Adding Standpoint Modalities to Non-Monotonic S4F: Preliminary Results

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NMR @ KR 2024, Hanoi Vietnam

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Motivation

Standpoint Logic

$coffee \rightarrow hot$

$coffee \rightarrow cold$

$\neg(cold \wedge hot)$

Motivation

Standpoint Logic

$\square$$\left[\mathit{coffee} \to \mathit{hot}\right]$

$\square$$\left[\mathit{coffee} \to \mathit{cold}\right]$

$\neg(\mathit{cold} \wedge \mathit{hot})$

Motivation

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$\Box$ $\left[\mathit{coffee} \rightarrow \mathit{hot}\right]$

$\Box$ $\left[\mathit{coffee} \rightarrow \mathit{cold}\right]$

$\neg(\mathit{cold} \wedge \mathit{hot})$

Non-Monotonic Reasoning

coffee

$\mathit{coffee} : \mathit{hot} / \mathit{hot}$

Motivation

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$\Box$ $\left[coffee \rightarrow hot\right]$

$\Box$ $\left[coffee \rightarrow cold\right]$

$\neg(cold \wedge hot)$

Non-Monotonic Reasoning

$$\left.\begin{array}{l} coffee \\ coffee : hot / hot \end{array}\right\} \models hot$$

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$\square$$\big[coffee \rightarrow hot \big]$

$\square$$\big[coffee \rightarrow cold \big]$

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Non-Monotonic Reasoning

$$\left.\begin{array}{l} coffee \\ coffee : hot / hot \\ \neg hot \end{array}\right\} \not\models hot$$

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$\Box$ $[coffee \to hot]$

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Non-Monotonic Reasoning

$$\left.\begin{array}{l} coffee \\ coffee : hot / hot \\ \neg hot \end{array}\right\} \not\models hot$$

Non-Monotonic Standpoint Logic

$coffee : hot / hot$

$coffee : cold / cold$

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$\Box$$\left[coffee \to hot \right]$

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$\Box$$\left[coffee : hot / hot \right]$

$\Box$$\left[coffee : cold / cold \right]$

Standpoint Logic [GR21]

$\Box$$\left[\underline{\mathbf{c}}\mathit{offee} \rightarrow \underline{\mathbf{h}}\mathit{ot}\right]$ "*Unequivocally [from*  *perspective]* $\underline{\mathbf{c}}$*offee* $\rightarrow$ $\underline{\mathbf{h}}$*ot*"

$\Box$$\left[\underline{\mathbf{c}}\mathit{offee} \rightarrow \neg\underline{\mathbf{h}}\mathit{ot}\right]$ "*Unequivocally [from*  *perspective]* $\underline{\mathbf{c}}$*offee* $\rightarrow \neg$$\underline{\mathbf{h}}$*ot*"

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$\Box$ $[\underline{\mathbf{c}}\mathit{offee} \to \underline{\mathbf{h}}\mathit{ot}]$ *"Unequivocally [from*  *perspective]* $\underline{\mathbf{c}}$*offee* $\to$ $\underline{\mathbf{h}}$*ot"*

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- Set of **standpoint names** $\mathcal{S}$.

Standpoint Logic [GR21]

$\square$$[\underline{\mathbf{c}}offee \to \underline{\mathbf{h}}ot]$ *"Unequivocally [from*  *perspective]* $\underline{\mathbf{c}}offee \to \underline{\mathbf{h}}ot$*"*

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- Set of **standpoint names** $\mathcal{S}$.
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  - Π – **precisifications**,
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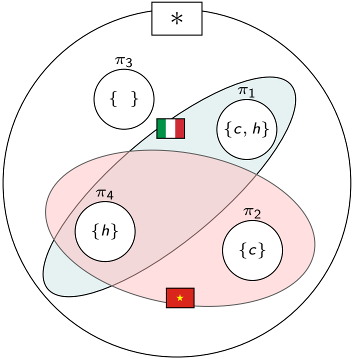

Standpoint Logic [GR21]

$\Box_{🇮🇹}[\underline{c}offee \to \underline{h}ot]$ *“Unequivocally [from 🇮🇹 perspective] $\underline{c}$offee → $\underline{h}$ot”*

$\Box_{🇻🇳}[\underline{c}offee \to \neg \underline{h}ot]$ *“Unequivocally [from 🇻🇳 perspective] $\underline{c}$offee → ¬$\underline{h}$ot”*

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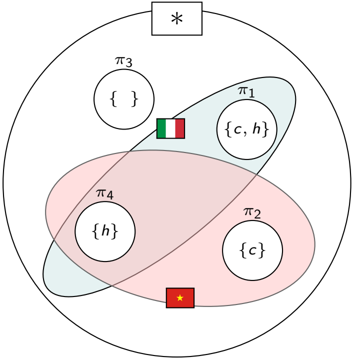
$$\mathcal{N}, \pi \Vdash \Box_{\mathsf{s}}\varphi :\Longleftrightarrow \mathcal{N}, \pi' \Vdash \varphi \text{ for all } \pi' \in \sigma(\mathsf{s})$$



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$\square_{🇮🇹}[\underline{c}offee \rightarrow \underline{h}ot]$ *"Unequivocally [from 🇮🇹 perspective] coffee → hot"*

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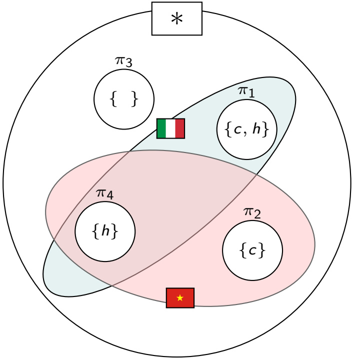

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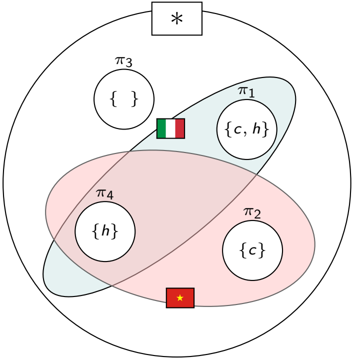

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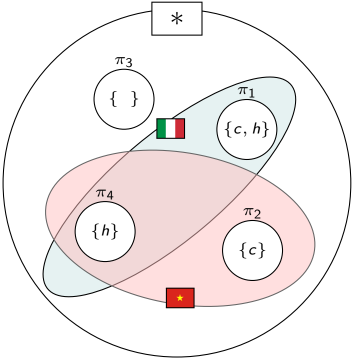

Objectives

Standpoint Propositional Logic [GR21]

