

# AGM Revision in Description Logics Under Fixed-Domain Semantics

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## Abstract

While semantic approaches for revising knowledge bases are fine-grained and independent of the syntactical forms, they are unable to be straightforwardly applied to description logics (DLs) under standard semantics. In this paper, we present a characterization of revision for (finite) knowledge bases in DLs under the fixed-domain semantics, where the domain is fixed and finite. We also introduce an instantiation of a model-based revision operator which satisfies all standard postulates using the notion of distance between interpretations. The model set of the revision result is shown to be expressible in a KB in our setting. In addition, by weakening the KB based on certain domain elements, an individual-based revision operator is provided as an alternative approach.

## Keywords

Belief Revision, Description Logics, Fixed-domain semantics


## 1. Introduction


Description logics (DLs) have been widely used to represent domain knowledge of the world in knowledge bases (KBs). As knowledge bases are not static entities but change over time, it is mandatory to effectively and efficiently manage such changes. One scenario is when a knowledge base has to incorporate new information while maintaining its consistency by performing changes as minimal as possible. This task is known in the literature as knowledge base *revision* and has been massively influenced by the AGM paradigm of Alchourrón, Gärdenfors, and Makinson [1]. A revision operator for knowledge bases is required to satisfy appropriate postulates (called AGM postulates) in order to qualify as a rational revision operator.


Approaches for revising DL knowledge bases are classified into *syntax-based* and *semantic-based* approaches. In syntax-based approaches, the operators directly modify the axioms in the knowledge bases. Existing work on syntactic approaches could not satisfy all AGM postulates [2, 3], considered only semi-revision [4, 5], or proposed additional postulates (different from the AGM's) for capturing the minimality principle [6, 7].


In contrast, semantic-based revision approaches investigate the models of KBs, search for the most plausible set of models to become the revision result, and generate a KB which corresponds to the produced model set. However, it has been shown that in DL with standard semantics, there are two main issues: (1) the models of the knowledge bases can be infinitely many and (2)

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even if we can somehow “compute” the model-based revision, the set of models as the result of the revision may not be expressible by a knowledge base (this is known as the inexpressibility problem [8]). Investigations were carried out to find alternative semantic characterizations [9, 10, 11] or to consider a hybrid approach for lightweight DL families [12]. However, these approaches required a new set of completely translated postulates to be satisfied, rather than the standard postulates for DL knowledge bases.

*Fixed-domain semantics* for DLs has been introduced to accommodate the scenario when the knowledge bases represent constraint-type or configuration problems [13, 14]. In this setting, the domain is explicitly given and thus is finite and fixed *a priori*. A reasoner called `wolpertinger`<sup>1</sup> has been developed to support typical reasoning tasks over knowledge bases under the fixed-domain semantics, which includes satisfiability checking and model enumeration.

In this article, we show the semantic representation theorem for knowledge base revision in DLs under the fixed-domain semantics. Alongside, we present two concrete approaches for revising knowledge bases. The first approach is a semantic-based revision approach, which is inspired from the approach by Katsuno and Mendelzon (KM) [15] for revising KBs in finite-signature propositional logic. We provide a representation theorem characterizing AGM revision operators via appropriate assignments. We also provide a concrete revision operator using the notion of distance between interpretations and show that the proposed operator satisfies all standard AGM postulates for DLs. The models as the outcome of this operation are expressed into a knowledge base using our axiom constructor. The second approach is a novel revision operator based on the notion of exceptional individual set. This individual set serves as a basis to weaken the prior KB whenever inconsistency occurs. The revision result of this approach is a union of the weakened prior KB with the new incoming KB.

The paper is organized as follows. We very briefly recap basic notions in the description logic *SR<sub>OIQ</sub>* in Section 2. In Section 3, we formally introduce the fixed-domain semantics and present an axiom construction from a given set of interpretations. Revision operator and postulates are introduced in Section 4, followed by a semantic characterization of the revision operator in DL under the fixed-domain semantics in Section 5. The instantiations of the revision approaches are presented in Section 6 for the semantic-based approach and in Section 7 for the individual-based approach.

## 2. Description Logics

We assume the readers are familiar with the description logic *SR<sub>OIQ</sub>* (which is the logical counterpart of the standard Web Ontology Language) with its standard syntax and semantics [16, 17]. Let  $N_I$ ,  $N_C$ , and  $N_R$  be finite and pairwise disjoint sets of individual names, concept names, and role names, respectively. Using these entities, *concept expressions* and *axioms* are built according to the standard *SR<sub>OIQ</sub>* constructors. A *SR<sub>OIQ</sub> knowledge base* is a (finite) set of *SR<sub>OIQ</sub>* axioms, which are in the form of ABox, TBox, or RBox axioms.

Given a *SR<sub>OIQ</sub>* knowledge base  $\mathcal{K}$ , we essentially determine the size of  $\mathcal{K}$  by counting the number of symbols it takes to write the knowledge base. We start by inductively defining the

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<sup>1</sup><https://github.com/wolpertinger-reasoner>

**Table 1**  
Size of concepts in a knowledge base

$\text{size}(A)$	$= 1$ for any $A \in N_C$ (including $\top, \perp$ , and <i>Self</i> )
$\text{size}(\{a_1, \dots, a_n\})$	$= n$ for any nominal concept, where $a_1, \dots, a_n \in \Delta$
$\text{size}(C \sqcap D) = \text{size}(C \sqcup D)$	$= 1 + \text{size}(C) + \text{size}(D)$ for any $C, D \in N_C$
$\text{size}(\neg C)$	$= 1 + \text{size}(C)$ for any $C \in N_C$
$\text{size}(\exists r.C) = \text{size}(\forall r.C)$	$= 2 + \text{size}(C)$ for any $C \in N_C$ and any $r \in N_R$
$\text{size}(\leq n r.C)$	$= \text{size}(\geq n r.C) = 2 + \log(n) + \text{size}(C)$ for any $C \in N_C$ and any $r \in N_R$
$\text{size}(r) = \text{size}(r^-)$	$= 1$ for any $r \in N_R$

**Table 2**  
Size of axioms in a knowledge base

$\text{size}(C \sqsubseteq D)$	$= 1 + \text{size}(C) + \text{size}(D)$ for any axiom $C \sqsubseteq D$ in TBox
$\text{size}(r \sqsubseteq s)$	$= 3$ for any $r \sqsubseteq s$ in RBox
$\text{size}(C(a))$	$= 1 + \text{size}(C)$ for any concept assertion in ABox
$\text{size}(r(a, b))$	$= 3$ for any role assertion in ABox
$\text{size}(r_1 \circ \dots \circ r_n \sqsubseteq r_{n+1})$	$= n + 2$ for any role chain axiom in RBox
$\text{size}(Dis(r, s))$	$= 3$ for any role disjointness axiom in RBox

size of *SRIOQ* concepts<sup>2</sup> and axioms as shown in Table 1 and Table 2. Then, the size of  $\mathcal{K}$  is the sum of the size of all axioms in  $\mathcal{K}$ , i.e.  $\text{size}(\mathcal{K}) = \sum_{\alpha \in \mathcal{K}} \text{size}(\alpha)$ .

