TECHNISCHE UNIVERSITÄT DRESDEN

NAVAS

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Navigating ASP Solution Spaces

Hanoi, 2nd November 2024

Motivation

Searching for a Bicycle

Motivation
Searching for a Bicycle

Motivation

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Searching for a Bicycle

Motivation
Combinatorial Search Problems

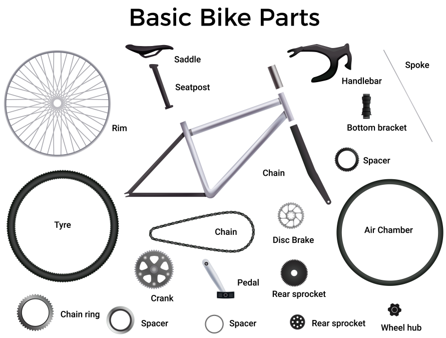
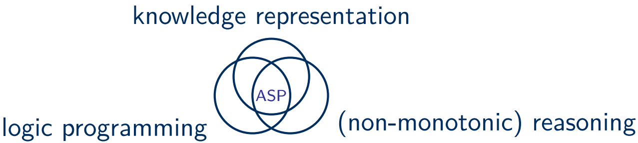


Figure 1: Bunt Vektor erstellt von macrovector - de.freepik.com

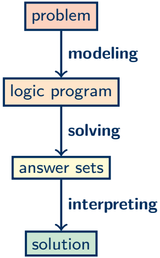
Answer Set Programming (ASP)



Declarative problem solving

- planning
- product configuration
- diagnosis

⋮



ASP Modelling and Solving

```
#const n=14.
{q(I, 1..n)} == 1 :- I = 1..n.
{q(1..n, J)} == 1 :- J = 1..n.
:- {q(D-J,J)} >=2, D=2..2*n.
:- {q(D+J,J)} >=2, D=1-n..n-1.
```

solver →

```
Answer: 1
q(5,13) q(7,14) q(2,8) q(6,11) q(4,7) q(1,3) q(9,10)
q(12,12) q(3,2) q(8,5) q(10,6) q(14,9) q(11,4) q(13,1)
Answer: 2
q(2,12) q(1,9) q(7,13) q(6,11) q(4,7) q(12,14) q(9,10)
q(3,3) q(5,4) q(8,5) q(10,6) q(14,8) q(11,1) q(13,2)
Answer: 3
q(1,13) q(7,14) q(3,9) q(6,11) q(4,7) q(2,4) q(9,10)
q(12,12) q(5,3) q(10,6) q(14,8) q(8,1) q(13,5) q(11,2)
⋮
Answer: 365596
q(4,13) q(1,9) q(7,14) q(3,8) q(2,6) q(8,11) q(11,12)
q(5,4) q(12,10) q(9,5) q(6,1) q(13,7) q(10,3) q(14,2)
SATISFIABLE
```

Navigating ASP Solution Spaces

```
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{q(I, 1..n)} == 1 :- I = 1..n.
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:- {q(D-J,J)} >=2, D=2..2*n.
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Diverse Solutions:

```
Solution: 1
q(1,12) q(2,8) q(3,6) q(4,14) q(5,9) q(6,2) q(7,5)
q(14,1) q(9,11) q(10,7) q(11,10) q(12,4) q(13,13) q(8,3)
Solution: 2
q(1,1) q(2,10) q(3,5) q(4,7) q(5,12) q(6,3) q(7,11)
q(8,2) q(9,14) q(10,9) q(11,4) q(12,13) q(13,8) q(14,6)
Solution: 3
q(1,11) q(2,2) q(3,10) q(4,6) q(5,3) q(6,1) q(7,13)
q(8,7) q(9,12) q(10,14) q(11,8) q(12,5) q(13,9) q(14,4)
```

Quantitative Reasoning:

* zoom in
* zoom out
* ...

Visual Approach:

* zoom in
* zoom out
* ...

Outline

- Preliminaries
- Weighted Faceted Navigation
- Diverse Answer Sets
- Representative Answer Sets
- Visual Approach for Solution Space Exploration
- Conclusion

Preliminaries

Definition (logic program)

A (normal disjunctive) *logic program* Π over a set of atoms $\{\alpha_0, \dots, \alpha_n\}$ is a finite set of rules r of the form:

$$\alpha_0 \mid \dots \mid \alpha_k \leftarrow \alpha_{k+1}, \dots, \alpha_m, {\sim}\alpha_{m+1}, \dots, {\sim}\alpha_n. \text{ where } 0 \leq k \leq m \leq n$$

Remark: We focus on ground programs without extended rules.

$\mathcal{AS}(\Pi) \dots$ **answer sets** (solutions)

$2^{\mathcal{AS}(\Pi)} \dots$ **solution space**

$\mathcal{BC}(\Pi) \coloneqq \bigcup \mathcal{AS}(\Pi) \dots$ **brave consequences**

$\alpha \in \mathcal{BC}(\Pi) \dots$ **partial solution**

$\mathcal{CC}(\Pi) \coloneqq \bigcap \mathcal{AS}(\Pi) \dots$ **cautious consequences**

Part 1 Weighted Faceted Navigation

Systematic Faceted Navigation

$\Pi: \quad a \,|\, b. \quad c \,|\, d \leftarrow b. \quad e.$

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Facets: $\mathcal{F}(\Pi) = \{a, b, c, d, \overline{a}, \overline{b}, \overline{c}, \overline{d}\}$

`ANSWER 1:` a, e

`ANSWER 2:` b, c, e

`ANSWER 3:` b, d, e

Systematic Faceted Navigation

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```
ANSWER 1:   a, e
ANSWER 2:   b, c, e
ANSWER 3:   b, d, e
```

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Routes: $\Delta^{\Pi} \coloneqq \{\langle f_0, \ldots, f_n \rangle \mid f_i \in \mathcal{F}(\Pi), 0 \leq i \leq n\} \cup \{\epsilon\}$

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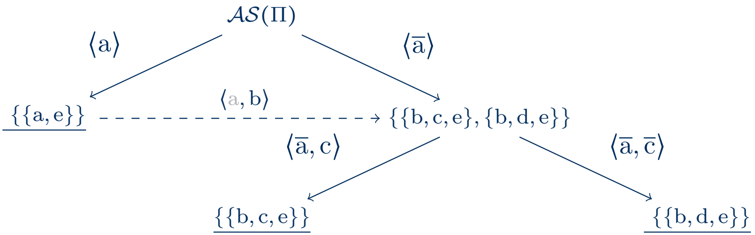