$\Box$$\left[coffee \to hot\right]$ $\Box$$\left[coffee \to cold\right]$

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Standpoint Description Logics [GRS22, GRS23b, GRS23a, ÁR24]

$\Box$$\left[\mathit{Coffee} \sqsubseteq \mathit{HotDrink}\right]$ $\Box$$\left[\mathit{Coffee} \sqsubseteq \mathit{ColdDrink}\right]$

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$\Box$$[Coffee \sqsubseteq HotDrink]$ $\Box$$[Coffee \sqsubseteq ColdDrink]$

Standpoint LTL [GGL23]

$\Box$$[coffee \to hot \wedge hot\,\mathbf{U}\,cold]$ $\Box$$[coffee \to \mathbf{G}\,cold]$

Objectives

Standpoint Propositional Logic [GR21]

$\square$🇮🇹 $\left[\mathit{coffee} \rightarrow \mathit{hot}\right]$ $\square$🇻🇳 $\left[\mathit{coffee} \rightarrow \mathit{cold}\right]$

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Standpoint NMR:

Objectives

Standpoint Propositional Logic [GR21]

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Standpoint NMR:

Choice of the underlying NMR formalism?
$\rightsquigarrow$ ideally as general as possible

Objectives

Standpoint Propositional Logic [GR21]

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Standpoint NMR:

Choice of the underlying NMR formalism?
$\rightsquigarrow$ ideally as general as possible

Complexity of reasoning?
$\rightsquigarrow$ ideally not harder than the underlying NMR formalism

Non-Monotonic Reasoning

Deafault Logic [Rei80]

$coffee : hot / hot$

Non-Monotonic Reasoning

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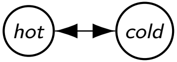
Argumentation [Dun95]



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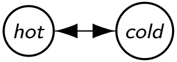

ASP [GL91]

$hot \leftarrow coffee, \sim cold$

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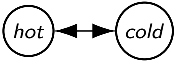

ASP [GL91]

$hot \leftarrow coffee, \sim cold$

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$$coffee : hot / hot$$
$$(\mathbf{K}coffee \land \mathbf{K}\neg\mathbf{K}\neg hot) \rightarrow \mathbf{K}hot$$

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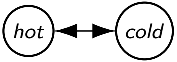

Non-Monotonic S4F

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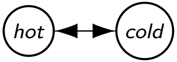

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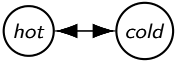
$$coffee : hot / hot$$
$$(\mathbf{K}coffee \land \mathbf{K}\neg\mathbf{K}\neg hot) \to \mathbf{K}hot$$

Argumentation [Dun95]



$$\mathbf{K}\neg\mathbf{K}\neg hot \to \mathbf{K}hot \qquad \mathbf{K}cold \to \mathbf{K}\neg hot$$
$$\mathbf{K}\neg\mathbf{K}\neg cold \to \mathbf{K}hot \qquad \mathbf{K}hot \to \mathbf{K}\neg cold$$

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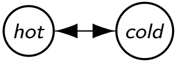

$\mathbf{K} \neg \mathbf{K} \neg hot \rightarrow \mathbf{K} hot$ $\quad$ $\mathbf{K} cold \rightarrow \mathbf{K} \neg hot$

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Modal Logic S4F [Seg71, ST94, Tru07]

$\mathbf{K}(\underline{\mathbf{c}}\mathit{offee} \to \underline{\mathbf{h}}\mathit{ot})$

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Semantics: $\mathcal{M} = (V, W, \xi)$:

- V, W – **worlds**,
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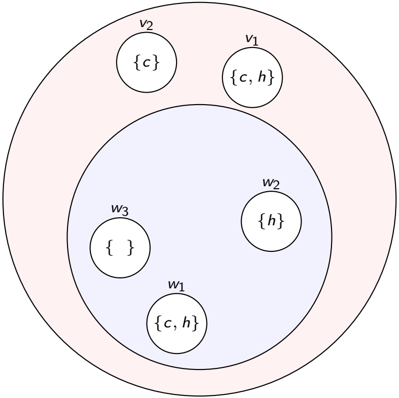
- $\xi : V \cup W \mapsto 2^{\mathcal{A}}$.



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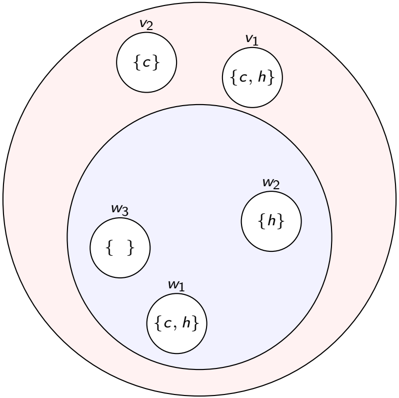

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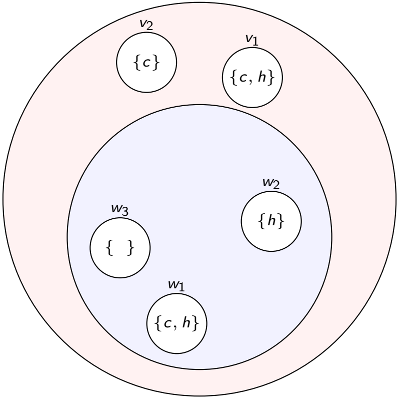
$$\mathcal{M}, w \Vdash \mathbf{K}\varphi :\Longleftrightarrow \begin{cases} \mathcal{M}, v \Vdash \varphi \text{ for all } v \in V \cup W & \text{if } w \in V, \\ \mathcal{M}, v \Vdash \varphi \text{ for all } v \in W & \text{otherwise} \end{cases}$$

Modal Logic S4F [Seg71, ST94, Tru07]

$\mathcal{M}, w_1 \Vdash \mathbf{K}(\underline{c}offee \rightarrow \underline{\mathbf{h}}ot)$



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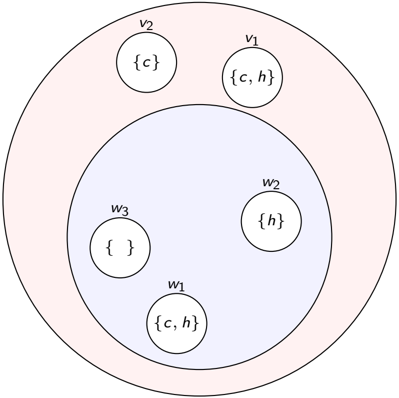