Now we briefly recall the *SRIOQ* semantics. Let  $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$  be a standard *SRIOQ* interpretation, where  $\Delta^{\mathcal{I}}$  is a non-empty set that is called domain of  $\mathcal{I}$  and  $\cdot^{\mathcal{I}}$  is a function that maps each individual  $a \in N_I$  to an element  $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ , each concept  $C \in N_C$  to a subset of  $\Delta^{\mathcal{I}}$ , and each role name  $r \in N_R$  to a subset of  $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ . We say that  $\mathcal{I}$  *satisfies* a knowledge base  $\mathcal{K}$  (or  $\mathcal{I}$  is a model of  $\mathcal{K}$ ) if it satisfies all axioms of  $\mathcal{K}$ , denoted as  $\mathcal{I} \models \mathcal{K}$ . A knowledge base  $\mathcal{K}$  *entails* an axiom  $\alpha$  if all models of  $\mathcal{K}$  are models of  $\alpha$ . We use  $\mathcal{L}$  to denote the DL language, i.e. the set of all possible DL axioms and  $\Omega$  to denote the set of all interpretations.

For describing belief revision on the semantic level, we endow the interpretation space  $\Omega$  with some structure. In particular, we will employ binary relations  $\preceq$  over  $\Omega$  (formally:  $\preceq \subseteq \Omega \times \Omega$ ), where the intuitive meaning of  $\mathcal{I}_1 \preceq \mathcal{I}_2$  is that  $\mathcal{I}_1$  is “equally good or better” than  $\mathcal{I}_2$  when it comes to serving as a model. We call  $\preceq$  *total* if  $\mathcal{I}_1 \preceq \mathcal{I}_2$  or  $\mathcal{I}_2 \preceq \mathcal{I}_1$  for any  $\mathcal{I}_1, \mathcal{I}_2 \in \Omega$ . We write  $\mathcal{I}_1 \prec \mathcal{I}_2$  as a shorthand, whenever  $\mathcal{I}_1 \preceq \mathcal{I}_2$  and  $\mathcal{I}_2 \not\preceq \mathcal{I}_1$  (the intuition being that  $\mathcal{I}_1$  is “strictly better” than  $\mathcal{I}_2$ ). For a selection  $\Omega' \subseteq \Omega$  of interpretations, an  $\mathcal{I} \in \Omega'$  is called  $\preceq$ -*minimal in*  $\Omega'$  if  $\mathcal{I} \preceq \mathcal{I}'$  for all  $\mathcal{I}' \in \Omega'$ .<sup>3</sup> We let  $\min(\Omega', \preceq)$  denote the set of  $\preceq$ -minimal interpretations in  $\Omega'$ . We call  $\preceq$  a *preorder* if it is transitive and reflexive.

### 3. Fixed-Domain Semantics

Let  $\Delta$  be a non-empty finite set called the fixed domain. An interpretation  $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$  is said to be  $\Delta$ -fixed, if  $\Delta^{\mathcal{I}} = \Delta$  and  $a^{\mathcal{I}} = a$  for all  $a \in \Delta$ . For a DL knowledge base  $\mathcal{K}$ , an interpretation

<sup>2</sup>We assume that the number  $n$  in the qualified number restriction concept is written in binary encoding.

<sup>3</sup>If  $\preceq$  is total, this definition is equivalent to the *absence* of any  $\mathcal{I}'' \in \Omega'$  with  $\mathcal{I}'' \prec \mathcal{I}$ .

$\mathcal{I}$  is a  $\Delta$ -model of  $\mathcal{K}$  ( $\mathcal{I} \models_{\Delta} \mathcal{K}$ ), if  $\mathcal{I}$  is a  $\Delta$ -fixed interpretation and  $\mathcal{I} \models \mathcal{K}$ . A knowledge base  $\mathcal{K}$  is called  $\Delta$ -consistent (or  $\Delta$ -satisfiable) if it has at least one  $\Delta$ -model. A knowledge base  $\mathcal{K}$   $\Delta$ -entails an axiom  $\alpha$  ( $\mathcal{K} \models_{\Delta} \alpha$ ) if  $\mathcal{I} \models \alpha$  for every  $\mathcal{I} \models_{\Delta} \mathcal{K}$ . Two KBs  $\mathcal{K}$  and  $\mathcal{K}'$  are  $\Delta$ -semantically equivalent (written as  $\mathcal{K} \equiv_{\Delta} \mathcal{K}'$ ) iff  $\mathcal{K} \models_{\Delta} \mathcal{K}'$  and  $\mathcal{K}' \models_{\Delta} \mathcal{K}$ . We will just say *consistent*, *entail*, or *equivalent*, and omit the subscript  $\Delta$ , if it is clear from the context. The set of all  $\Delta$ -models of  $\mathcal{K}$  is denoted by  $Mod_{\Delta}(\mathcal{K})$ . Note that we only consider finite knowledge bases in the process of revision.

Fixed-domain semantics can be seen as a further restriction of finite-model reasoning [13]. This approach restricts reasoning to a domain that is known *a priori*. This restriction gives us not only an advantage in terms of computational complexity, but arguably more intuitive models of a knowledge base in some cases (for more about the reasoning complexity, see [18, 14]). Previous studies have provided a practical reasoner [18], SPARQL querying [19], and justification framework [20] under this approach.

We are working with *SRIOQ* knowledge bases under some assumptions on the axiom side. The original definition of *SRIOQ* RBox contains axioms expressing role hierarchy ( $r \sqsubseteq s$ ), role chains ( $r_1 \circ \dots \circ r_n \sqsubseteq r$ ), role disjointness ( $Dis(r, s)$ ), transitivity ( $Tra(r)$ ), symmetry ( $Sym(r)$ ), asymmetry ( $Asy(r)$ ), reflexivity ( $Ref(r)$ ), and irreflexivity ( $Irr(r)$ ). In this article, we will only consider the first three axiom expressions since the remaining forms can be syntactically rewritten into other known axioms:  $Sym(r)$  can be translated as  $r^{-} \sqsubseteq r$ ,  $Asy(r)$  can be expressed as  $Dis(r, r^{-})$ , and  $Tra(r)$  can be rewritten into the role chain axiom  $r \circ r \sqsubseteq r$ . For (ir)reflexivity axioms,  $Ref(r)$  and  $Irr(r)$  can be translated as  $\top \sqsubseteq \exists r.Self$  and  $\top \sqsubseteq \neg \exists r.Self$ , respectively. Moreover, as opposed to the standard *SRIOQ* definition, we do not impose the global restriction called *regularity* since reasoning in KBs with unrestricted role hierarchies is always guaranteed to be decidable under the fixed-domain semantics [13].

In the following, we introduce a method to construct a *SRIOQ* axiom under fixed-domain semantics from a given set of interpretations such that the models of the axiom are exactly the given interpretations. This construction is useful to express the result of our model-based revision approach (see Section 6) into a DL knowledge base and to show that our semantic characterization is indeed compatible with the revision operator (see Definition 4.1). Let  $\{\mathcal{I}_1, \dots, \mathcal{I}_n\} \subseteq \Omega$  be a set of  $\Delta$ -interpretations and  $\mathcal{I}_i \in \{\mathcal{I}_1, \dots, \mathcal{I}_n\}$  be one of the interpretations, we define

$$\begin{aligned} \tau(\mathcal{I}_i) = & \left( \prod_{C \in N_C} \prod_{d \in \Delta \text{ and } d \in C^{\mathcal{I}_i}} \exists u.(\{d\} \sqcap C) \right) \sqcap \left( \prod_{r \in N_R} \prod_{d, e \in \Delta \text{ and } (d, e) \in R^{\mathcal{I}_i}} \exists u.(\{d\} \sqcap \exists r.\{e\}) \right) \sqcap \\ & \left( \prod_{C \in N_C} \prod_{d \in \Delta \text{ and } d \notin C^{\mathcal{I}_i}} \exists u.(\{d\} \sqcap \neg C) \right) \sqcap \left( \prod_{r \in N_R} \prod_{d, e \in \Delta \text{ and } (d, e) \notin r^{\mathcal{I}_i}} \exists u.(\{d\} \sqcap \neg \exists r.\{e\}) \right) \sqcap \\ & \left( \prod_{a \in N_I(\mathcal{K}) \setminus \Delta, d \in \Delta \text{ and } a^{\mathcal{I}_i} = d} \exists u.(\{a\} \sqcap \{d\}) \right), \end{aligned}$$

where  $u$  is the universal role. Then, we construct a *SRIOQ* axiom as follows:

$$form_{\Delta}(\{\mathcal{I}_1, \dots, \mathcal{I}_n\}) = \top \sqsubseteq \bigsqcup_{1 \leq i \leq n} (\tau(\mathcal{I}_i)) \quad (1)$$

**Proposition 3.1.** Let  $\{\mathcal{I}_1, \dots, \mathcal{I}_n\}$  be a set of  $\Delta$ -interpretations.  $\text{Mod}_\Delta(\text{form}_\Delta(\mathcal{I}_1, \dots, \mathcal{I}_n)) = \{\mathcal{I}_1, \dots, \mathcal{I}_n\}$  holds.

