}, \\
 & \text{man} \sqsubseteq \text{person}, \\
 & \text{mother} = \text{woman} \sqcap \exists \text{child} : \text{person}, \\
 & \text{father} = \text{man} \sqcap \exists \text{child} : \text{person}, \\
 & \text{parent} = \text{mother} \sqcup \text{father}, \\
 & \text{grandparent} = \text{parent} \sqcap \exists \text{child} : \text{parent}, \\
 & \text{father_without_son} = \text{father} \sqcap \forall \text{child} : \neg \text{man} \\
 \mathcal{K}_A : & \text{parent}(\text{carl}), \text{parent}(\text{conny}), \\
 & \text{child}(\text{conny}, \text{joe}), \text{child}(\text{conny}, \text{carl}), \\
 & \text{man}(\text{joe}), \text{man}(\text{carl}), \text{woman}(\text{conny}).
 \end{array}$$

Are the following consequences valid? **Justify** your answers.

1. $\mathcal{K}_T \cup \mathcal{K}_A \models \text{grandparent}(\text{conny})$
2. $\mathcal{K}_T \cup \mathcal{K}_A \models \text{father}(\text{carl})$
3. $\mathcal{K}_T \cup \mathcal{K}_A \models \text{father_without_son}(\text{carl})$

Problem 1.2

Prove that $F \sqsubseteq G \equiv F \sqcap \neg G = \perp$

Problem 1.3

Show that the $\text{grandparent} \sqsubseteq_T \text{parent}$ by reducing subsumption into concept satisfiability, where T is the T-Box from the lectures.

Problem 1.4

Is the concept $(\text{father} \sqcap \text{mother})$ satisfiable w.r.t. T of the lectures?

Problem 1.5

1. Which generalized concept axioms must be added to prevent that a person is female and male?

2. Is there a single generalized concept axiom that prevents that a person is female and male?

Problem 1.6

Give an equivalent concept without the construct \sqcap and $\exists r.C$ for $(\text{woman} \sqcap \exists \text{child}.\text{person})$

Problem 1.7

Prove that $K \models (\forall r.C)(a)$ and $K \models r(a, b)$, then $K \models C(b)$