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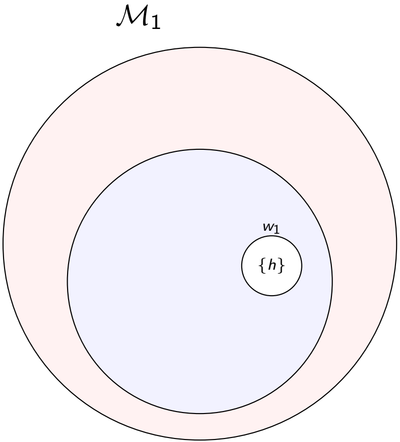
Minimal S4F models [ST94]

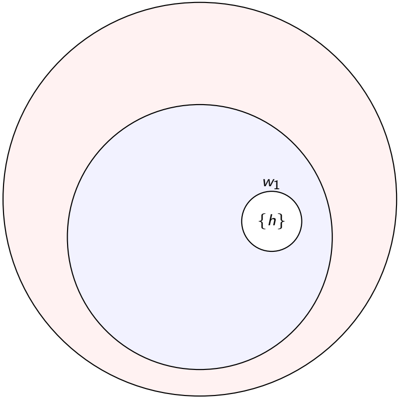


$$T = \mathbf{K}(\underline{\mathbf{h}}ot \vee \underline{\mathbf{co}}ld)$$

$$\mathcal{M}_1 \Vdash T$$

Minimal S4F models [ST94]

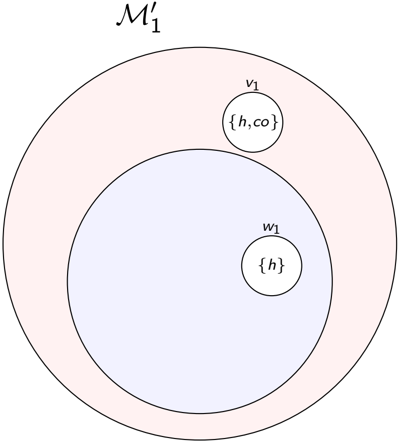
$\mathcal{M}_1$



$T = \mathbf{K}(\underline{\mathbf{h}}ot \vee \underline{\mathbf{co}}ld)$

$\mathcal{M}_1 \Vdash T \quad \mathcal{M}_1 \Vdash \neg\underline{\mathbf{co}}ld$

Minimal S4F models [ST94]



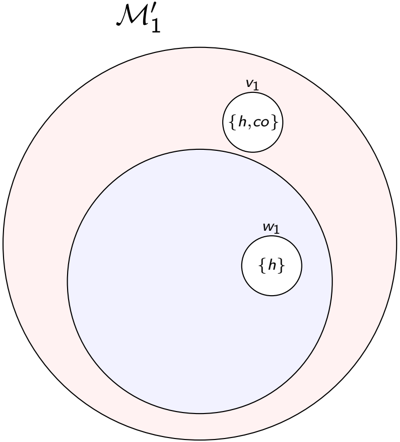
$T = \mathbf{K}(\underline{\mathbf{h}}ot \lor \underline{\mathbf{co}}ld)$

$\mathcal{M}_1' \lhd \mathcal{M}_1$

$\mathcal{M}_1 \Vdash T \quad \mathcal{M}_1 \Vdash \neg\underline{\mathbf{co}}ld$

$\mathcal{M}_1' \nVdash \neg\underline{\mathbf{co}}ld$

Minimal S4F models [ST94]



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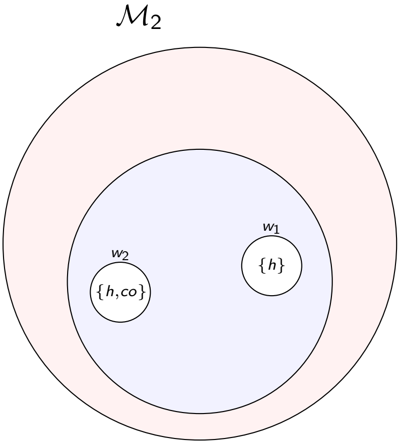


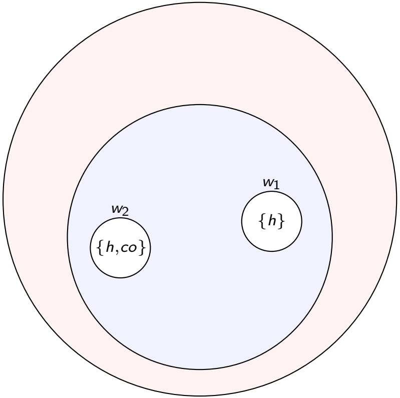
$T = \mathbf{K}(\underline{\mathbf{h}}ot \vee \underline{\mathbf{co}}ld)$

$\mathcal{M}_1 \Vdash T \quad \mathcal{M}_1 \Vdash \neg\underline{\mathbf{co}}ld$
$\mathcal{M}_2 \Vdash T$

$\mathcal{M}_1' \Vdash T \quad \mathcal{M}_1' \nVdash \neg\underline{\mathbf{co}}ld$

Minimal S4F models [ST94]

$\mathcal{M}_2$

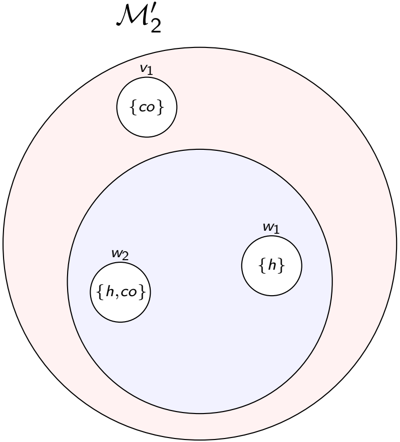


$$T = \mathbf{K}(\underline{\mathbf{h}}ot \lor \underline{\mathbf{co}}ld)$$

$$\mathcal{M}_1 \Vdash T \quad \mathcal{M}_1 \Vdash \neg\underline{\mathbf{co}}ld$$
$$\mathcal{M}_2 \Vdash T \quad \mathcal{M}_2 \Vdash \underline{\mathbf{h}}ot$$

$$\mathcal{M}_1' \Vdash T \quad \mathcal{M}_1' \nVdash \neg\underline{\mathbf{co}}ld$$

Minimal S4F models [ST94]



$T = \mathbf{K}(\underline{\mathbf{h}}ot \lor \underline{\mathbf{co}}ld)$

$\mathcal{M}_2' \lhd \mathcal{M}_2$

$\mathcal{M}_1 \Vdash T \quad \mathcal{M}_1 \Vdash \neg\underline{\mathbf{co}}ld$