## 4. Revision Operator and Postulates

In this article, we use knowledge base revision operators to model multiple revision, which is the process of incorporating multiple new beliefs (axioms) into the present beliefs (axioms) held by an agent, in a consistent way (whenever that is possible). We define revision operators over knowledge bases as follows.

**Definition 4.1** (Revision operator). Let  $\Delta$  be a fixed domain and  $\mathcal{L}$  be the set of all axioms of a given DL. A function  $\circ : \mathcal{P}_{\text{fin}}(\mathcal{L}) \times \mathcal{P}_{\text{fin}}(\mathcal{L}) \rightarrow \mathcal{P}_{\text{fin}}(\mathcal{L})$  is called a (multiple) revision operator.

We consider multiple revision, which is that all given axioms have to be incorporated, i.e. given a knowledge base  $\mathcal{K}$  and new information  $\mathcal{K}'$  (also a knowledge base here), we demand success of revision, i.e.  $\mathcal{K} \circ \mathcal{K}' \models \mathcal{K}'$ . Besides the success condition, the belief change community has brought up and discussed several further requirements for revision operators to make them *rational* (for summaries, see [21, 22]).

We will make use of the AGM postulates for Description Logics [23, 24, 25], which are adapted from a version of Katsuno and Mendelzon postulates for propositional logic with a finite signature [15]:

- (G1)  $\mathcal{K} \circ \mathcal{K}' \models \mathcal{K}'$ .
- (G2) If  $\text{Mod}(\mathcal{K} \cup \mathcal{K}') \neq \emptyset$  then  $\mathcal{K} \circ \mathcal{K}' \equiv \mathcal{K} \cup \mathcal{K}'$ .
- (G3) If  $\text{Mod}(\mathcal{K}') \neq \emptyset$  then  $\text{Mod}(\mathcal{K} \circ \mathcal{K}') \neq \emptyset$ .
- (G4) If  $\mathcal{K}_1 \equiv \mathcal{K}_2$  and  $\mathcal{K}' \equiv \mathcal{K}''$  then  $\mathcal{K}_1 \circ \mathcal{K}' \equiv \mathcal{K}_2 \circ \mathcal{K}''$ .
- (G5)  $(\mathcal{K} \circ \mathcal{K}') \cup \mathcal{K}'' \models \mathcal{K} \circ (\mathcal{K}' \cup \mathcal{K}'')$ .
- (G6) If  $\text{Mod}((\mathcal{K} \circ \mathcal{K}') \cup \mathcal{K}'') \neq \emptyset$  then  $\mathcal{K} \circ (\mathcal{K}' \cup \mathcal{K}'') \models (\mathcal{K} \circ \mathcal{K}') \cup \mathcal{K}''$ .

(G1) guarantees that the newly added belief must be a logical consequence of the result of the revision. (G2) says that if the expansion of  $\mathcal{K}$  by  $\mathcal{K}'$  is consistent, then the result of the revision is equivalent to the expansion of  $\mathcal{K}$  by  $\mathcal{K}'$ . (G3) guarantees the consistency of the revision result if the newly added belief is consistent. (G4) is the principle of the irrelevance of the syntax, stating that the revision operation is independent of the syntactic form of the bases. (G5) and (G6) ensure more careful handling of unions of belief bases. In particular, together, they enforce that  $\mathcal{K} \circ (\mathcal{K}' \cup \mathcal{K}'') \equiv (\mathcal{K} \circ \mathcal{K}') \cup \mathcal{K}''$ , unless  $\mathcal{K}''$  contradicts  $\mathcal{K} \circ \mathcal{K}'$ .

## 5. Semantic Characterizations of Knowledge Base Revision

One central notion for the characterization of revisions is the notion of *faithful assignment*, which was introduced by Katsuno and Mendelzon [15].

**Definition 5.1** (assignment, faithful). Let  $\mathcal{L}$  be a set of all axioms of a given DL. An assignment is a function  $\preceq_{(\cdot)} : \mathcal{P}_{\text{fin}}(\mathcal{L}) \rightarrow \mathcal{P}(\Omega \times \Omega)$  that assigns to each knowledge base  $\mathcal{K}$  a total binary relation  $\preceq_{\mathcal{K}}$  over  $\Omega$ . An assignment  $\preceq_{(\cdot)}$  is called faithful if it satisfies the following conditions for all  $\mathcal{I}, \mathcal{I}' \in \Omega$  and all knowledge bases  $\mathcal{K}$  and  $\mathcal{K}'$ :

- (F1) If  $\mathcal{I}, \mathcal{I}' \models \mathcal{K}$ , then  $\mathcal{I} \prec_{\mathcal{K}} \mathcal{I}'$  does not hold.
- (F2) If  $\mathcal{I} \models \mathcal{K}$  and  $\mathcal{I}' \not\models \mathcal{K}$ , then  $\mathcal{I} \prec_{\mathcal{K}} \mathcal{I}'$ .
- (F3) If  $\mathcal{K} \equiv \mathcal{K}'$ , then  $\preceq_{\mathcal{K}} = \preceq_{\mathcal{K}'}$ .

An assignment  $\preceq_{(\cdot)}$  is a preorder assignment if  $\preceq_{\mathcal{K}}$  is a preorder for every knowledge base  $\mathcal{K}$ .

Intuitively, faithful assignments provide information about which of the two interpretations is “closer to  $\mathcal{K}$ -modelhood”. Consequently, the actual  $\mathcal{K}$ -models are  $\preceq_{\mathcal{K}}$ -minimal. The next definition captures the idea of an assignment adequately representing the behavior of a revision operator.

