Reasoning with Attributed Description Logics

Markus Krötzsch *Maximilian Marx* Ana Ozaki Veronika Thost

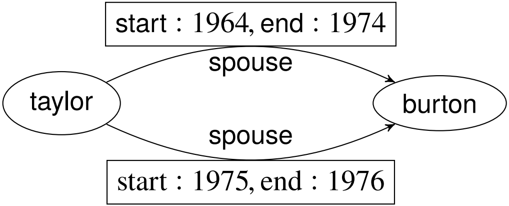
Center for Advancing Electronics Dresden (cfaed), TU Dresden

30th International Workshop on Description Logics

Full paper: `https://iccl.inf.tu-dresden.de/web/Inproceedings3154`

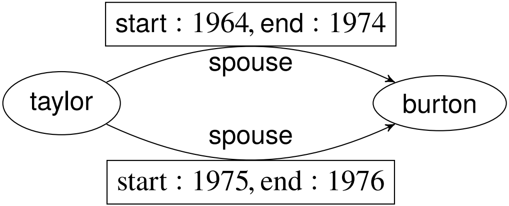
Why Attributed DLs?

- Liz Taylor & Richard Burton in Wikidata:



Why Attributed DLs?

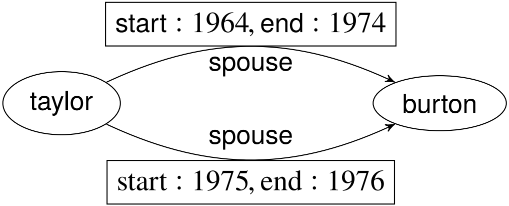
- Liz Taylor & Richard Burton in Wikidata:



- edges may occur with multiple distinct annotations
- annotations: finite attribute–value sets, attached to concept & role names

Why Attributed DLs?

- Liz Taylor & Richard Burton in Wikidata:



- edges may occur with multiple distinct annotations
- annotations: finite attribute–value sets, attached to concept & role names
- e.g., spouse is symmetric, so inverses should coincide on start & end

$$X : \lfloor\ \rfloor \qquad (\mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@\lfloor \mathsf{start} : X.\mathsf{start}, \mathsf{end} : X.\mathsf{end} \rfloor)$$

Specifiers: constraining annotations

- two flavours of annotations: open & closed specifiers

consider $\mathsf{spouse}(\mathsf{taylor}, \mathsf{burton})@[\mathsf{start}: 1964, \mathsf{end}: 1974]$

$\lfloor\;\rfloor$ ✓	$[\;]$ ✗
$\lfloor \mathsf{start}: 1964 \rfloor$ ✓	$[\mathsf{start}: 1964]$ ✗
$\lfloor \mathsf{start}: 1964, \mathsf{end}: 1974 \rfloor$ ✓	$[\mathsf{start}: 1964, \mathsf{end}: 1974]$ ✓
$\lfloor \mathsf{start}: 1964, \mathsf{loc}: \mathsf{Montreal} \rfloor$ ✗	$[\mathsf{start}: 1964, \mathsf{end}: 1974, \mathsf{loc}: *]$ ✓
$\lfloor \mathsf{start}: 1964, \mathsf{end}: + \rfloor$ ✓	$[\mathsf{start}: 1964, \mathsf{end}: 1974, \mathsf{loc}: +]$ ✗

- simplification: instead of $C@\lfloor\;\rfloor$, write C

Attributed DL axioms

Axioms may use variables in annotation positions:

- all variables are universally quantified

$$\mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X$$

Attributed DL axioms

Axioms may use variables in annotation positions:

- all variables are universally quantified

$$\mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X$$

- variables may be constrained by using a specifier

$$X\mathbin{:}\lfloor \mathsf{start} : 1964 \rfloor \quad (\mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X)$$

Attributed DL axioms

Axioms may use variables in annotation positions:

- all variables are universally quantified

$$\mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X$$

- variables may be constrained by using a specifier

$$X\mathbin{:}\lfloor \mathsf{start} : 1964 \rfloor \quad (\mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X)$$