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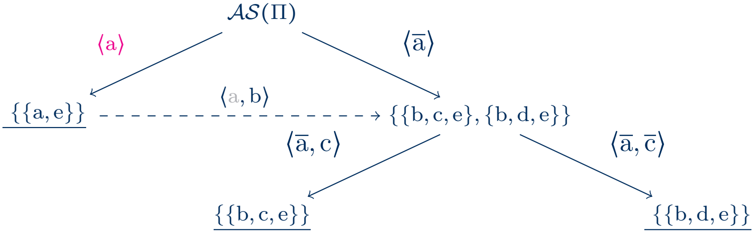

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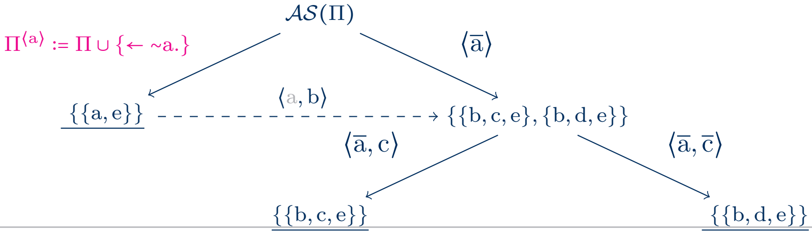

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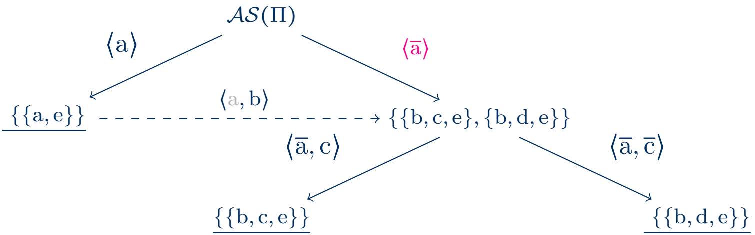

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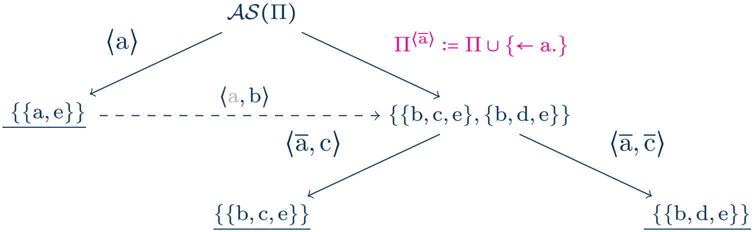
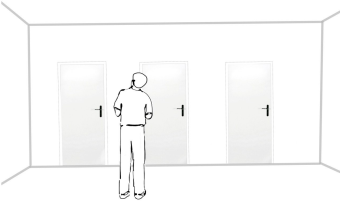


What is the effect of taking a certain navigation step?

Can we somehow characterize sub-spaces beforehand?



[1] Johannes Klaus Fichte, S.A.G., Dominik Rusovac. **Rushing and Strolling among Answer Sets - Navigation Made Easy**
Proceedings of the 36th AAAI Conference on Artificial Intelligence (AAAI 2022), 2022.

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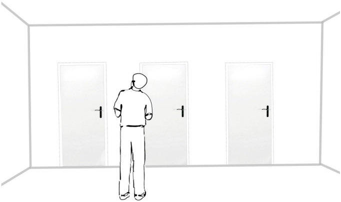

Let's do some counting!

Quantifying effects of navigation steps

The Weight of a Facet

Definition (weighting function)

We call $\# : \{\Pi^\delta \mid \delta \in \Delta^\Pi\} \rightarrow \mathbb{N}$ *weighting function*, whenever $\#(\Pi^\delta) > 0$, if $|\mathcal{AS}(\Pi)| \geq 2$.

Definition (weight)

Let $\delta \in \Delta^\Pi$, $\mathrm{f} \in \mathcal{F}(\Pi)$ and δ' be a redirection of δ w.r.t. f. The *weight* of f w.r.t. $\#$, Π^δ and δ' is defined as:

$$\omega_\#(\mathrm{f}, \Pi^\delta, \delta') \coloneqq \begin{cases} \#(\Pi^\delta) - \#(\Pi^{\delta'}), & \text{if } \langle \delta, \mathrm{f} \rangle \notin \Delta^\Pi_\mathrm{s} \text{ and } \delta' \neq \epsilon; \\ \#(\Pi^\delta) - \#(\Pi^{\langle \delta, \mathrm{f} \rangle}), & \text{otherwise.} \end{cases}$$

The Weight of a Facet

Definition (weight)

Let $\delta \in \Delta^{\Pi}$, $f \in \mathcal{F}(\Pi)$ and δ' be a redirection of δ w.r.t. f. The *weight* of f w.r.t. $\#$, Π^{δ} and δ' is defined as:

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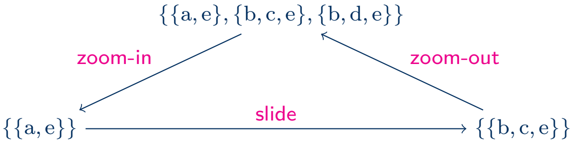
Effects:



Which Weighting Function? Absolute vs. Relative Weights

Natural choice?

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Natural choice?

- Absolute Weight:
  Count Answer Sets with $\omega_{\#\mathcal{AS}}$

$$\mathcal{AS}(\Pi) = \{\{a, e\}, \{b, c, e\}, \{b, d, e\}\}$$

$$\omega_{\#\mathcal{AS}}(b, \Pi, \epsilon) = 1 \;\downarrow$$

$$\mathcal{AS}(\Pi^{\langle b \rangle}) = \{\{b, c, e\}, \{b, d, e\}\}$$

Which Weighting Function? Absolute vs. Relative Weights

Natural choice?

- Absolute Weight:
  Count Answer Sets with $\omega_{\#_{\mathcal{AS}}}$

  Counting answer sets is hard ☹ [3]

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[2] Johannes K Fichte, Markus Hecher, Michael Morak, and Stefan Woltran. **Answer set solving with bounded treewidth revisited.** In LPNMR 2017.

Which Weighting Function? Absolute vs. Relative Weights

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Counting answer sets is hard ☹ [3]

Relative Weights:
cheaper methods to quantify effects

- Count Supported Models with $\omega_{\#_{\mathcal{S}}}$

$$\mathcal{AS}(\Pi) = \{\{a, e\}, \{b, c, e\}, \{b, d, e\}\}$$

$$\downarrow \; \omega_{\#_{\mathcal{S}}}(b, \Pi, \epsilon) = 1$$

$$\mathcal{AS}(\Pi^{\langle b \rangle}) = \{\{b, c, e\}, \{b, d, e\}\}$$

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Which Weighting Function? Absolute vs. Relative Weights

Natural choice?