**Definition 5.2** (compatible). *A revision operator  $\circ$  is called compatible with some assignment  $\preceq_{(\cdot)}$  if  $\text{Mod}(\mathcal{K} \circ \mathcal{K}') = \min(\text{Mod}(\mathcal{K}'), \preceq_{\mathcal{K}})$  for all knowledge bases  $\mathcal{K}$  and  $\mathcal{K}'$ .*

**Theorem 5.3** (Adaptation of Theorem 3.3. in [15]). *Let  $\circ$  be a revision operator for  $SR\text{OIQ}$  under fixed-domain semantics. Then,  $\circ$  satisfies (G1)–(G6) if and only if it is compatible with some faithful preorder assignment.*

*Proof.* The proof is similar to the one of the Representation Theorem by Katsuno and Mendelzon [15, Theorem 3.3.]. For the “if” direction, the arguments are similar and straightforward. For the “only if” direction, we assume the existence of a revision operator  $\circ$  which satisfies postulates (G1)–(G6). Then, for any knowledge base  $\mathcal{K}$ , one can obtain a faithful preorder assignment compatible with  $\circ$  by employing relation encoding  $\preceq_{\mathcal{K}}$  as:  $\mathcal{I} \preceq_{\mathcal{K}} \mathcal{I}'$  if and only if either  $\mathcal{I} \in \text{Mod}(\mathcal{K})$  or  $\mathcal{I} \in \text{Mod}(\mathcal{K} \circ \text{form}_{\Delta}(\mathcal{I}, \mathcal{I}'))$  for any interpretations  $\mathcal{I}$  and  $\mathcal{I}'$ .  $\square$

## 6. Model-based Approach

In this section, we present our first approach to perform model-based revision in the fixed-domain semantics setting. Our concrete revision operator is adapted from Dalal’s operator [26]. The original operator works for two propositional formulas  $\psi$  and  $\mu$ . The *difference set* between their models consists of propositional variables that are interpreted differently by them. Then, the *distance* between them is defined as the minimal cardinality of the *difference sets* between models of  $\psi$  and  $\mu$ . The set of models of revising  $\psi$  by  $\mu$  consists of models of  $\mu$  such that there exists a model of  $\psi$  such that the cardinality of the difference set between the two models is the same as the distance between  $\psi$  and  $\mu$ . In [15], it has been shown that Dalal’s revision operator can be defined as the set of minimal models of  $\mu$  w.r.t a faithful preorder relation  $\preceq_{\psi}$ .

To adapt Dalal’s revision operator to DLs under fixed-domain semantics, we need to define the “difference set” between two models. Thanks to the elements in the domain being finite and known, we can characterize the  $\Delta$ -models of the knowledge bases and then we can define the difference between two  $\Delta$ -models in a similar way as the difference set between two models in propositional logic based on the grounded form of the interpretations.

**Definition 6.1** (Grounded interpretation). *Let  $\mathcal{K}$  be a KB and  $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$  be a  $\Delta$ -fixed interpretation. The ground representation of  $\mathcal{I}$  is the following:  $\text{Gr}(\mathcal{I}) = \{C(d) \mid d \in \Delta \text{ and } d \in C^{\mathcal{I}}\} \cup \{r(d, e) \mid d, e \in \Delta \text{ and } (d, e) \in r^{\mathcal{I}}\} \cup \{a = d \mid a \in N_{\mathcal{I}}(\mathcal{K}), d \in \Delta \text{ and } a^{\mathcal{I}} = d\}$ .*

In the following, we introduce a distance between two models based on the operator of symmetric difference, denoted with  $\oplus$ , which is defined as  $S \oplus S' = (S \cup S') \setminus (S \cap S')$  for any set  $S$  and  $S'$ .

**Definition 6.2** (Distance). *Let  $\mathcal{K}$  be a KB and  $\mathcal{I}$  be a  $\Delta$ -fixed interpretation. The distance between  $\text{Mod}_\Delta(\mathcal{K})$  and  $\mathcal{I}$  is defined as:  $\text{dist}(\text{Mod}_\Delta(\mathcal{K}), \mathcal{I}) = \min_{\mathcal{I}' \in \text{Mod}_\Delta(\mathcal{K})} \text{dist}(\mathcal{I}', \mathcal{I})$ , where  $\text{dist}(\mathcal{I}', \mathcal{I}) = |\text{diff}(\mathcal{I}', \mathcal{I})|$  and  $\text{diff}(\mathcal{I}', \mathcal{I}) = \text{Gr}(\mathcal{I}') \oplus \text{Gr}(\mathcal{I})$ .*

Now we are ready to introduce a *model-based revision operator* for Description Logic under fixed-domain semantics.

**Definition 6.3.** *Let  $\mathcal{K}$  and  $\mathcal{K}'$  be any two knowledge bases. Let  $\preceq_{(\cdot)}^\Delta: \mathcal{K} \mapsto \preceq_{\mathcal{K}}^\Delta$  be an assignment, where the binary relation  $\preceq_{\mathcal{K}}^\Delta$  is defined by letting  $\mathcal{I}_1 \preceq_{\mathcal{K}}^\Delta \mathcal{I}_2$  if and only if  $\text{dist}(\text{Mod}_\Delta(\mathcal{K}), \mathcal{I}_1) \leq \text{dist}(\text{Mod}_\Delta(\mathcal{K}), \mathcal{I}_2)$  for all interpretations  $\mathcal{I}_1$  and  $\mathcal{I}_2$ . We define the model-based revision operator  $\circ_\Delta$  as follows:*

$$\mathcal{K} \circ_\Delta \mathcal{K}' = \{\text{form}_\Delta(\min(\text{Mod}_\Delta(\mathcal{K}'), \preceq_{\mathcal{K}}^\Delta))\}.$$

**Proposition 6.4.** *The model-based change operator  $\circ_\Delta$  satisfies the postulates (G1)-(G6).*

*Proof.* Similar to the assignment presented in [15, Section 4.1.], we have that the assignment  $\preceq_{(\cdot)}^\Delta$  is a faithful preorder assignment. From Proposition 3.1, we obtain  $\text{Mod}_\Delta(\mathcal{K} \circ_\Delta \mathcal{K}') = \min(\text{Mod}_\Delta(\mathcal{K}'), \preceq_{\mathcal{K}}^\Delta)$ , which shows compatibility. Finally from Theorem 5.3, we obtain that  $\circ_\Delta$  satisfies the postulates (G1)-(G6).  $\square$

## 7. Individual-based Approach

In this section, we present the second approach to revise our DL knowledge bases. The main idea is that instead of removing the whole axiom(s) whenever inconsistency occurs, the axioms are *weakened*, that is, modified by adding some exceptions. Different from the previous approach (cf. Section 6) which computes the interpretations, this approach focuses on the elements of the fixed domain. In particular, we will work with sets of exceptional individuals, which serve as a basis to weaken the knowledge base. For the weakening process, we impose the assumption that the knowledge base  $\mathcal{K}$  is free of RBox axioms. This assumption enables simpler weakening steps as we only consider TBox and ABox axioms. To this end, we introduce an equivalent transformation for an arbitrary knowledge base into a KB without RBox axioms. This transformation is possible as we are working with fixed-domain semantics. The idea is to keep the TBox and ABox axioms unchanged and to “partially ground” any RBox axiom into a set of GCIs involving existential restriction with nominal concepts.

**Definition 7.1** (KB transformation). *Let  $\Delta$  be a fixed domain and  $\mathcal{K} = (\mathcal{A}, \mathcal{T}, \mathcal{R})$  be a KB under the fixed-domain semantics, where  $\mathcal{A}$  is an ABox,  $\mathcal{T}$  is a TBox, and  $\mathcal{R}$  is an RBox. The KB transformation is  $\text{trans}_\Delta(\mathcal{K}) = \bigcup_{\alpha \in \mathcal{K}} \text{trans}_\Delta(\alpha)$ , where:*

- $\text{trans}_\Delta(\alpha) = \{\alpha\}$  for any  $\alpha \in \mathcal{T} \cup \mathcal{A}$ .
- $\text{trans}_\Delta(\alpha) = \bigcup_{d \in \Delta} \{\exists r.\{d\} \sqsubseteq \exists s.\{d\}\}$  for any  $\alpha = r \sqsubseteq s \in \mathcal{R}$ .