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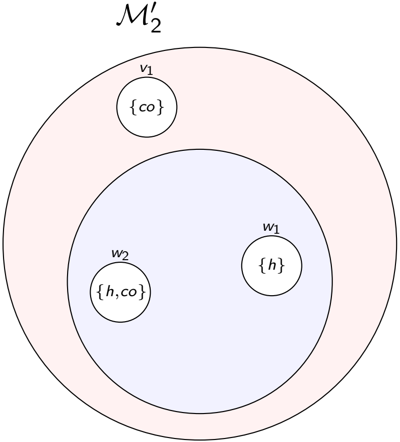

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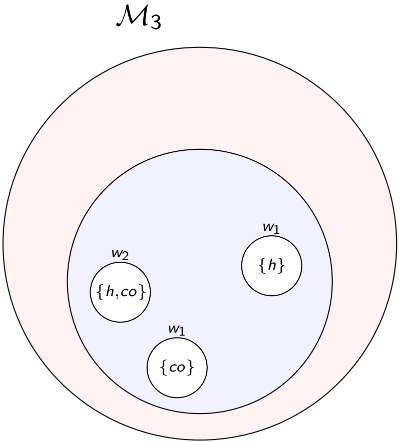

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$$\mathcal{M}_2 \Vdash T \quad \mathcal{M}_2 \Vdash \underline{\mathbf{h}}ot$$
$$\mathcal{M}_3 \Vdash T$$

$$\mathcal{M}'_1 \Vdash T \quad \mathcal{M}'_1 \nVdash \neg\underline{\mathbf{co}}ld$$
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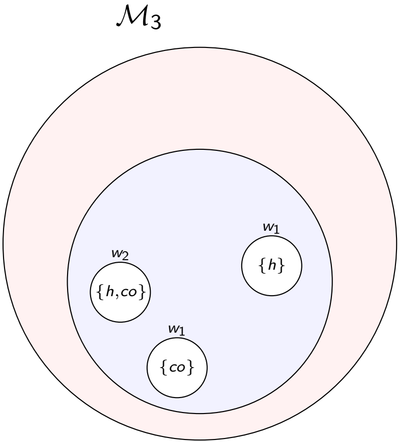

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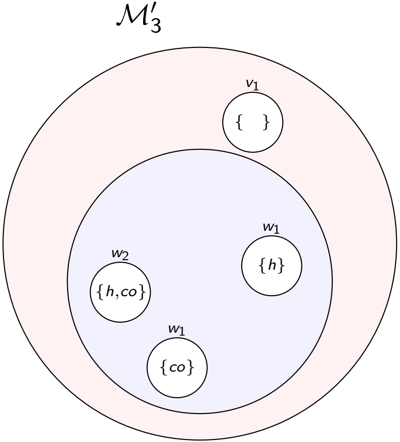

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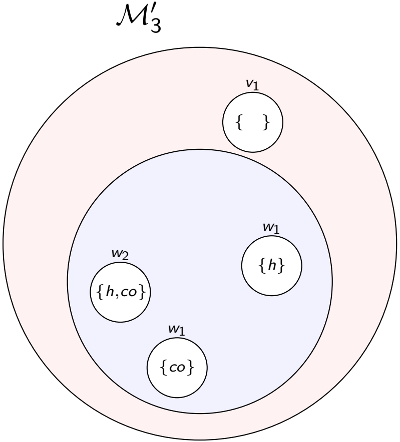
Minimal S4F models [ST94]



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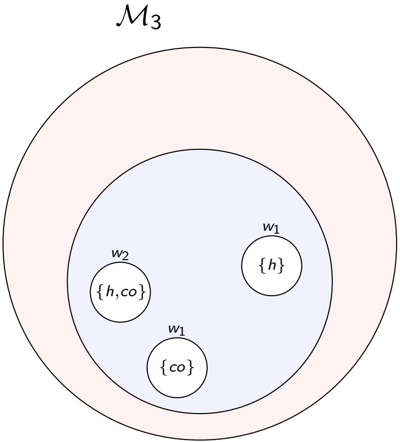
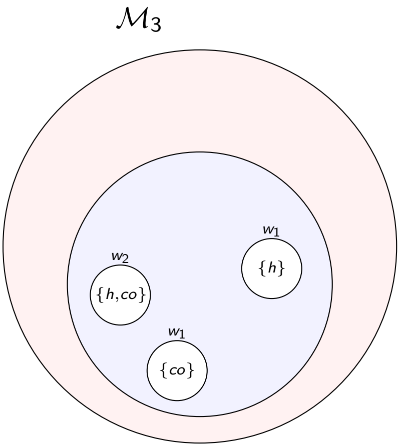
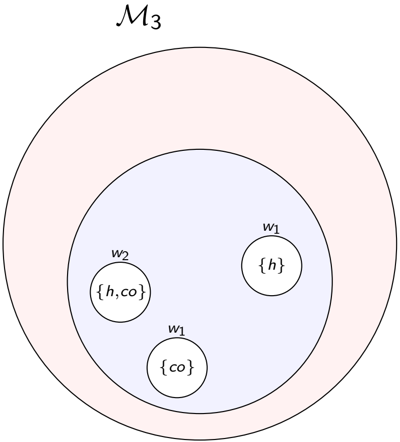


Minimal S4F models [ST94]



$\mathcal{M}$ is a **minimal S4F model** of T if:
- $\mathcal{M} \Vdash T$,
- for every $\mathcal{M}'$ such that $\mathcal{M}' \lhd \mathcal{M}$, $\mathcal{M}' \nVdash T$

Minimal S4F models [ST94]



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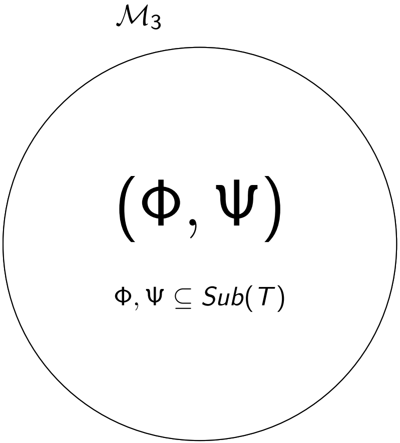
EXISTENCE$_{\mathsf{S4F}}$
- does there exist a minimal S4F model of T?

Minimal S4F models [ST94]



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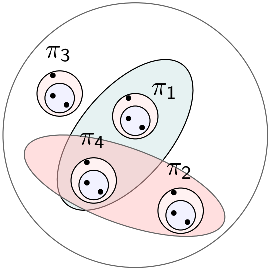
EXISTENCE$_{\mathsf{S4F}}$
- does there exist a minimal S4F model of T?