- Absolute Weight:
  Count Answer Sets with $\omega_{\#_{\mathcal{AS}}}$

Counting answer sets is hard ☹ [fichte2017answer]

Relative Weights:
cheaper methods to quantify effects

- Count Supported Models with $\omega_{\#_{\mathcal{S}}}$
- Count Facets with $\omega_{\#_{\mathcal{F}}}$

$$\mathcal{AS}(\Pi) = \{\{a, e\}, \{b, c, e\}, \{b, d, e\}\}$$

$$\omega_{\#_{\mathcal{F}}}(b, \Pi, \epsilon) = 4 \quad \downarrow$$

$$\mathcal{AS}(\Pi^{\langle b \rangle}) = \{\{b, c, e\}, \{b, d, e\}\}$$

$$\mathcal{AS}(\Pi) = \{\{a, e\}, \{b, c, e\}, \{b, d, e\}\}$$

$$\omega_{\#_{\mathcal{F}}}(\overline{c}, \Pi, \epsilon) = 2 \quad \downarrow$$

$$\mathcal{AS}(\Pi^{\langle \overline{c} \rangle}) = \{\{a, e\}, \{b, d, e\}\}$$

[2] Johannes K Fichte, Markus Hecher, Michael Morak, and Stefan Woltran. **Answer set solving with bounded treewidth revisited.** In LPNMR 2017.

Rushing and Strolling among Answer Sets

Definition (strictly goal-oriented navigation mode)

The *stricly goal-oriented* navigation mode $\nu^{\#}_{\mathrm{sgo}} : \Delta^{\Pi}_{\mathrm{s}} \times \mathcal{F}(\Pi) \rightarrow 2^{\mathcal{AS}(\Pi)}$ is defined by:

$$\nu^{\#}_{\mathrm{sgo}}(\delta, \mathrm{f}) := \begin{cases} \mathcal{AS}(\Pi^{\langle \delta, \mathrm{f} \rangle}), & \text{if } \mathrm{f} \in \max_{\omega_{\#}}(\Pi^{\delta}); \\ \mathcal{AS}(\Pi^{\delta}), & \text{otherwise.} \end{cases}$$

Definition (explore navigation mode)

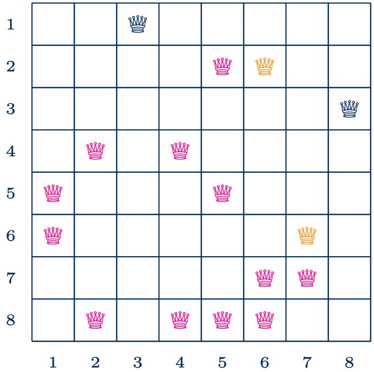
The *explore* navigation mode $\nu^{\#}_{\mathrm{expl}} : \Delta^{\Pi}_{\mathrm{s}} \times \mathcal{F}(\Pi) \rightarrow 2^{\mathcal{AS}(\Pi)}$ is defined by:

$$\nu^{\#}_{\mathrm{expl}}(\delta, \mathrm{f}) := \begin{cases} \mathcal{AS}(\Pi^{\langle \delta, \mathrm{f} \rangle}), & \text{if } \mathrm{f} \in \min_{\omega_{\#}}(\Pi^{\delta}); \\ \mathcal{AS}(\Pi^{\delta}), & \text{otherwise.} \end{cases}$$

Quantitative Arguments

	2		5		1		9	
8			2		3			6
	3			6			7	
6	*6*	1		*2*	*2*∣*6*			
5	4			*2*		*8*	1	9
				2∣*5*	*2*∣*5*	7		
	9			3			8	
2			8		4			7
	1		9		7		6	

How to solve this Sudoku as quick as possible?

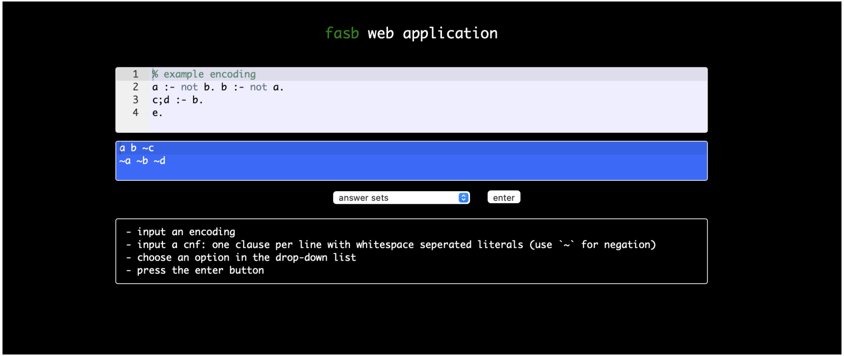


Which moves (queens) have the least (1/4)/most (3/4) impact?

fasb – Faceted Answer Set Browser

REPL on top of clingo solver implementing: ν_{go}, $\nu^{\#}_{\text{sgo}}$, $\nu^{\#}_{\text{expl}}$ for $\# \in \{\#_{\mathcal{AS}}, \#_{\mathcal{F}}\}$

https://github.com/drwadu/fasb



https://drwadu.github.io/web-fasb.github.io/

Concluding on Weighted Faceted Navigation

- Concept is rather easy to understand
- Answer set counting is hard
  - IASCAR [3]
  - Quantum faceted navigation [4]
- Facet counting is easier (practical feasability) [5]

[3] Johannes Klaus Fichte, S.A.G., Markus Hecher, Dominik Rusovac. **IASCAR: Incremental Answer Set Counting by Anytime Refinement** Theory and Practice of Logic Programming, 24(3):505-532, May 2024.
[4] Riccardo Romanello, Davide Della Giustina, Stefano Pessotto, Carla Piazza. **Speeding up Answer Set Programming by Quantum Computing** QUASAR '24: Proceedings of the 2024 Workshop on Quantum Search and Information Retrieval Pages 1 - 8, 2024.

[5] Dominik Rusovac, Markus Hecher, Martin Gebser, S.A.G., Johannes K. Fichte. **Navigating and Querying Answer Sets: How Hard Is It Really and Why?** Proceedings of the 21st International Conference on Principles of Knowledge Representation and Reasoning (KR 2024).