- $\text{trans}_\Delta(\alpha) = \bigcup_{d \in \Delta} \{\exists r_1 \dots \exists r_n. \{d\} \sqsubseteq \exists r_{(n+1)}. \{d\}\}$  for any  $\alpha = r_1 \circ \dots \circ r_n \sqsubseteq r_{(n+1)} \in \mathcal{R}$ .
- $\text{trans}_\Delta(\alpha) = \bigcup_{d \in \Delta} \{(\exists r. \{d\}) \sqcap (\exists s. \{d\}) \sqsubseteq \perp\}$  for any  $\alpha = \text{Dis}(r, s) \in \mathcal{R}$ .

We observe that the new RBox-free KB is semantically equivalent to the original one.

**Lemma 7.2.** *Let  $\Delta$  be a fixed domain and  $\mathcal{K} = (\mathcal{A}, \mathcal{T}, \mathcal{R})$  be a KB under the fixed-domain semantics and  $\text{trans}(\mathcal{K})$  be the transformation of  $\mathcal{K}$  (cf. Definition 7.1).  $\text{trans}(\mathcal{K}) \equiv_\Delta \mathcal{K}$  holds.*

While preserving the semantics of the original KB  $\mathcal{K}$ , one might notice that the new KB  $\text{trans}_\Delta(\mathcal{K})$  is “bigger” than  $\mathcal{K}$ . Let  $n_{\mathcal{K}}$  be the size of some KB  $\mathcal{K} = (\mathcal{A}, \mathcal{T}, \mathcal{R})$ . Since  $\text{trans}_\Delta(\alpha)$  produces the same axiom for each axiom  $\alpha \in \mathcal{A}$  or  $\alpha \in \mathcal{T}$ , the size of the transformed ABox and TBox are equal to the size of the original ABox and TBox in  $\mathcal{K}$ . For an RBox axiom  $\alpha \in \mathcal{R}$ ,  $\text{trans}_\Delta(\alpha)$  generates  $|\Delta|$  number of transformed axioms. Then, the size of  $\text{trans}_\Delta(\mathcal{K})$  is linearly bounded by  $n_{\mathcal{K}} \times |\Delta|$ .

Given the knowledge base in the transformed form, now we are ready to weaken the axioms in the knowledge base.

**Definition 7.3** (Weakened knowledge base). *Let  $\Delta$  be a fixed domain and  $\mathcal{K}$  be a transformed knowledge base,  $C, D$  be any two concept names,  $r$  be a role name, and  $\Delta' = \{a_1, \dots, a_n\}$  be a set of individual elements with  $\Delta' \subseteq \Delta$ . Consider an axiom  $\sigma \in \mathcal{K}$ :*

- (1) *If  $\sigma$  is a general concept inclusion  $C \sqsubseteq D$ , then the weakened GCI  $\sigma^{-\Delta'}$  w.r.t  $\Delta'$  is  $C \sqcap \neg\{a_1\} \sqcap \dots \sqcap \neg\{a_n\} \sqsubseteq D$ .*
- (2) *If  $\sigma$  is a concept assertion  $C(a_i)$ , then the weakened concept assertion  $\sigma^{-\Delta'}$  w.r.t  $\Delta'$  is  $\top(a_i)$  if  $a_i \in \Delta'$  and  $C(a_i)$  otherwise.*
- (3) *If  $\sigma$  is a role assertion  $r(a, b)$ , then the weakened role assertion  $\sigma^{-\Delta'}$  w.r.t  $\Delta'$  is  $u(a, b)$  if  $a \in \Delta'$ , and  $r(a, b)$  otherwise. The same rule also applies for any inverse role assertion  $r^-(a, b)$ .*

The weakened knowledge base  $\mathcal{K}^{-\Delta'}$  of  $\mathcal{K}$  w.r.t.  $\Delta'$  is  $\mathcal{K}^{-\Delta'} = \{\sigma^{-\Delta'} \mid \sigma \in \mathcal{K}\}$ , i.e., the set of all weakened axioms of  $\mathcal{K}$ .

Definition 7.3 describes the way to weaken any axiom in a KB  $\mathcal{K}$  given the individual set  $\Delta' \subseteq \Delta$ . We note that our definition of weakening is syntax-dependent. For two semantically equivalent knowledge bases, the weakening process might produce two non-equivalent results, even if we weaken both knowledge bases based on the exact same individuals. For instance, let  $\Delta = \{c, d\}$ ,  $\mathcal{K}_1 = \{A \sqsubseteq \forall r. B, A(c), r(c, d)\}$  and  $\mathcal{K}_2 = \{\exists r^-. A \sqsubseteq B, A(c), r^-(d, c)\}$ . It can be checked that  $\mathcal{K}_1 \equiv_\Delta \mathcal{K}_2$ . Suppose we weaken the two KBs w.r.t.  $\Delta' = \{c\}$ , then the results are  $\mathcal{K}_1^{-\Delta'} = \{A \sqcap \neg\{c\} \sqsubseteq \forall r. B, \top(c), u(c, d)\}$  and  $\mathcal{K}_2^{-\Delta'} = \{\exists r^-. A \sqcap \neg\{c\} \sqsubseteq B, \top(c), r^-(d, c)\}$ . Consider a  $\Delta$ -interpretation  $\mathcal{I}$  such that  $A^{\mathcal{I}} = \{c, d\}$ ,  $B^{\mathcal{I}} = \{c\}$ , and  $r^{\mathcal{I}} = \{(c, d)\}$ . We observe that  $\mathcal{I}$  is a model of  $\mathcal{K}_1^{-\Delta'}$ , but it is not a model of  $\mathcal{K}_2^{-\Delta'}$ . This shows that  $\mathcal{K}_1^{-\Delta'}$  and  $\mathcal{K}_2^{-\Delta'}$  are not semantically equivalent. Next, we proceed by defining the notion of an exceptional individual set as follows.

**Definition 7.4** (Exceptional individual set). *Let  $\mathcal{K}$  and  $\mathcal{K}'$  be two knowledge bases. A set of exceptional individuals w.r.t.  $\mathcal{K}$  and  $\mathcal{K}'$  is a set  $Exc \subseteq \Delta$  such that  $\mathcal{K}^{-Exc} \cup \mathcal{K}'$  is consistent. We use  $\mathcal{E}(\mathcal{K}, \mathcal{K}')$  to denote the set of all sets of exceptional individuals w.r.t.  $\mathcal{K}$  and  $\mathcal{K}'$ .*



The following lemma shows that an exceptional individual set always exists w.r.t. any two consistent knowledge bases.

**Lemma 7.5.** *Let  $\Delta = \{a_1, \dots, a_n\}$  be a set of fixed-domain elements. For any two knowledge bases  $\mathcal{K}$  and  $\mathcal{K}'$  which are consistent and in the transformed forms (w.l.o.g), we have  $\mathcal{E}(\mathcal{K}, \mathcal{K}') \neq \emptyset$ .*

We show that our exceptional-individual-based weakening is monotonic in terms of  $\Delta$ -entailment between two weakened knowledge bases.

**Lemma 7.6.** *Let  $\mathcal{K}$  be a knowledge base that is consistent and w.l.o.g in a transformed form. Let  $\Delta_1, \Delta_2 \subseteq \Delta$  be two sets of individuals. If  $\Delta_1 \subseteq \Delta_2$ , then  $\mathcal{K}^{-\Delta_1} \models_{\Delta} \mathcal{K}^{-\Delta_2}$ .*

Using the notion of the exceptional individual set, we present the individual-based revision operator for any two knowledge bases under the fixed-domain semantics. Whenever the incoming KB is inconsistent with the prior KB, the operator chooses one of the minimal exceptional individual sets so that the weakened prior KB is consistent with the incoming one.