EXISTENCE$_{\mathsf{S4F}}$ is Σ_2^{P}-complete [ST93]
- small (polynomial w.r.t. $\|T\|$), syntactic representation of a minimal S4F model is used.

Standpoint S4F ($\mathbb{S}$S4F)

Semantics: $\mathfrak{S} = (\Pi, \Omega, \sigma, \zeta, \gamma)$:



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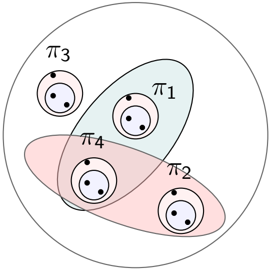

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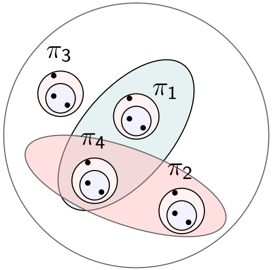

Standpoint S4F ($\mathbb{S}$S4F)

Semantics: $\mathfrak{S} = (\Pi, \Omega, \sigma, \zeta, \gamma)$:

- Π – **precisifications**,
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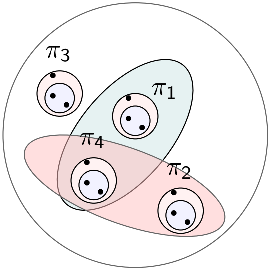
- $\sigma : \mathcal{S} \mapsto 2^{\Pi}$,



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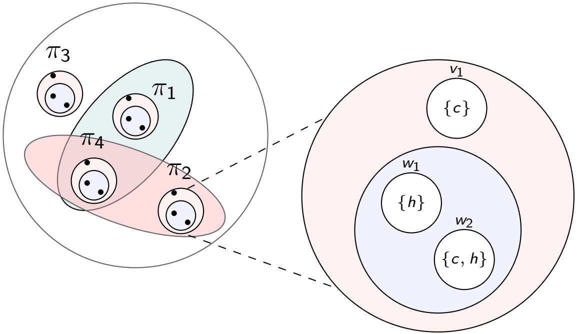

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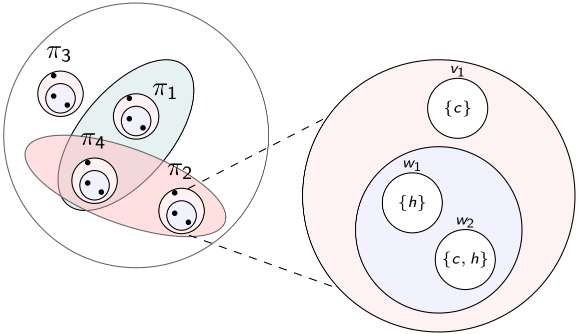
- $\gamma : \Omega \mapsto 2^{\mathcal{A}}$



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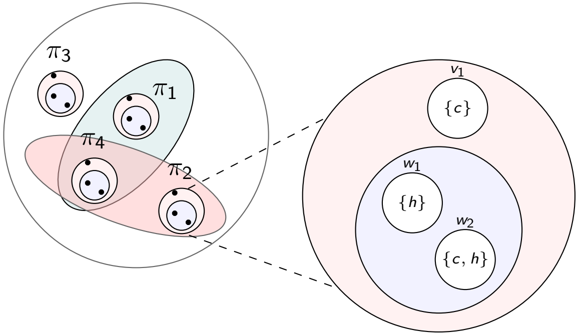

$$\mathfrak{S}, \pi, w \Vdash \mathbf{K}\varphi \quad :\Longleftrightarrow \begin{cases} \mathfrak{S}, \pi, w' \Vdash \varphi \text{ for all } w' \in \zeta_o(\pi) \cup \zeta_i(\pi) & \text{if } w \in \zeta_o(\pi) \\ \mathfrak{S}, \pi, w' \Vdash \varphi \text{ for all } w' \in \zeta_i(\pi) & \text{if } w \in \zeta_i(\pi) \end{cases}$$

$$\mathfrak{S}, \pi, w \Vdash \Box_s\varphi \quad :\Longleftrightarrow \mathfrak{S}, \pi', w' \Vdash \varphi \text{ for all } \pi' \in \sigma(s) \text{ and } w' \in \zeta_o(\pi') \cup \zeta_i(\pi')$$

Standpoint S4F ($\mathbb{S}$S4F)

Semantics: $\mathfrak{S} = (\Pi, \Omega, \sigma, \zeta, \gamma)$:

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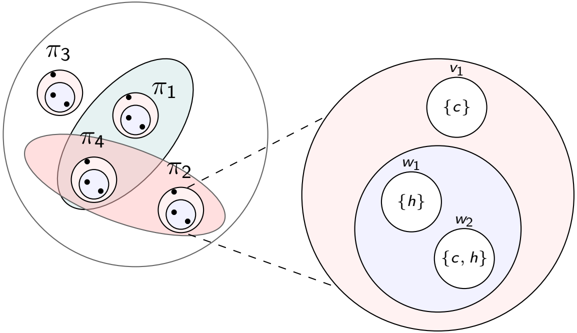

$\mathfrak{S}$ is a **minimal $\mathbb{S}$S4F model** of $T \subseteq \mathbb{S}$S4F if:

- $\mathfrak{S} \Vdash T$,
- for every $\pi \in \Pi$ there is an S4F theory $\Xi_\pi \subseteq Sub(T) \cap \mathcal{L}_{\mathbf{K}}$ such that $\left(\zeta_o(\pi), \zeta_i(\pi), \gamma|_{\zeta(\pi)}\right)$ is a minimal S4F model of Ξ_π.

Restrictions on the $\mathbb{S}$S4F Syntax

A **modal default** [Tru07] $\mathcal{L}_{\mathbf{K}}$:

$$\psi ::= \mathbf{K}\alpha \mid \neg\psi \mid \psi \wedge \psi \mid \mathbf{K}\psi$$

with $\alpha \in \mathcal{L}$ (propositional formula.)