Part 2 Diverse Answer Sets

Similar/Diverse Solutions in ASP [6]

Divers Answer Sets

- n k-DIVERSE SOLUTIONS
- $n = |S|$, S ... collection of AS
- $\Delta(S) \geq k$
- $\Delta : 2^{AS} \rightarrow \mathbb{N}^0$

[6] Thomas Eiter, Esra Erdem, Halit Erdogan, Michael Fink: Finding similar/diverse solutions in answer set programming. Theory Pract. Log. Program. 13(3): 303-359 (2013)

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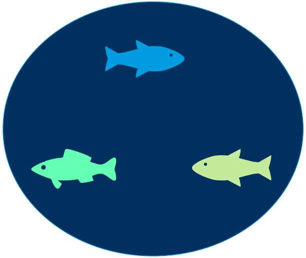

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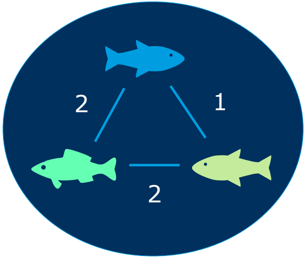

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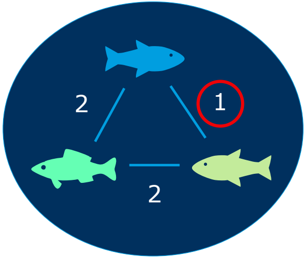

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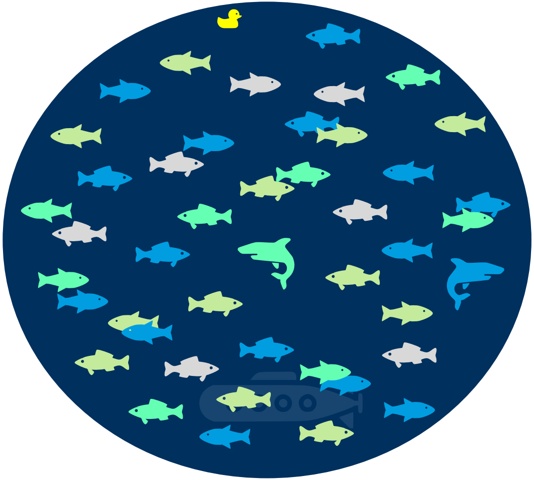
Methods:

- enumeration and postprocessing
- parallel solving
- iterative solving

[6] Thomas Eiter, Esra Erdem, Halit Erdogan, Michael Fink: Finding similar/diverse solutions in answer set programming. Theory Pract. Log. Program. 13(3): 303-359 (2013)

Reworking Methods [7]



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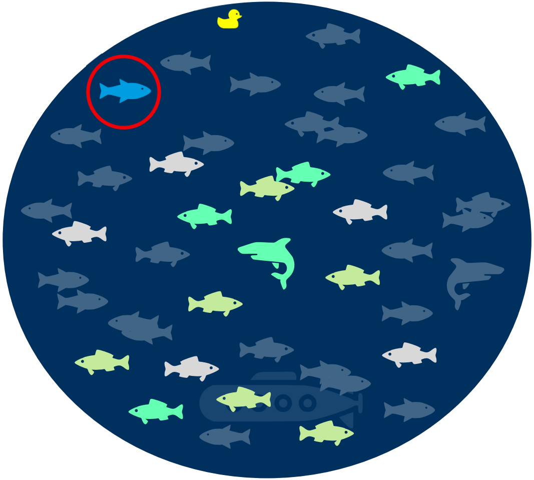

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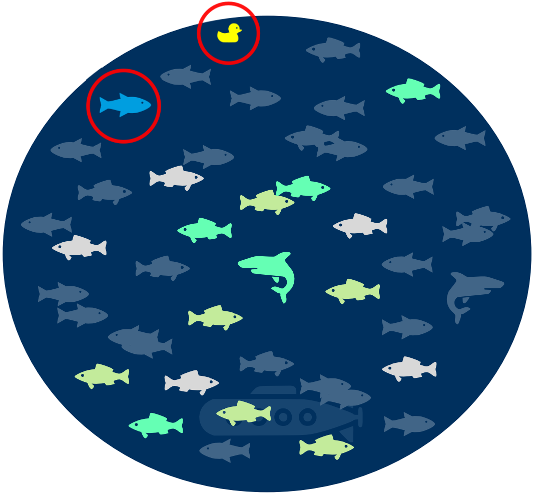

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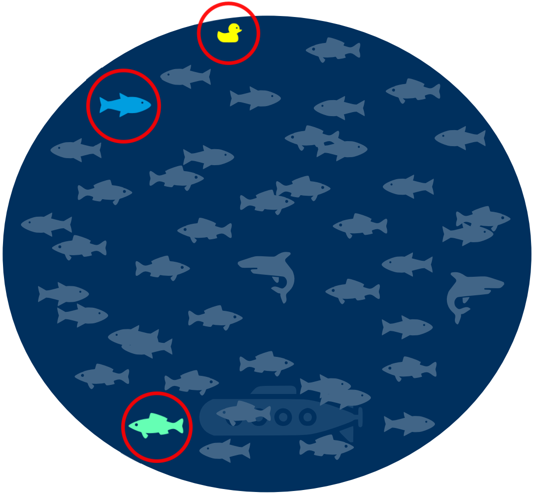

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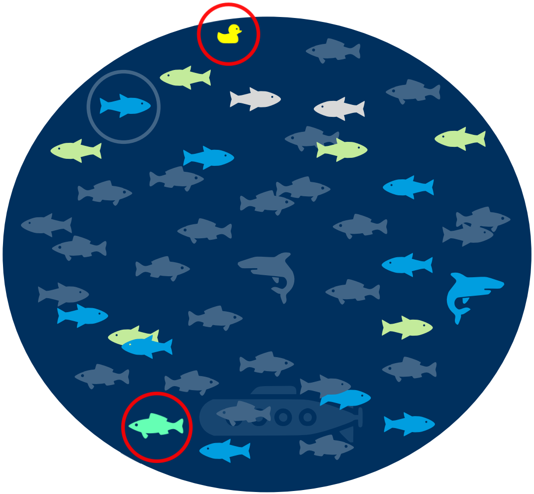

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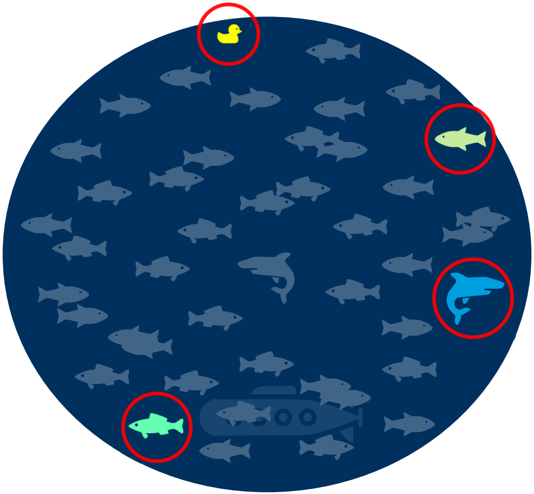

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Reworking Methods [7]