**Definition 7.7** (Individual-based Revision). *Let  $\mathcal{K}$  and  $\mathcal{K}'$  be two knowledge bases. An individual-based revision operator is a revision operator  $\circ_{\Delta}^{\pi}$  such that for any knowledge base  $\mathcal{K}$  and  $\mathcal{K}'$ :*

$$\mathcal{K} \circ_{\Delta}^{\pi} \mathcal{K}' = \begin{cases} \text{trans}_{\Delta}(\mathcal{K})^{-\pi(\mathcal{E}(\mathcal{K}, \mathcal{K}'))} \cup \mathcal{K}' & \text{if } \mathcal{K}' \text{ is consistent,} \\ \mathcal{K}' & \text{otherwise,} \end{cases}$$

where  $\pi : \mathcal{P}(\mathcal{P}(\Delta)) \rightarrow \mathcal{P}(\Delta)$  is a selection function retrieving subset-minimal elements, i.e.  $\pi(\mathcal{X}) \in \mathcal{X}$  and there is no  $Y \in \mathcal{X}$  such that  $Y \subset \pi(\mathcal{X})$ .

The result of the revision  $\mathcal{K} \circ_{\Delta}^{\pi} \mathcal{K}'$  is linearly bigger than the inputs  $\mathcal{K}$  and  $\mathcal{K}'$ . The only size change is for the prior KB  $\mathcal{K}$  (i.e. to be transformed and weakened), while  $\mathcal{K}'$  remains unchanged. In the weakening process, the size of the axioms changes only whenever the GCIs are weakened. As  $\pi(\mathcal{E}(\mathcal{K}, \mathcal{K}')) \subseteq \Delta$ , every GCI weakening adds at most  $n_{\Delta}$  negated nominal concepts which represent exceptional individuals, where  $n_{\Delta} = |\Delta|$ . Hence, the size growth from  $\text{trans}_{\Delta}(\mathcal{K})$  to  $\text{trans}_{\Delta}(\mathcal{K})^{-\pi(\mathcal{E}(\mathcal{K}, \mathcal{K}'))}$  is only linearly bounded by  $n_{\Delta}$ . Overall, in the worst case scenario, when we revise an arbitrary knowledge base  $\mathcal{K}$  (with the size of  $n_{\mathcal{K}}$ ) by some KB  $\mathcal{K}'$  (with the size of  $n_{\mathcal{K}'}$ ), the result of the revision  $\mathcal{K} \circ_{\Delta}^{\pi} \mathcal{K}'$  has the size of  $(n_{\mathcal{K}} \times n_{\Delta}^2) + n_{\mathcal{K}'}$ . Note that  $n_{\Delta}^2$  comes from transformation and weakening procedures.

This individual-based revision operator works on the syntactic level by weakening the axioms of the original knowledge base. Recall that the weakening process is syntax-dependent, therefore, this revision operation also depends on the syntax of the knowledge base. For two knowledge bases which are semantically equivalent but syntactically different, there is no guarantee that the revision would result in two equivalent weakened knowledge bases. For instance, assume we have  $\Delta = \{c, d\}$  and two equivalent knowledge bases as previously defined  $\mathcal{K}_1 = \{A \sqsubseteq \forall r.B, A(c), r(c, d)\}$  and  $\mathcal{K}_2 = \{\exists r^{-}.A \sqsubseteq B, A(c), r^{-}(d, c)\}$ . Suppose we want to revise each  $\mathcal{K}_1$  and  $\mathcal{K}_2$  by an incoming KB  $\mathcal{K}_3 = \{\neg B(d)\}$ . Since both union  $\mathcal{K}_1 \cup \mathcal{K}_3$  and  $\mathcal{K}_2 \cup \mathcal{K}_3$  are inconsistent, we search for the minimal set of exceptional individuals that would make the weakened version of the two prior KBs consistent with  $\mathcal{K}_3$ . Then, we find  $\pi(\mathcal{E}(\mathcal{K}_1, \mathcal{K}_3)) = \{c\}$  and  $\pi(\mathcal{E}(\mathcal{K}_2, \mathcal{K}_3)) = \{d\}$ . The result of the revision  $\mathcal{K}_1 \circ_{\Delta}^{\pi} \mathcal{K}_3 = \{A \sqcap \neg\{c\} \sqsubseteq \forall r.B, \neg B(d)\}$ ,

while for the other one  $\mathcal{K}_2 \circ_{\Delta}^{\pi} \mathcal{K}_3 = \{\exists r^{-}.A \sqcap \neg\{d\} \sqsubseteq B, A(c), \neg B(d)\}$ . Hence, we observe that  $\mathcal{K}_1 \circ_{\Delta}^{\pi} \mathcal{K}_3 \not\equiv_{\Delta} \mathcal{K}_2 \circ_{\Delta}^{\pi} \mathcal{K}_3$ . This observation can be considered as a counter example to show that the revision operator  $\circ_{\Delta}^{\pi}$  fails to satisfy postulate (G4) which would guarantee syntax-independence. For the satisfaction of the five remaining postulates, the following proposition shows positive results.

**Proposition 7.8.** *The individual-based change operator  $\circ_{\Delta}^{\pi}$  satisfies postulates (G1)-(G3), (G5), and (G6).*

## 8. Related Work

Syntax-based approaches for revision in DLs directly modify the axioms occurring in the knowledge base. The modification may include dropping axioms [4, 5, 27, 28] or weakening them [2, 3]. However, applying the original AGM postulates [1] to a syntax-based approach for revision in DL is found to have a main issue: while AGM used axiom negation for their syntax-based revision construction, DLs are typically not closed under negation of axioms. Earlier approaches [4, 5] implemented *semi-revision* in the DL family *SHOIN*, where the *consistency* postulate (corresponding to (G3)) and the *success* postulate (corresponding to (G1)) can not be guaranteed simultaneously. Later, Ribeiro and Wasserman [6, 7] introduced alternative constructions for revision in general negation-free logics. However, they did not consider postulates (G5) and (G6) in their representation theorem. Instead, they proposed some special postulates for base change inspired by Hansson [21], namely *core-retainment* and *relevance* to capture the minimal change principle. Our individual-based approach can be regarded as a syntax-based approach since the outcome of the revision is generated by axiom weakening. This approach is in a similar vein to some previous works [29, 30, 31] in the spirit of finding “more general” axioms to accommodate changes, even though those deal primarily with contraction and repairs.

To deal with the possibility of infinitely many models in DL knowledge bases under standard semantics, many studies in semantic-based approaches [32, 33, 34, 9, 10, 11] investigate alternative semantic characterizations for specific DL families. As a consequence, their model-based revision operators work with finitely many “characterized” interpretations. To address the inexpressibility problem, the notion of a *maximal approximation* was introduced to capture the revision result by a knowledge base [35, 9, 11, 12]. A maximal approximation of a result of revision  $\mathcal{K} \circ \mathcal{K}'$  is a new knowledge base  $\mathcal{K}''$  such that  $Mod(\mathcal{K} \circ \mathcal{K}') \subseteq Mod(\mathcal{K}'')$  and there is no other  $\mathcal{K}^*$  with  $Mod(\mathcal{K} \circ \mathcal{K}') \subseteq Mod(\mathcal{K}^*)$  and  $Mod(\mathcal{K}') \subset Mod(\mathcal{K}^*)$ . In our fixed-domain semantics setting, both above issues can be resolved naturally. The most plausible (the minimal) models can be computed as the interpretations are finite and the revised knowledge base can be obtained as these models can be expressed into axioms. Table 3 summarizes the related approaches and compares them with our model-based and individual-based approach.