- annotations may refer to assignments in other annotations

$$X\mathbin{:}\lfloor \mathsf{start} : 1964 \rfloor, Y\mathbin{:}\lfloor \mathsf{start} : X.\mathsf{start}, \mathsf{end} : Y.\mathsf{end} \rfloor \\ (\mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X)$$

Attributed DL axioms

Axioms may use variables in annotation positions:

- all variables are universally quantified

$$\mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X$$

- variables may be constrained by using a specifier

$$X\!:\![\mathsf{start} : 1964] \quad (\mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X)$$

- annotations may refer to assignments in other annotations

$$X\!:\![\mathsf{start} : 1964], Y\!:\![\mathsf{start} : X.\mathsf{start}, \mathsf{end} : Y.\mathsf{end}]$$
$$(\mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X)$$

- note: cyclic references are allowed

Complexity of Reasoning in Attributed DLs

DL	ground	restricted	unrestricted
$\mathcal{EL}_@$	PTime	PTime/PSpace-hard*	ExpTime
$\mathcal{ALCH}_@$	ExpTime	ExpTime	2ExpTime
$\mathcal{SROIQ}_@$	N2ExpTime	N2ExpTime	N2ExpTime
$\mathcal{EL}_{@+}$	PTime	ExpTime	undecidable
$\mathcal{ALCH}_{@+}$	ExpTime	2ExpTime	undecidable

- except for PSpace-hardness, bounds are tight
- Nominals require special handling (bounds on domain size)
- $\mathcal{SROIQ}_@$ results from M. Krötzsch, M. Marx, A. Ozaki and V. Thost. 'Attributed Description Logics: Ontologies for Knowledge Graphs'. In: *Proc. 16th Int. Semantic Web Conf. (ISWC'17)*. to appear. Springer, 2017

Reasoning for ground KBs

Introduce fresh concept/role names for each annotated concept/role

- yields polynomially larger KB in underlying, classical DL:

$$\mathsf{spouse}(\mathsf{taylor}, \mathsf{burton})@[\mathsf{start}: 1964, \mathsf{end}: 1974] \quad (1)$$

$$\mathsf{spouse}@\lfloor \mathsf{start}: 1964 \rfloor \sqsubseteq \mathsf{spouse}^{-}@\lfloor \mathsf{start}: 1964 \rfloor \quad (2)$$

$$\rightsquigarrow \quad \mathsf{spouse}_{[\mathsf{start}:1964, \mathsf{end}:1974]}(\mathsf{taylor}, \mathsf{burton})$$

$$\mathsf{spouse}_{\lfloor \mathsf{start}:1964 \rfloor} \sqsubseteq \mathsf{spouse}^{-}_{\lfloor \mathsf{start}:1964 \rfloor}$$

Reasoning for ground KBs

Introduce fresh concept/role names for each annotated concept/role

- yields polynomially larger KB in underlying, classical DL:

$$\mathsf{spouse}(\mathsf{taylor},\mathsf{burton})@[\mathsf{start}:1964,\mathsf{end}:1974] \quad (1)$$

$$\mathsf{spouse}@\lfloor\mathsf{start}:1964\rfloor \sqsubseteq \mathsf{spouse}^{-}@\lfloor\mathsf{start}:1964\rfloor \quad (2)$$

$$\rightsquigarrow \quad \mathsf{spouse}_{[\mathsf{start}:1964,\mathsf{end}:1974]}(\mathsf{taylor},\mathsf{burton})$$

$$\mathsf{spouse}_{\lfloor\mathsf{start}:1964\rfloor} \sqsubseteq \mathsf{spouse}^{-}_{\lfloor\mathsf{start}:1964\rfloor}$$

- interactions between open & closed specifiers: (1), (2) entails

$$\mathsf{spouse}(\mathsf{burton},\mathsf{taylor})@\lfloor\mathsf{start}:1964\rfloor,$$

but we do not get $\mathsf{spouse}_{\lfloor\mathsf{start}:1964\rfloor}(\mathsf{burton},\mathsf{taylor})$