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$\mathcal{L}_{\mathbb{S}\mathbf{K}}$ built via:

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An $\mathbb{S}$S4F theory T is **simple** if:

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Default Logic

$(\mathbf{K}\textit{coffee} \wedge \mathbf{K}\neg\mathbf{K}\neg\textit{hot}) \rightarrow \mathbf{K}\textit{hot}$

Argumentation

$\mathbf{K}\neg\mathbf{K}\neg\textit{hot} \rightarrow \mathbf{K}\textit{hot}$

$\mathbf{K}\textit{hot} \rightarrow \mathbf{K}\neg\textit{cold}$

ASP

$(\mathbf{K}\textit{coffee} \wedge \mathbf{K}\neg\mathbf{K}\neg\textit{cold}) \rightarrow \mathbf{K}\textit{hot}$

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Argumentation

$$\mathbf{K}\neg\mathbf{K}\neg\mathit{hot} \rightarrow \mathbf{K}\mathit{hot}$$
$$\mathbf{K}\mathit{hot} \rightarrow \mathbf{K}\neg\mathit{cold}$$

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$(\mathbf{K}\mathit{coffee} \wedge \mathbf{K}\neg\mathbf{K}\neg\mathit{hot}) \rightarrow \mathbf{K}\mathit{hot}$
$\in \mathcal{L}_{\mathbf{K}}$

Argumentation

$\mathbf{K}\neg\mathbf{K}\neg\mathit{hot} \rightarrow \mathbf{K}\mathit{hot} \quad \in \mathcal{L}_{\mathbf{K}}$
$\mathbf{K}\mathit{hot} \rightarrow \mathbf{K}\neg\mathit{cold} \quad \in \mathcal{L}_{\mathbf{K}}$

ASP

$(\mathbf{K}\mathit{coffee} \wedge \mathbf{K}\neg\mathbf{K}\neg\mathit{cold}) \rightarrow \mathbf{K}\mathit{hot}$
$\in \mathcal{L}_{\mathbf{K}}$

Restrictions on the $\mathbb{S}$S4F Syntax

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$$\varphi ::= \psi \mid \neg\varphi \mid \varphi \wedge \varphi \mid \square_{\mathsf{s}}\varphi$$

where $\psi \in \mathcal{L}_{\mathbf{K}}$.

An $\mathbb{S}$S4F theory T is **simple** if:

- every $\varphi \in T$ is of the form $\varphi = \square_{\mathsf{s}}\psi$

with $\psi \in \mathcal{L}_{\mathbf{K}}$.

Standpoint Default Logic

$\square$🇮🇹$\big[(\mathbf{K}\mathit{coffee} \wedge \mathbf{K}\neg\mathbf{K}\neg\mathit{hot}) \rightarrow \mathbf{K}\mathit{hot}\big] \in \mathcal{L}_{\mathbb{S}\mathbf{K}}$

Standpoint Argumentation

$\square$🇮🇹$\big[\mathbf{K}\neg\mathbf{K}\neg\mathit{hot} \rightarrow \mathbf{K}\mathit{hot}\big] \in \mathcal{L}_{\mathbb{S}\mathbf{K}}$

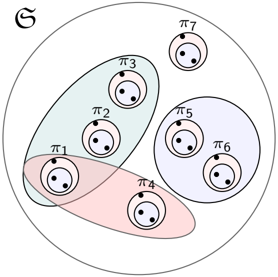
$\square$🇮🇹$\big[\mathbf{K}\mathit{hot} \rightarrow \mathbf{K}\neg\mathit{cold}\big] \in \mathcal{L}_{\mathbb{S}\mathbf{K}}$

Standpoint ASP

$\square$🇮🇹$\big[(\mathbf{K}\mathit{coffee} \wedge \mathbf{K}\neg\mathbf{K}\neg\mathit{cold}) \rightarrow \mathbf{K}\mathit{hot}\big] \in \mathcal{L}_{\mathbb{S}\mathbf{K}}$

Small Model Property of $\mathbb{S}$S4F

$T \subseteq \mathcal{L}_{\mathbb{S}\mathsf{K}}$, $\mathfrak{S} = (\Pi, \Omega, \sigma, \zeta, \gamma)$, $\mathfrak{S} \Vdash T$.



Small Model Property of $\mathbb{S}$S4F

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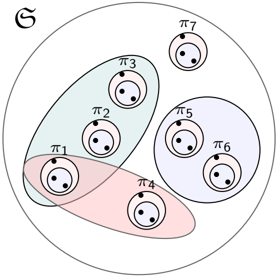
If $\neg\Box_{\mathsf{s}}\varphi \in Sub(T)$ and $\mathfrak{S} \Vdash \neg\Box_{\mathsf{s}}\varphi$
there is $\pi \in \sigma(\mathsf{s})$ with $\mathfrak{S}, \pi \nVdash \varphi$



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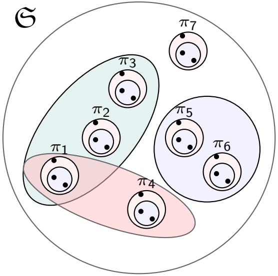
- $\Pi' = \{ \qquad \}$



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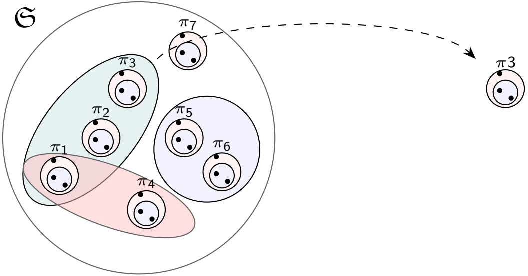

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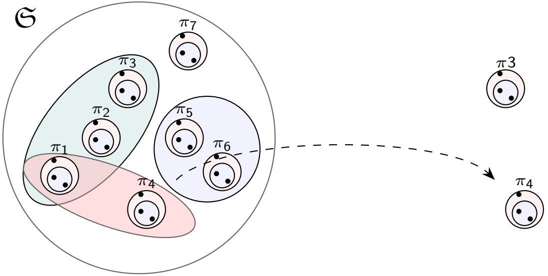
- $\Pi' = \{\pi_3,\ \pi_4 \qquad \}$



Small Model Property of $\mathbb{S}$S4F

$T \subseteq \mathcal{L}_{\mathbb{S}\mathsf{K}}$, $\mathfrak{S} = (\Pi, \Omega, \sigma, \zeta, \gamma)$, $\mathfrak{S} \Vdash T$.