## 9. Conclusion and Future Work

We have presented two approaches for revising knowledge bases in Description Logics under the fixed-domain semantics, where the models of the knowledge bases are guaranteed to be

**Table 3**

Overview of our approach and comparison with related work.

Approach	Class	DL setting	Postulates	Method for generating result
Qi et al. [2]	syntax-based	$\mathcal{ALC}$	(G1)-(G3), (G5), (G6)	axiom weakening
Aiguier et al. [3]		$\mathcal{ALC}$	(G1)-(G3), (G5), (G6)	axiom relaxation
Halaschek-Wiener et al. [4]		$\mathcal{SHOIN}$	semi-revision postulates	axiom removal
Ribeiro and Wassermann [5, 27]		$\mathcal{SROIQ}$	semi-revision postulates	axiom removal
Zheleznyakov et al. [28]		DL-Lite	customized postulates	axiom removal
Our individual-based approach		$\mathcal{SROIQ}$ ; fixed domain	(G1)-(G3), (G5), (G6)	axiom weakening
Wang et al. [32, 9]	semantic-based	DL-Lite $_{bool}^N$	(G1)-(G5)	distance between features; approximation
Chang et al. [33]		$\mathcal{EL}_{\perp}$	(G1)-(G5)	graph-based justification; axiom removal
Zhuang et al. [34, 10]		DL-Lite $_{core}$	customized postulates	type-based axiom removal
Dong et al. [11]		$\mathcal{SHIQ}$	customized postulates	distance between completion graphs; approximation
Our model-based approach	$\mathcal{SROIQ}$ ; fixed domain	(G1)-(G6)	distance between models; direct axiom construction from models (c.f. Equation (1))	

finite. For our model-based approach, we provided an axiom construction from a given set of interpretations where the axiom’s models are exactly the given interpretation set. We adapted KM’s semantic approach and provided a representation theorem for AGM revision operators in  $\mathcal{SROIQ}$  under fixed-domain semantics, as well as a concrete model-based revision operation using the notion of distance. The second approach is a novel revision technique for this particular DL by axiom weakening based on exceptional individual sets.

For future work, we want to find a new axiom construction from a given set of  $\Delta$ -interpretations, as the current construction is arguably rather technical (cf. Equation (1)). In addition, we also plan to implement both revision approaches in ASP (Answer Set Programming) and evaluate their performances, following the use of ASP in earlier work for reasoning [18] and justification [20] in DL under the fixed-domain semantics.

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## References

- [1] C. E. Alchourrón, P. Gardenfors, D. Makinson, On the logic of theory change: Partial meet contraction and revision functions, *Journal of Symbolic Logic* 50 (1985) 510–530. doi:10.2307/2274239.
- [2] G. Qi, W. Liu, D. A. Bell, Knowledge base revision in description logics, in: M. Fisher, W. van der Hoek, B. Konev, A. Lisitsa (Eds.), *Logics in Artificial Intelligence*, Springer Berlin Heidelberg, 2006, pp. 386–398.
- [3] M. Aiguier, J. Atif, I. Bloch, C. Hudelot, Belief revision, minimal change and relaxation: A general framework based on satisfaction systems, and applications to description logics, *Artificial Intelligence* 256 (2018) 160–180. URL: <http://www.sciencedirect.com/science/article/pii/S0004370217301686>. doi:<https://doi.org/10.1016/j.artint.2017.12.002>.
- [4] C. Halaschek-Wiener, Y. Katz, B. Parsia, Belief base revision for expressive description logics, *CEUR Workshop Proceedings* 216 (2006) 1–10.
- [5] M. M. Ribeiro, R. Wassermann, Base revision for ontology debugging, *Journal of Logic and Computation* 19 (2009) 721–743. doi:10.1093/logcom/exn048.
- [6] M. M. Ribeiro, R. Wassermann, AGM Revision in Description Logics, *Proceedings the IJCAI Workshop on Automated Reasoning about Context and Ontology Evolution (ARCOE)* (2009).
- [7] M. M. Ribeiro, R. Wassermann, Minimal change in AGM revision for non-classical logics, in: C. Baral, G. D. Giacomo, T. Eiter (Eds.), *Principles of Knowledge Representation and Reasoning: Proceedings of the Fourteenth International Conference, KR 2014, Vienna, Austria, July 20-24, 2014*, AAAI Press, 2014. URL: <http://www.aaai.org/ocs/index.php/KR/KR14/paper/view/8008>.
- [8] H. Liu, C. Lutz, M. Milicic, F. Wolter, Updating description logic aboxes, in: P. Doherty, J. Mylopoulos, C. A. Welty (Eds.), *Proceedings, Tenth International Conference on Principles of Knowledge Representation and Reasoning, Lake District of the United Kingdom, June 2-5, 2006*, AAAI Press, 2006, pp. 46–56. URL: <http://www.aaai.org/Library/KR/2006/kr06-008.php>.
- [9] Z. Wang, K. Wang, R. W. Topor, DI-lite ontology revision based on an alternative semantic characterization, *ACM Trans. Comput. Log.* 16 (2015) 31:1–31:37. URL: <https://doi.org/10.1145/2786759>. doi:10.1145/2786759.
- [10] Z. Zhuang, Z. Wang, K. Wang, G. Qi, Contraction and revision over dl-lite tboxes, in: C. E. Brodley, P. Stone (Eds.), *Proceedings of the Twenty-Eighth AAAI Conference on Artificial Intelligence, July 27 -31, 2014, Québec City, Québec, Canada*, AAAI Press, 2014, pp. 1149–1156. URL: <http://www.aaai.org/ocs/index.php/AAAI/AAAI14/paper/view/8510>.
- [11] T. Dong, C. L. Duc, M. Lamolle, Tableau-based revision for expressive description logics with individuals, *Journal of Web Semantics* 45 (2017) 63–79. URL: <http://www.sciencedirect.com/science/article/pii/S1570826817300331>. doi:<https://doi.org/10.1016/j.websem.2017.09.001>.
- [12] D. Zheleznyakov, E. Kharlamov, W. Nutt, D. Calvanese, On expansion and contraction of dl-lite knowledge bases, *J. Web Semant.* 57 (2019). URL: <https://doi.org/10.1016/j.websem.2018.12.002>. doi:10.1016/j.websem.2018.12.002.