Problem Definition

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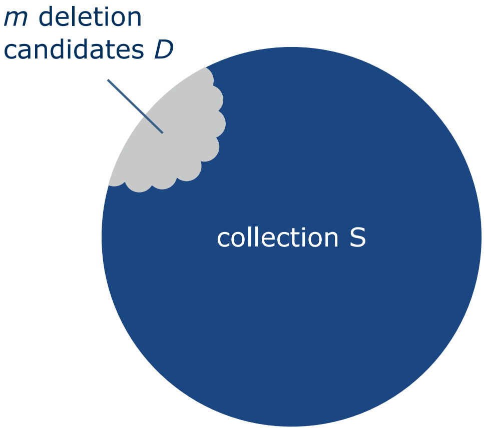
Reworking Methods [7]

Problem Definition

- $|S \setminus S'| \leq m$



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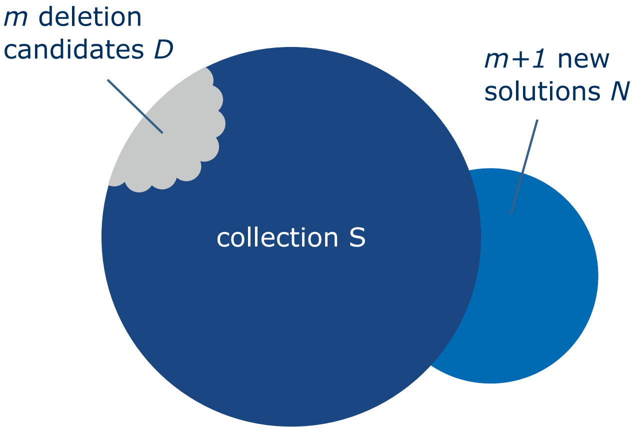
Reworking Methods [7]

Problem Definition

- $|S'| > |S|$
- $|S \setminus S'| \leq m$



[7] Elisa Böhl, S.A.G. **Tunas - Fishing for Diverse Answer Sets: A Multi-Shot Trade up Strategy**; LPNMR 2022.

Reworking Methods [7]

Problem Definition

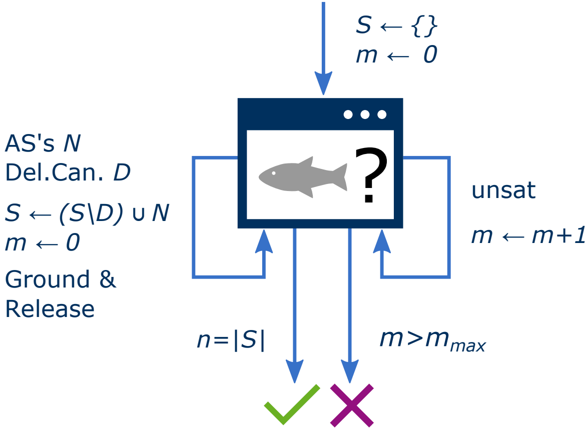
- $|S'| > |S|$
- $|S \setminus S'| \leq m$
- $\Delta(S') \geq k$
- NP-complete

[7] Elisa Böhl, S.A.G. **Tunas - Fishing for Diverse Answer Sets: A Multi-Shot Trade up Strategy**; LPNMR 2022.

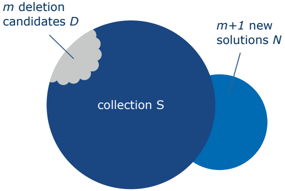
Tunas - Trade Up Navigation for Answer Sets

Properties

- comparatively fast
- anytime approach
- multi-shot
  - additive grounding
  - concealment and deletion of atoms
  - counting over changing domain



Tunas - Elaborations



	Nondeterministic Choosing (ND)	**Iterative Choosing (IT)**
	ASP guesses deletion candidates	*wrapper* iterates over deletion candidates
Generating Multiple(M) parallel	TunasMND	TunasMIT
Generating Single (S) iterativ	X	TunasS

Concluding on Diverse Answer Sets

- Diverse collections give a nice overview on the solutionspace
- Multi-shot ASP is good choice to iteratively improve collections
- User needs to specify diversity measure (and provide it as ASP encoding)

Part 3 Representative Answer Sets

Collecting Something of Everything

$\mathcal{AS}(\Pi_0)$:

s_1 = {■, ▲}　　s_2 = {●, ▲}　　s_3 = {◆, ✖}

s_4 = {■, ◆}　　s_5 = {✖, ▼, ✚}　　s_6 = {●, ▼, ✚}

$T = \{at(\Pi_0)\}$

Collecting Something of Everything

$\mathcal{AS}(\Pi_0)$:

s_1 = {■, ▲} s_2 = {●, ▲} s_3 = {◆, ✖}

s_4 = {■, ◆} s_5 = {✖, ▼, ✚} s_6 = {●, ▼, ✚}

$T = \{at(\Pi_0)\}$

– $S = \{s_1, s_2, s_4, s_5\}$ is sound over T,

Collecting Something of Everything

$\mathcal{AS}(\Pi_0)$:

$s_1 = \{$■, ▲$\}$ $s_2 = \{$●, ▲$\}$ $s_3 = \{$◆, ✖$\}$

$s_4 = \{$■, ◆$\}$ $s_5 = \{$✖, ▼, ✚$\}$ $s_6 = \{$●, ▼, ✚$\}$

$T = \{at(\Pi_0)\}$

- $S = \{s_1, s_2, s_4, s_5\}$ is sound over T,
- S is not a packing over T as atoms ▲ and ■ appear twice.

Collecting Something of Everything

$\mathcal{AS}(\Pi_0)$:

s_1 = {■, ▲}	s_2 = {●, ▲}	s_3 = {◆, ✖}
s_4 = {■, ◆}	s_5 = {✖, ▼, ✚}	s_6 = {●, ▼, ✚}

$T = \{at(\Pi_0)\}$

- $S = \{s_1, s_2, s_4, s_5\}$ is sound over T,
- S is not a packing over T as atoms ▲ and ■ appear twice.
- By removing s_1 from S, we obtain a perfect collection over T.

Representative Collections [8]

- collection S: set of answer sets; target atoms T
- Soundness: all target atoms covered; $T \subseteq \bigcup S$
- Diversity: self information / Shannon entropy [1], $p_S(a_i) = \frac{m_i}{\sum_{j=1}^{n} m_j}$, with m_i frequency of atom a_i:

$$H[T|_S] \coloneqq \sum_{a \in T} p_S(a) \log_2 \frac{1}{p_S(a)}$$

$$D(T|_S) \coloneqq 2^{H[T|_S]} \in [0, |T|]$$

[1] T. Leinster, 'Entropy and diversity: the axiomatic approach', Cambridge university press, 2021

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- normalisation into representativeness:

$$R(T|_S) \coloneqq \frac{D(T|_S)}{|T|} \in [0, 1]$$

[8] Elisa Böhl, S.A.G, Dominik Rusovac. Representative Answer Sets: Collecting Something of Everything. Proceedings of the 26th European Conference on Artificial Intelligence (ECAI 2023), 271–278, September 2023.

