Foundations of Semantic Web Technologies

Tutorial 4

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Exercise 4.1. Transform the following concepts into negation normal form:

(a) \( \neg \left(A \sqcap \forall r.B\right)\)

(b) \( \neg \forall r.\exists s.\neg B \sqcup \exists r.A\)

(c) \( \neg \left(\neg (A \sqcap \exists r.\top) \sqcup \ni 3 s.\left(A \sqcup \neg B\right)\right)\)

Exercise 4.2. Apply the tableau algorithm in order to check if the axiom \( A \sqsubseteq B \) is a logical consequence of the TBox \( \{\neg C \sqsubseteq B, A \sqcap C \sqsubseteq \bot\} \).

Exercise 4.3. Apply the tableau algorithm in order to check satisfiability of the concept \( A \sqcap \forall r.B \) w.r.t. the TBox \( \{A \sqsubseteq \exists r.A, B \sqsubseteq \exists r^-\cdot C, C \sqsubseteq \forall r.\forall r.B\} \).

Exercise 4.4. Markus wants to apply the tableau algorithm for checking the satisfiability of the concept \( B \sqcap \exists r^-\cdot A \) w.r.t. the TBox \( \{A \sqsubseteq \exists r^-\cdot A \sqcap \exists r.B, \top \sqsubseteq \leq 1 r\} \). He arrives at the situation depicted below and concludes that no further rules are applicable, since \( v_2 \) is blocked by \( v_1 \). What is Markus’ error? Continue the algorithm until its termination. (You don’t have to illustrate all intermediate steps, just provide the final state.)

\[
\begin{align*}
v_0 & \downarrow r^- \\
v_1 & \downarrow r^- \\
v_2 & \\
L(v_0) &= \{B \sqcap \exists r^-\cdot A, B, \exists r^-\cdot A, C_T, \neg A, \leq 1 r\} \\
L(v_1) &= \{A, C_T, \exists r^-\cdot A, \exists r.B, \leq 1 r\} \\
L(v_2) &= \{A, C_T, \exists r^-\cdot A, \exists r.B, \leq 1 r\}.
\end{align*}
\]

Exercise 4.5. Extend the \( \leq 1 \) rule in a way that also qualified functionality axioms of the form \( \top \sqsubseteq \leq 1 r.A \) can be treated correctly, where \( A \) is an atomic concept. Can you also treat arbitrary axioms of the form \( C \sqsubseteq \leq 1 r.D \)?