If $\neg\Box_{\mathsf{s}}\varphi \in Sub(T)$ and $\mathfrak{S} \Vdash \neg\Box_{\mathsf{s}}\varphi$
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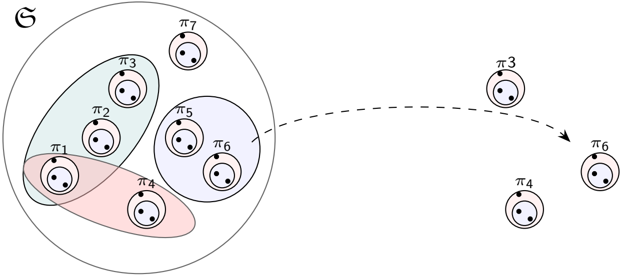

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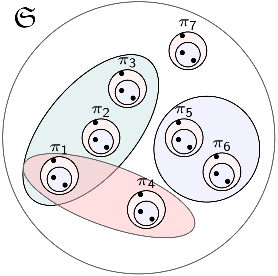

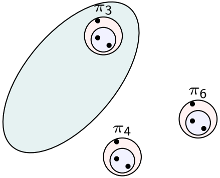

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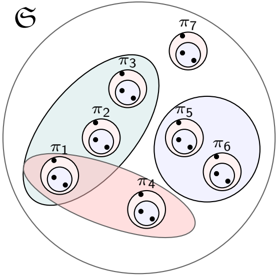

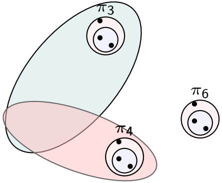

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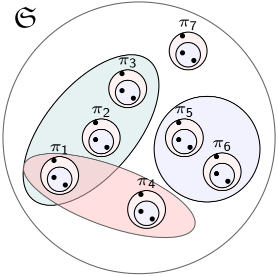

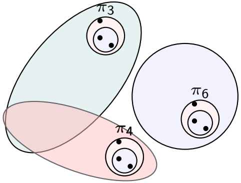

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Construct $\mathfrak{S}' = (\Pi', \Omega', \sigma', \zeta', \gamma')$:

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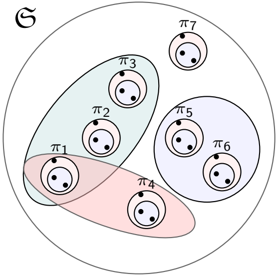

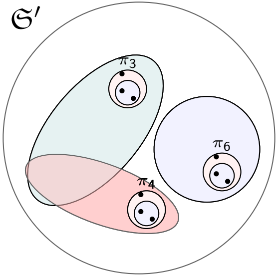

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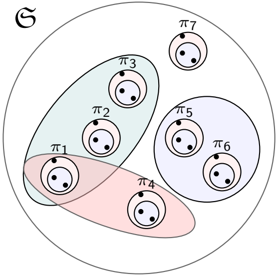
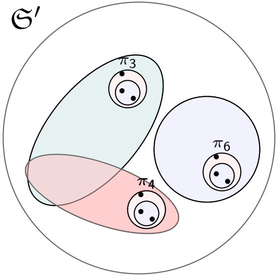
$|\Pi'| \leq poly(\|T\|)$ and $\mathfrak{S}' \Vdash T$





Complexity of $\mathbb{S}$S4F Reasoning

$T \subseteq \mathcal{L}_{\mathbb{S}\mathbf{K}}$.

Construct

$\mathcal{P} = (\qquad)$

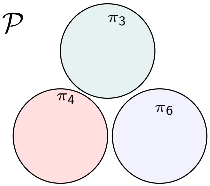
Complexity of $\mathbb{S}$S4F Reasoning

$T \subseteq \mathcal{L}_{\mathbb{S}\mathbf{K}}$.

1. create π_φ with $\pi_\varphi \in \sigma(\mathsf{s})$ for every $\neg\Box_{\mathsf{s}}\varphi \in Sub(T)$. (i.e. construct Π and σ.)

Construct
$\mathcal{P} = (\Pi, \sigma \quad)$

$\mathcal{P}$



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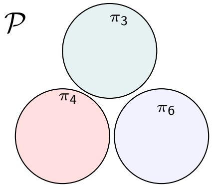

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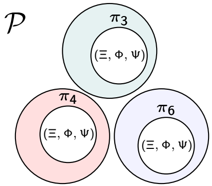
Construct
$\mathcal{P} = \left(\Pi, \sigma\, (\Xi, \Phi, \Psi)_{\pi \in \Pi}\right)$



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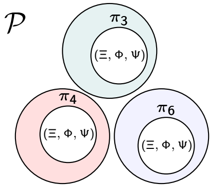

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Construct

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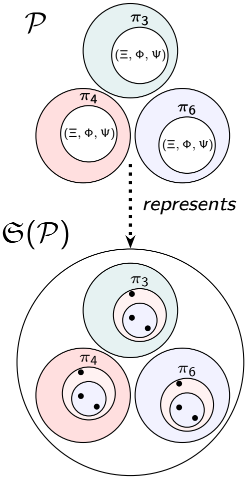

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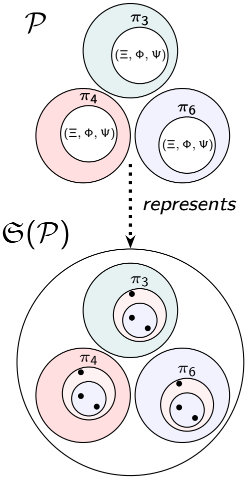

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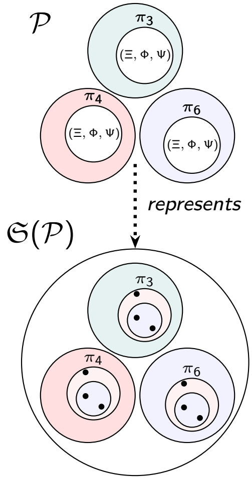

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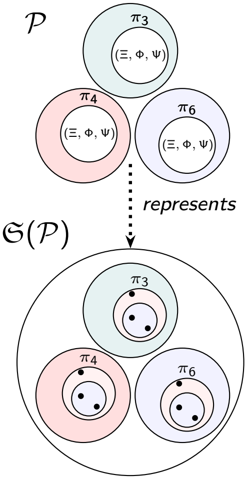

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EXISTENCE$_{\mathbb{S}\mathsf{S4F}}$ is in Σ_2^{P}.

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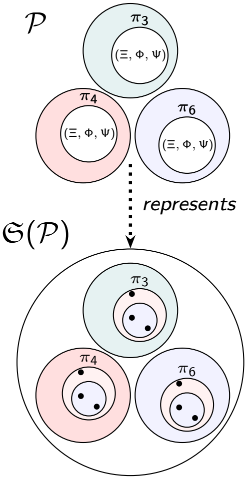

Wrap-Up

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Take-aways

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Worth considering

- lifting the simple theory restriction,
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- "global" knowledge minimization.