[1] T. Leinster, 'Entropy and diversity: the axiomatic approach', Cambridge university press, 2021

Example

$s_1 = \{\blacksquare, \blacktriangle\}$ $\quad s_2 = \{\bullet, \blacktriangle\}$ $\quad s_3 = \{\blacklozenge, \boldsymbol{\times}\}$

$s_4 = \{\blacksquare, \blacklozenge\}$ $\quad s_5 = \{\boldsymbol{\times}, \blacktriangledown, \boldsymbol{+}\}$ $\quad s_6 = \{\bullet, \blacktriangledown, \boldsymbol{+}\}$

$$T = \{at(\Pi_0)\}, S = \{s_1, s_2, s_4, s_5\}$$

$$S|_T^m = \{\blacklozenge^1, \blacksquare^2, \bullet^1, \blacktriangle^2, \boldsymbol{\times}^1, \boldsymbol{+}^1, \blacktriangledown^1\}$$

$$p_S(a) : \frac{1}{9}, \frac{2}{9}, \frac{1}{9}, \frac{2}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}$$

Entropy	$H[T\|_S]$	$= \sum_{a \in T} p_S(a) \log_2 \frac{1}{p_S(a)}$	≈ 2.7165
Diversity	$D(T\|_S)$	$= 2^{H[T\|_S]}$	≈ 6.573
Representativeness	$R(T\|_S)$	$= \frac{D(T\|_S)}{\|T\|}$	≈ 0.94

Example cont.

$s_1 = \{■, ▲\}$ $s_2 = \{●, ▲\}$ $s_3 = \{◆, ✖\}$

$s_4 = \{■, ◆\}$ $s_5 = \{✖, ▼, ✚\}$ $s_6 = \{●, ▼, ✚\}$

$$T = \{at(\Pi_0)\}, S' = \{s_2, s_4, s_5\}$$

$$S|_T^m = \{◆^1, ■^1, ●^1, ▲^1, ✖^1, ✚^1, ▼^1\}$$

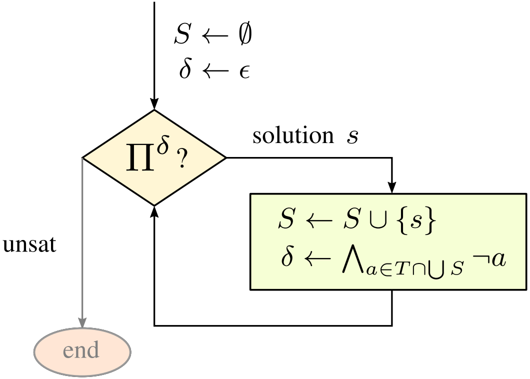
$$p_S(a) : \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7} \text{(uniform distribution)}$$

Entropy	$H[T\|_S]$	$= \sum_{a \in T} p_S(a) \log_2 \frac{1}{p_S(a)}$	$= 7\frac{1}{7} \log_2 7 = \log_2 7$
Diversity	$D(T\|_S)$	$= 2^{H[T\|_S]}$	$= 2^{\log_2 7} = 7$
Representativeness	$R(T\|_S)$	$= \frac{D(T\|_S)}{\|T\|}$	$= \frac{7}{7} = 1$

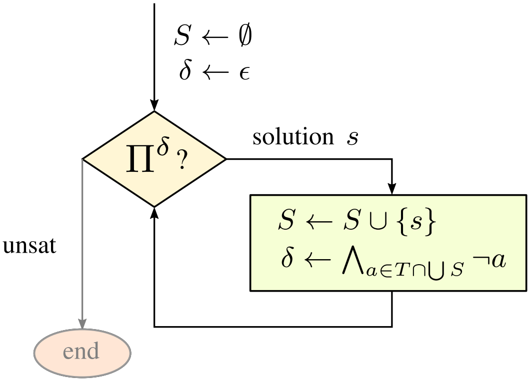
Obtaining Representative Collections

- Approach: Faceted Answer Set Navigation
  - activating a facet: propagation of a truth value
  - counting facets enables to measure uncertainty
- Algorithms: Greedy for diversity (D) and soundness (S)

Algorithm: D-Greedy

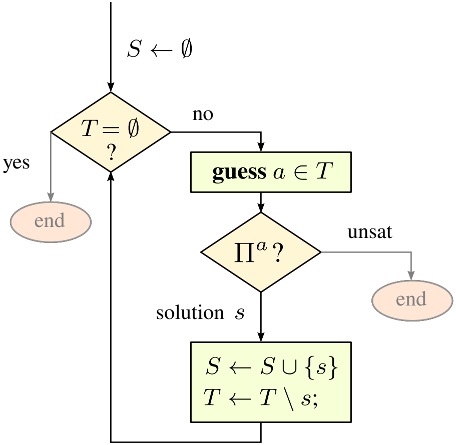


Algorithm: D-Greedy

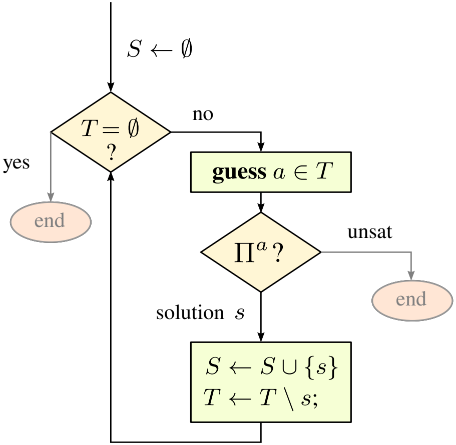


- D-Greedy-max
- D-Greedy-max$_+$
- D-Greedy-all-max
- D-Greedy-all-max$_+$

Algorithm: S-Greedy



Algorithm: S-Greedy



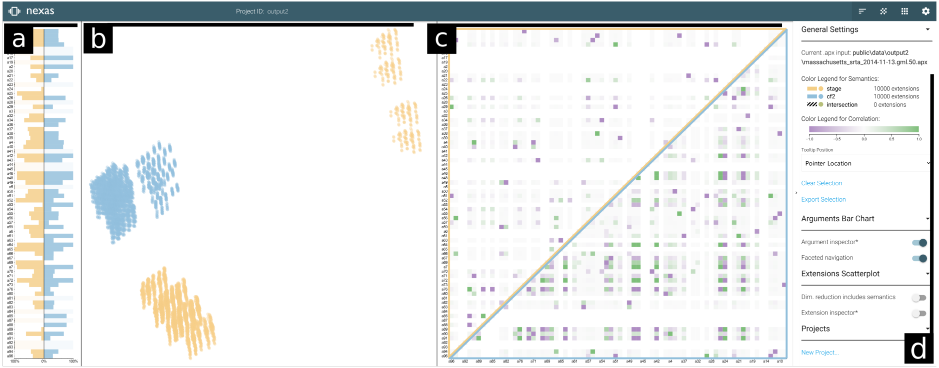
- S-Greedy
- S-Greedy-Sieve
  (search on route $\bigvee_{i=1} t_i$)