- [13] S. A. Gaggl, S. Rudolph, L. Schweizer, Fixed-domain reasoning for description logics, in: G. A. Kaminka, M. Fox, P. Bouquet, E. Hüllermeier, V. Dignum, F. Dignum, F. van Harmelen (Eds.), Proceedings of the 22nd European Conference on Artificial Intelligence (ECAI 2016), volume 285 of *Frontiers in Artificial Intelligence and Applications*, IOS Press, 2016, pp. 819–827.
- [14] S. Rudolph, L. Schweizer, Not too big, not too small... complexities of fixed-domain reasoning in first-order and description logics, in: A. Artale, B. Glimm, R. Kontchakov (Eds.), Proceedings of the 30th International Workshop on Description Logics, Montpellier, France, July 18–21, 2017, volume 1879 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2017. URL: <http://ceur-ws.org/Vol-1879/paper28.pdf>.
- [15] H. Katsuno, A. O. Mendelzon, Propositional knowledge base revision and minimal change, *Artificial Intelligence* 52 (1991) 263–294. doi:[https://doi.org/10.1016/0004-3702\(91\)90069-V](https://doi.org/10.1016/0004-3702(91)90069-V).
- [16] I. Horrocks, O. Kutz, U. Sattler, The even more irresistible sroiq, in: Proceedings of the Tenth International Conference on Principles of Knowledge Representation and Reasoning, KR’06, AAAI Press, 2006, p. 57–67.
- [17] S. Rudolph, Foundations of description logics, in: A. Polleres, C. d’Amato, M. Arenas, S. Handschuh, P. Kroner, S. Ossowski, P. F. Patel-Schneider (Eds.), Reasoning Web. Semantic Technologies for the Web of Data - 7th International Summer School 2011, Galway, Ireland, August 23–27, 2011, Tutorial Lectures, volume 6848 of *Lecture Notes in Computer Science*, Springer, 2011, pp. 76–136. URL: [https://doi.org/10.1007/978-3-642-23032-5\\_2](https://doi.org/10.1007/978-3-642-23032-5_2). doi:10.1007/978-3-642-23032-5\_2.
- [18] S. Rudolph, L. Schweizer, S. Tirtarasa, Wolpertinger: A fixed-domain reasoner, in: N. Nikitina, D. Song (Eds.), ISWC 2017 Posters & Demonstration Track, 2017.
- [19] S. Rudolph, L. Schweizer, Z. Yao, Sparql queries over ontologies under the fixed-domain semantics, in: A. C. Nayak, A. Sharma (Eds.), PRICAI 2019: Trends in Artificial Intelligence, Springer International Publishing, Cham, 2019, pp. 486–499.
- [20] S. Rudolph, L. Schweizer, S. Tirtarasa, Justifications for description logic knowledge bases under the fixed-domain semantics, in: C. Benz Müller, F. Ricca, X. Parent, D. Roman (Eds.), Rules and Reasoning - Second International Joint Conference, RuleML+RR 2018, Luxembourg, September 18–21, 2018, Proceedings, volume 11092 of *Lecture Notes in Computer Science*, Springer, 2018, pp. 185–200. URL: [https://doi.org/10.1007/978-3-319-99906-7\\_12](https://doi.org/10.1007/978-3-319-99906-7_12).
- [21] S. O. Hansson, A Textbook of Belief Dynamics: Theory Change and Database Updating, Springer, 1999.
- [22] E. L. Fermé, S. O. Hansson, Belief Change - Introduction and Overview, Springer Briefs in Intelligent Systems, Springer, 2018.
- [23] G. Qi, W. Liu, D. A. Bell, Knowledge base revision in description logics, in: M. Fisher, W. van der Hoek, B. Konev, A. Lisitsa (Eds.), Proceedings of the 10th European Conference of Logics in Artificial Intelligence (JELIA 2006), volume 4160 of *LNCS*, Springer, 2006, pp. 386–398. URL: [https://doi.org/10.1007/11853886\\_32](https://doi.org/10.1007/11853886_32). doi:10.1007/11853886\_32.
- [24] G. Qi, F. Yang, A survey of revision approaches in description logics, in: International Conference on Web Reasoning and Rule Systems, Springer, 2008, pp. 74–88.
- [25] G. Qi, J. Du, Model-based revision operators for terminologies in description logics., in: IJCAI, 2009, pp. 891–897.

- [26] M. Dalal, Investigations into a theory of knowledge base revision : Preliminary report, in: AAI, 1988.
- [27] M. M. Ribeiro, R. Wassermann, The ontology reviser plug-in for protégé, in: F. L. G. de Freitas, H. Stuckenschmidt, H. S. Pinto, A. Malucelli, Ó. Corcho (Eds.), Proceedings of the 3rd Workshop on Ontologies and their Applications, Salvador, Bahia, Brazil, October 26, 2008, volume 427 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2008. URL: <http://ceur-ws.org/Vol-427/paper1.pdf>.
- [28] D. Zheleznyakov, E. Kharlamov, W. Nutt, D. Calvanese, On expansion and contraction of dl-lite knowledge bases, *J. Web Semant.* 57 (2019). URL: <https://doi.org/10.1016/j.websem.2018.12.002>. doi:10.1016/j.websem.2018.12.002.
- [29] V. B. Matos, R. Guimarães, Y. D. Santos, R. Wassermann, Pseudo-contractions as gentle repairs, in: C. Lutz, U. Sattler, C. Tinelli, A. Turhan, F. Wolter (Eds.), *Description Logic, Theory Combination, and All That - Essays Dedicated to Franz Baader on the Occasion of His 60th Birthday*, volume 11560 of *Lecture Notes in Computer Science*, Springer, 2019, pp. 385–403. URL: [https://doi.org/10.1007/978-3-030-22102-7\\_18](https://doi.org/10.1007/978-3-030-22102-7_18). doi:10.1007/978-3-030-22102-7\_18.
- [30] F. Baader, F. Kriegel, A. Nuradiansyah, R. Peñaloza, Making repairs in description logics more gentle, in: M. Thielscher, F. Toni, F. Wolter (Eds.), *Principles of Knowledge Representation and Reasoning: Proceedings of the Sixteenth International Conference, KR 2018, Tempe, Arizona, 30 October - 2 November 2018*, AAAI Press, 2018, pp. 319–328. URL: <https://aaai.org/ocs/index.php/KR/KR18/paper/view/18056>.
- [31] N. Troquard, R. Confalonieri, P. Galliani, R. Peñaloza, D. Porello, O. Kutz, Repairing ontologies via axiom weakening, in: S. A. McIlraith, K. Q. Weinberger (Eds.), *Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence, (AAAI-18), the 30th innovative Applications of Artificial Intelligence (IAAI-18), and the 8th AAAI Symposium on Educational Advances in Artificial Intelligence (EAAI-18)*, New Orleans, Louisiana, USA, February 2-7, 2018, AAAI Press, 2018, pp. 1981–1988. URL: <https://www.aaai.org/ocs/index.php/AAAI/AAAI18/paper/view/17189>.
- [32] Z. Wang, K. Wang, R. W. Topor, A new approach to knowledge base revision in dl-lite, in: M. Fox, D. Poole (Eds.), *Proceedings of the Twenty-Fourth AAAI Conference on Artificial Intelligence, AAAI 2010, Atlanta, Georgia, USA, July 11-15, 2010*, AAAI Press, 2010. URL: <http://www.aaai.org/ocs/index.php/AAAI/AAAI10/paper/view/1786>.
- [33] L. Chang, U. Sattler, T. Gu, An abox revision algorithm for the description logic el\_bot, in: M. Bienvenu, M. Ortiz, R. Rosati, M. Simkus (Eds.), *Informal Proceedings of the 27th International Workshop on Description Logics, Vienna, Austria, July 17-20, 2014*, volume 1193 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2014, pp. 459–470. URL: [http://ceur-ws.org/Vol-1193/paper\\_64.pdf](http://ceur-ws.org/Vol-1193/paper_64.pdf).
- [34] Z. Wang, K. Wang, G. Qi, Z. Zhuang, Y. Li, Instance-driven tbox revision in dl-lite, in: M. Bienvenu, M. Ortiz, R. Rosati, M. Simkus (Eds.), *Informal Proceedings of the 27th International Workshop on Description Logics, Vienna, Austria, July 17-20, 2014*, volume 1193 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2014, pp. 734–745. URL: [http://ceur-ws.org/Vol-1193/paper\\_39.pdf](http://ceur-ws.org/Vol-1193/paper_39.pdf).
- [35] G. D. Giacomo, M. Lenzerini, A. Poggi, R. Rosati, On the update of description logic ontologies at the instance level, in: *Proceedings, The Twenty-First National Conference on*

Artificial Intelligence and the Eighteenth Innovative Applications of Artificial Intelligence Conference, July 16-20, 2006, Boston, Massachusetts, USA, AAAI Press, 2006, pp. 1271–1276.  
URL: <http://www.aaai.org/Library/AAAI/2006/aaai06-199.php>.