Reasoning for ground KBs

Introduce fresh concept/role names for each annotated concept/role

- yields polynomially larger KB in underlying, classical DL:

$$\mathsf{spouse}(\mathsf{taylor}, \mathsf{burton})@[\mathsf{start}: 1964, \mathsf{end}: 1974] \quad (1)$$

$$\mathsf{spouse}@\lfloor \mathsf{start}: 1964 \rfloor \sqsubseteq \mathsf{spouse}^-@\lfloor \mathsf{start}: 1964 \rfloor \quad (2)$$

$$\rightsquigarrow \quad \mathsf{spouse}_{[\mathsf{start}:1964, \mathsf{end}:1974]}(\mathsf{taylor}, \mathsf{burton})$$

$$\mathsf{spouse}_{\lfloor \mathsf{start}:1964 \rfloor} \sqsubseteq \mathsf{spouse}^-_{\lfloor \mathsf{start}:1964 \rfloor}$$

- interactions between open & closed specifiers: (1), (2) entails

$$\mathsf{spouse}(\mathsf{burton}, \mathsf{taylor})@\lfloor \mathsf{start}: 1964 \rfloor,$$

  but we do not get $\mathsf{spouse}_{\lfloor \mathsf{start}:1964 \rfloor}(\mathsf{burton}, \mathsf{taylor})$

- axiomatise these inclusions: $\mathsf{spouse}_{[\mathsf{start}:1964, \mathsf{end}:1974]} \sqsubseteq \mathsf{spouse}_{\lfloor \mathsf{start}:1964 \rfloor}$

Dealing with non-ground KBs

Transform KB into a ground KB:

- instantiate each axiom for every possible annotation

$$\mathsf{spouse}(\mathsf{taylor},\mathsf{burton})@\lfloor \mathsf{start}: 1964\rfloor \qquad \mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X$$

$$\mathsf{spouse}(\mathsf{taylor},\mathsf{burton})@\lfloor \mathsf{start}: 1974\rfloor$$

$$\rightsquigarrow \quad \mathsf{spouse}_{\lfloor \mathsf{start}:1964\rfloor}(\mathsf{taylor},\mathsf{burton}) \quad \mathsf{spouse}_{\lfloor \mathsf{start}:1974\rfloor}(\mathsf{taylor},\mathsf{burton})$$

$$\mathsf{spouse}_{\lfloor \mathsf{start}:1964\rfloor} \sqsubseteq \mathsf{spouse}^{-}_{\lfloor \mathsf{start}:1964\rfloor}$$

$$\mathsf{spouse}_{\lfloor \mathsf{start}:1974\rfloor} \sqsubseteq \mathsf{spouse}^{-}_{\lfloor \mathsf{start}:1974\rfloor}$$

Dealing with non-ground KBs

Transform KB into a ground KB:

- instantiate each axiom for every possible annotation

$$\mathsf{spouse}(\mathsf{taylor}, \mathsf{burton})@\lfloor \mathsf{start} : 1964 \rfloor \qquad \mathsf{spouse}@X \sqsubseteq \mathsf{spouse}^{-}@X$$

$$\mathsf{spouse}(\mathsf{taylor}, \mathsf{burton})@\lfloor \mathsf{start} : 1974 \rfloor$$

$$\rightsquigarrow \quad \mathsf{spouse}_{\lfloor \mathsf{start}:1964 \rfloor}(\mathsf{taylor}, \mathsf{burton}) \quad \mathsf{spouse}_{\lfloor \mathsf{start}:1974 \rfloor}(\mathsf{taylor}, \mathsf{burton})$$

$$\mathsf{spouse}_{\lfloor \mathsf{start}:1964 \rfloor} \sqsubseteq \mathsf{spouse}^{-}_{\lfloor \mathsf{start}:1964 \rfloor}$$

$$\mathsf{spouse}_{\lfloor \mathsf{start}:1974 \rfloor} \sqsubseteq \mathsf{spouse}^{-}_{\lfloor \mathsf{start}:1974 \rfloor}$$