Concluding on Representative Collections

- Experiments showed, if quality of outcome matters, the more complex heuristics pay off (D-Greedy-all-$\max_+$)
- Otherwise, less complex methods (S-Greedy-Sieve) are much faster → hybrid approach
- Entropy a reasonable measure for diversity of solutions

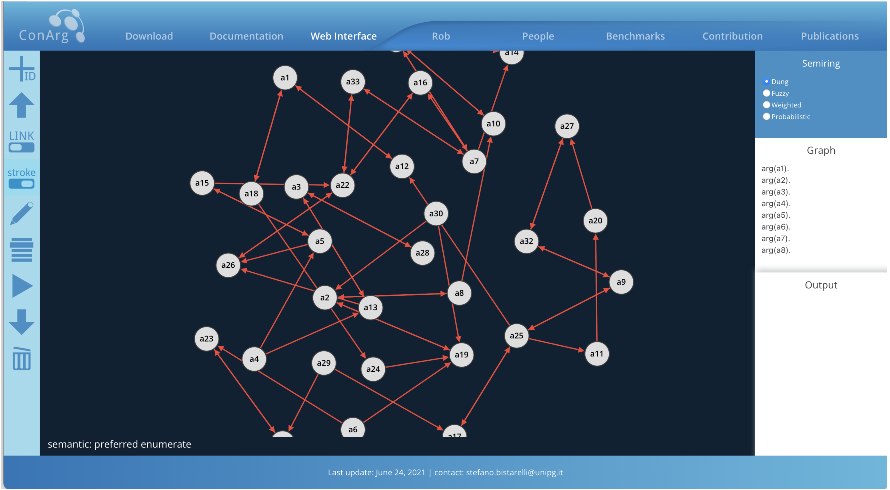
Part 4 Visual Approach for Solution Space Exploration

NEXAS: A Visual Tool for Navigating and Exploring Argumentation Solution Spaces [9]



[9] Raimund Dachselt, S.A.G., Markus Krötzsch, Julián Méndez, Dominik Rusovac, Mei Yang. **NEXAS: A Visual Tool for Navigating and Exploring Argumentation Solution Spaces**; COMMA 2022. `https://imld.de/nexas`

Visualization of AF Extensions



Use Case 1: Compare two semantics

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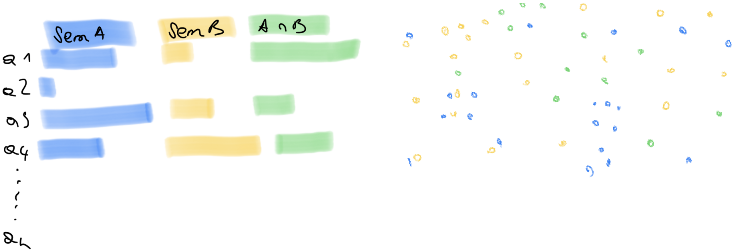
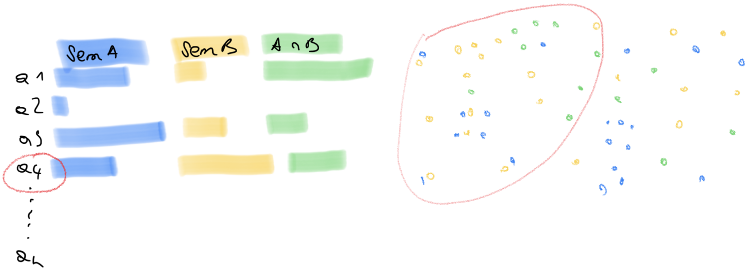


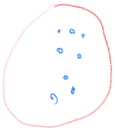
Use Case 2: Very large solution space

Only compute particular sub-space of the whole solution space, where some arguments are either contained in all extensions or in none.



Use Case 3: Navigate towards desired solution sub-space

Show for which arguments one can zoom-in (arguments that are credulously but not skeptically accepted)



Design Goals

DG-1: Intuitive and Familiar Representations. We aim to foster intuitive understanding of the views by using traditional representations of the AF components while also encoding relevant information that the users can obtain insights from.

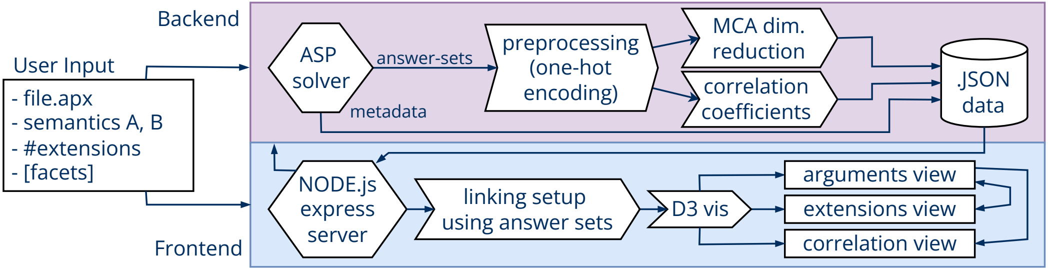
DG-2: Highlight Component Relations. A major challenge is to understand how components affect others. Thus, we aim to make these relations visible through linked interactions to foster understanding of the underlying framework.

DG-3: Maintainable and Customizable. The system design must be flexible and allow incorporation of further components in future iterations.

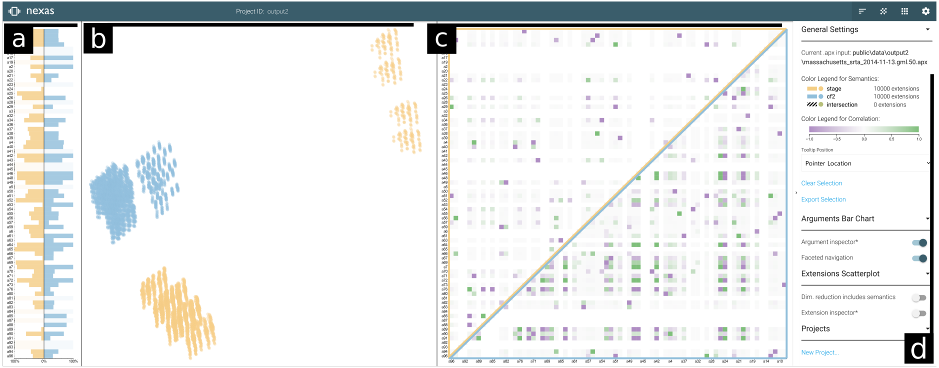
DG-4: Support Several Tasks and Workflows. We aim to support tasks with disjoint purposes and thus the available interactions must reflect such purposes.