Instantiation – Standpoint Default Logic

$T = \{\mathbf{K}\mathit{coffee},$

$\mathbf{K}\neg(\mathit{cold} \wedge \mathit{hot}),$

$\mathbf{K}\neg(\mathit{espresso} \wedge \mathit{low\_caffeine})$

$\square_{*}\left[\mathit{coffee} : \mathit{hot} / \mathit{hot}\right]$

$\square$ $\left[\mathit{coffee} : \mathit{cold} / \mathit{cold}\right]$

$\square$ $\left[\mathit{coffee} : \mathit{espresso} / \mathit{espresso}\right]$

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$T \mathrel{|\!\!\approx_{\text{cred}}} \Box$$\left[\mathbf{K}\mathit{espresso}\right]$

Instantiation – Standpoint Default Logic

$$
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\end{aligned}
$$

$$T \mathrel{|\!\approx_{\text{cred}}} \Box_{\text{🇮🇹}}\left[\mathbf{K}\mathit{espresso}\right]$$

$$T \mathrel{|\!\approx_{\text{cred}}} \Box_{\text{🇮🇹}}\left[\mathbf{K}\mathit{hot}\right]$$

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\end{aligned}
$$

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$$T \mathrel{|\!\approx_{\text{cred}}} \Box_{\text{🇺🇸}}\left[\mathbf{K}\mathit{low\_caffeine}\right]$$
$$T \mathrel{|\!\approx_{\text{cred}}} \Box_{\text{🇺🇸}}\left[\mathbf{K}\mathit{hot}\right]$$
$$T \mathrel{|\!\approx_{\text{cred}}} \Box_{\text{🇻🇳}}\left[\mathbf{K}\mathit{cold}\right]$$

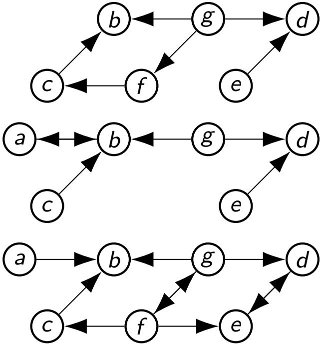
Instantiation – Standpoint Default Logic

$$
\begin{aligned}
T = \{ & \mathbf{K}\,\mathit{coffee}, \\
& \mathbf{K}\neg(\mathit{cold} \wedge \mathit{hot}), \\
& \mathbf{K}\neg(\mathit{espresso} \wedge \mathit{low\_caffeine}) \\
& \Box_{*}\left[\mathit{coffee} : \mathit{hot} / \mathit{hot}\right] \\
& \Box_{\text{🇻🇳}}\left[\mathit{coffee} : \mathit{cold} / \mathit{cold}\right] \\
& \Box_{\text{🇮🇹}}\left[\mathit{coffee} : \mathit{espresso} / \mathit{espresso}\right] \\
& \Box_{\text{🇺🇸}}\left[\mathit{coffee} : \mathit{low\_caffeine} / \mathit{low\_caffeine}\right] \quad \}
\end{aligned}
$$

$$
\begin{aligned}
& T \mathrel{|}\!\!\approx_{\text{cred}} \Box_{\text{🇮🇹}}\left[\mathbf{K}\,\mathit{espresso}\right] \\
& T \mathrel{|}\!\!\approx_{\text{cred}} \Box_{\text{🇮🇹}}\left[\mathbf{K}\,\mathit{hot}\right] \\
& T \mathrel{|}\!\!\approx_{\text{cred}} \Box_{\text{🇺🇸}}\left[\mathbf{K}\,\mathit{low\_caffeine}\right] \\
& T \mathrel{|}\!\!\approx_{\text{cred}} \Box_{\text{🇺🇸}}\left[\mathbf{K}\,\mathit{hot}\right] \\
& T \mathrel{|}\!\!\approx_{\text{cred}} \Box_{\text{🇻🇳}}\left[\mathbf{K}\,\mathit{cold}\right] \\
& T \mathrel{|}\!\!\approx_{\text{cred}} \Box_{\text{🇻🇳}}\left[\mathbf{K}\,\mathit{hot}\right]
\end{aligned}
$$

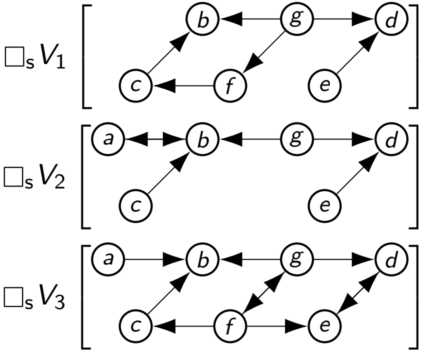
Instantiation – Standpoint Argumentation

(Example by [BNR21])
$T_f F$



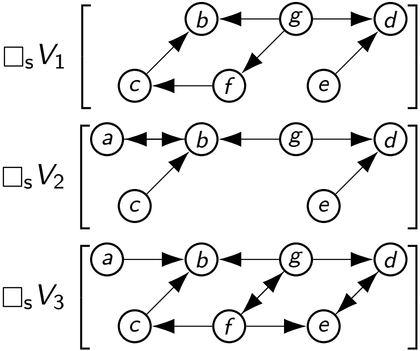
Instantiation – Standpoint Argumentation

(Example by [BNR21])
$T_f F$



Instantiation – Standpoint Argumentation

(Example by [BNR21])
$T_f F$

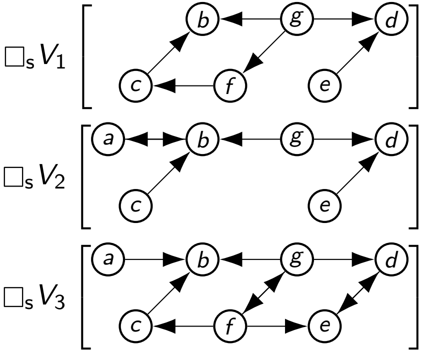


Stable extensions:
$V_1 : \{c, e, g\}$
$V_2 : \{a, c, e, g\}$
$V_3 : \{a, c, e, g\}$ and $\{a, d, f\}$

Instantiation – Standpoint Argumentation

(Example by [BNR21])

$T_f F$



Stable extensions:

$V_1 : \{c, e, g\}$

$V_2 : \{a, c, e, g\}$

$V_3 : \{a, c, e, g\}$ and $\{a, d, f\}$

$T_F \mid\!\approx_{\text{cred}} \Box_{V_2}\mathbf{K}a$

$T_F \mid\!\approx_{\text{cred}} \Box_{*}\mathbf{K}c$

$T_F \not\mid\!\approx_{\text{scep}} \Box_{*}\mathbf{K}c$

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