- unfortunately, the grounding is exponential in the size of KB:

$$C(a)@[\,] \qquad C(a)@[\mathsf{b} : \mathsf{b}]$$

$$C@X \sqcap C@Y \sqcap C@Z \sqsubseteq C@X$$

- syntactic restrictions ensure a polynomial grounding

Regaining Tractability for $\mathcal{EL}_{@}$

Sufficient conditions for polynomial grounding:

(A) number of variables per axiom is bounded,

(B) number of 'dots' $X.a$ is bounded, and

(C) no merging with 'dots': if $a : X.b$ occurs in some annotation $\mathcal{S}$, then there is no further assignment for a in $\mathcal{S}$

Regaining Tractability for $\mathcal{EL}_@$

Sufficient conditions for polynomial grounding:

(A) number of variables per axiom is bounded,

(B) number of 'dots' $X.a$ is bounded, and

(C) no merging with 'dots': if $a : X.b$ occurs in some annotation $\mathcal{S}$, then there is no further assignment for a in $\mathcal{S}$

- violating any condition yields intractability for $\mathcal{EL}_@$
- violating (C) results in PSpace-hardness

Regaining Tractability for $\mathcal{EL}_{@}$

Sufficient conditions for polynomial grounding:

(A) number of variables per axiom is bounded,

(B) number of 'dots' $X.a$ is bounded, and

(C) no merging with 'dots': if $a : X.b$ occurs in some annotation $\mathcal{S}$, then there is no further assignment for a in $\mathcal{S}$

- violating any condition yields intractability for $\mathcal{EL}_{@}$
- violating (C) results in PSpace-hardness
- reasoning for $\mathcal{ALCH}_{@}$ KBs satisfying the conditions is ExpTime-complete

An Undecidable Case

Without restrictions, Attributed DLs with + are undecidable:

- interaction of $X.a$ and + admits an encoding of Existential Rules in quantifier-free attributed $\mathcal{EL}$
- forbidding either $X.a$ or + is sufficient to recover decidability
- practically, $X.a$ is more relevant

An Undecidable Case

Without restrictions, Attributed DLs with + are undecidable:

- interaction of $X.a$ and + admits an encoding of Existential Rules in quantifier-free attributed $\mathcal{EL}$
- forbidding either $X.a$ or + is sufficient to recover decidability
- practically, $X.a$ is more relevant, but + adds expressive power:

$$\mathsf{educatedAt@}\lfloor\mathsf{degree} : +\rfloor \sqsubseteq \mathsf{obtainedDegreeFrom}$$

- decidability results for Existential Rules suggest that a weaker condition may suffice for decidability

An Undecidable Case

Without restrictions, Attributed DLs with + are undecidable:

- interaction of $X.a$ and + admits an encoding of Existential Rules in quantifier-free attributed $\mathcal{EL}$
- forbidding either $X.a$ or + is sufficient to recover decidability
- practically, $X.a$ is more relevant, but + adds expressive power:

$$\mathsf{educatedAt@}\lfloor\mathsf{degree} : +\rfloor \sqsubseteq \mathsf{obtainedDegreeFrom}$$

- decidability results for Existential Rules suggest that a weaker condition may suffice for decidability
- Corollary: Attributed DLs (without +) capture Datalog

Summary & Outlook

- Summary:
  - we add annotations (sets of attribute–value pairs) to concept and role names
  - specifiers allow to constrain variables in axioms
  - ‘ground and rename’ reasoning approach
  - attributed reasoning is exponentially harder
  - syntactic conditions ensure that we avoid this blowup

Summary & Outlook

- Summary:
  - we add annotations (sets of attribute–value pairs) to concept and role names
  - specifiers allow to constrain variables in axioms
  - ‘ground and rename’ reasoning approach
  - attributed reasoning is exponentially harder
  - syntactic conditions ensure that we avoid this blowup
- Future Work:
  - data complexities
  - extension to further DL constructs ($\mathcal{EL}^{++}_{@}$?)
  - annotation-aware reasoning algorithms