DG-5: Ready-to-use. We aim to minimize setup complexity of the tool to account for various user environments.

Technical Design

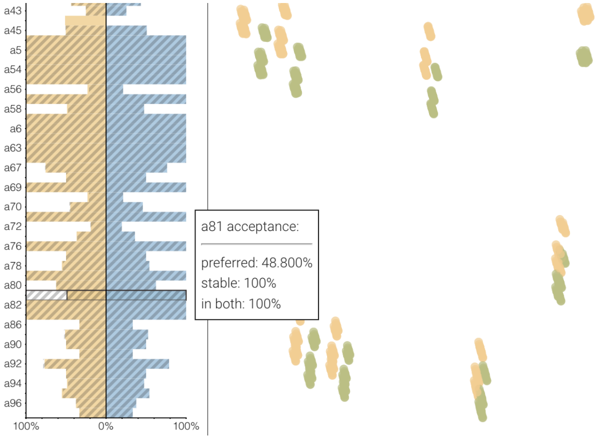


Visualization Design

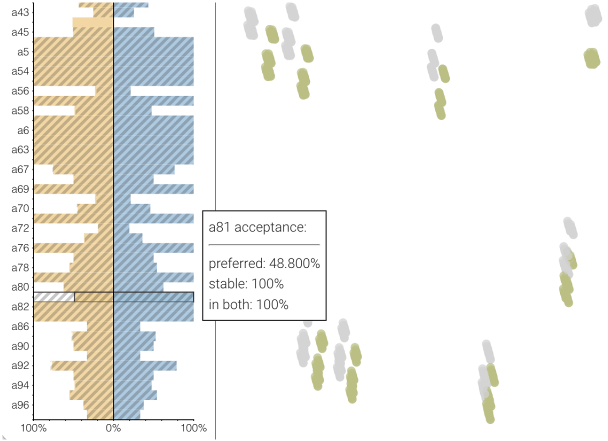


https://imld.de/nexas

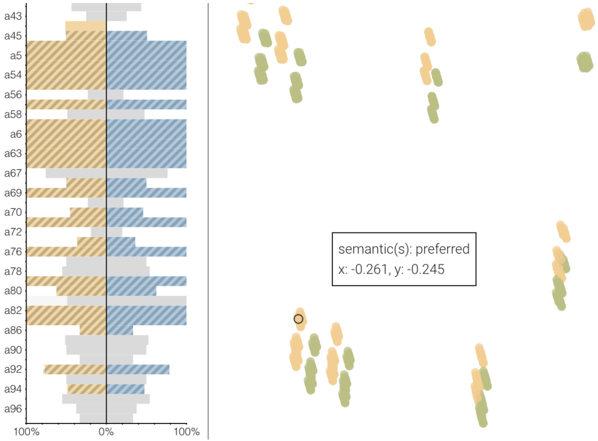
Argument View



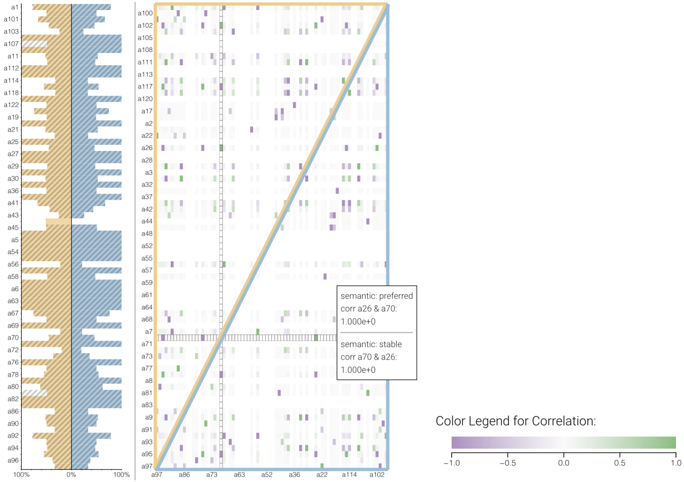
Argument View



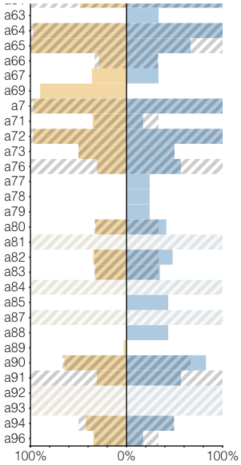
Extension View

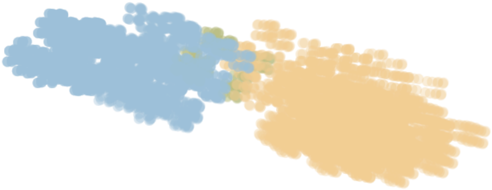


Correlation View

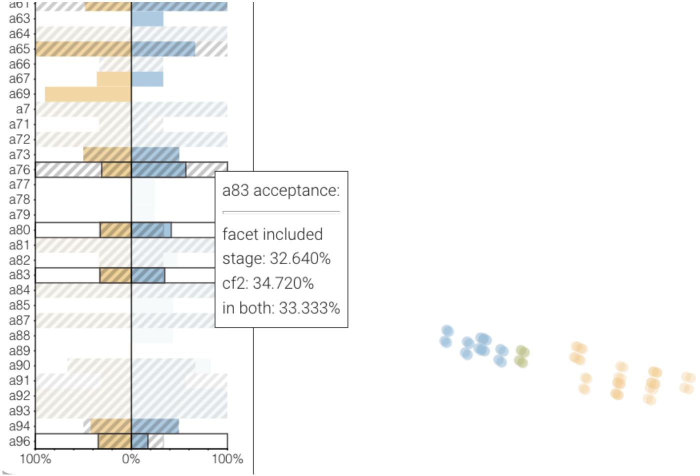


Faceted Navigation





Faceted Navigation



Sum up on Visualization

– Visual approach clearly supports solution space exploration
– Limits are in the representation of large solution spaces, if dimentional reduction should be used
– General: visual representation of large collections of sets is challenging

Summary & Future Work

Summary:

- Weighted faceted navigation allows to quantitatively explore the solution space
- Iterative reworking strategies to comupte diverse answer sets
- Representative answer sets based on the concept of entropy
- Visual exploration of solution space

Future Work:

- Generalize to solution space navigation (not only for ASP relevant)
- Concrete applications
- Generalize visualization approach for answer sets

Thanks to all Collaborators


Elisa Böhl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Markus Krötzsch


Julián Méndez

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