Problem 1.1

In the lectures the following example from Description Logics was presented:

\[ K_T : \]
\[ \text{woman} \sqsubseteq \text{person}, \]
\[ \text{man} \sqsubseteq \text{person}, \]
\[ \text{mother} = \text{woman} \sqcap \exists \text{child} : \text{person}, \]
\[ \text{father} = \text{man} \sqcap \exists \text{child} : \text{person}, \]
\[ \text{parent} = \text{mother} \sqcup \text{father}, \]
\[ \text{grandparent} = \text{parent} \sqcap \exists \text{child} : \text{parent}, \]
\[ \text{father\_without\_son} = \text{father} \sqcap \forall \text{child} : \neg \text{man} \]

\[ K_A : \]
\[ \text{parent}(\text{carl}), \text{parent}(\text{conny}), \]
\[ \text{child}(\text{conny}, \text{joe}), \text{child}(\text{conny}, \text{carl}), \]
\[ \text{man}(\text{joe}), \text{man}(\text{carl}), \text{woman}(\text{conny}). \]

Are the following consequences valid? \textbf{Justify} your answers.

1. \( K_T \cup K_A \models \text{grandparent}(\text{conny}) \)
2. \( K_T \cup K_A \models \text{father}(\text{carl}) \)
3. \( K_T \cup K_A \models \text{father\_without\_son}(\text{carl}) \)

Problem 1.2

Prove that \( F \sqsubseteq G \equiv F \sqcap \neg G = \bot \)

Problem 1.3

Show that the \( \text{grandparent} \sqsubseteq T \text{ parent} \) by reducing subsumption into concept satisfiability, where \( T \) is the T-Box from the lectures.

Problem 1.4

Is the concept \( (\text{father} \sqcap \text{mother}) \) satisfiable w.r.t. \( T \) of the lectures?

Problem 1.5

1. Which generalized concept axioms must be added to prevent that a person is female and male?
2. Is there a single generalized concept axiom that prevents that a person is female and male?

Problem 1.6
Give an equivalent concept without the construct \( \sqcap \) and \( \exists r.C \) for \((\text{woman} \sqcap \exists \text{child.person})\)

Problem 1.7
Prove that \( K \models (\forall r.C)(a) \) and \( K \models r(a,b) \), then \( K \models C(b) \)