Exercise 4 — Algebraic Properties of Bisimilarity

Let $Names$ be a set of names and derive the set of actions by $\mathsf{Act} = Names \cup \overline{Names} \cup \{\tau\}$ ($\tau \notin Names \cup \overline{Names}$). The process calculus $\mathsf{CCS}(\mathsf{Act}, \mathcal{K}, \mathcal{T}_{\mathcal{K}})$ is the smallest set of processes Pr derived from the following grammar.

$$P ::= \mathbf{0} \ | \ \mu.P \ | \ P \,|\, P \ | \ P + P \ | \ \boldsymbol{\nu} a\, P \ | \ K$$

where $\mu \in \mathsf{Act}$, $a \in Names$, and $K \in \mathcal{K}$. The semantics of $\mathsf{CCS}(\mathsf{Act}, \mathcal{K}, \mathcal{T}_{\mathcal{K}})$ is given as the smallest transition relation $\longrightarrow \subseteq \mathsf{Pr} \times \mathsf{Act} \times \mathsf{Pr}$ satisfying the rules below and $\mathcal{T}_{\mathcal{K}}$ (i.e., $\mathcal{T}_{\mathcal{K}} \subseteq \longrightarrow$).

$$\textsc{Pre}\ \frac{}{\mu.P \xrightarrow{\mu} P}$$

$$\textsc{Sum}_\mathrm{L}\ \frac{P \xrightarrow{\mu} P'}{P + Q \xrightarrow{\mu} P'} \qquad \textsc{Sum}_\mathrm{R}\ \frac{Q \xrightarrow{\mu} Q'}{P + Q \xrightarrow{\mu} Q'} \qquad \textsc{Res}\ \frac{P \xrightarrow{\mu} P'}{\boldsymbol{\nu} a\, P \xrightarrow{\mu} \boldsymbol{\nu} a\, P'}\ \mu \notin \{a, \overline{a}\}$$

$$\textsc{Par}_\mathrm{L}\ \frac{P \xrightarrow{\mu} P'}{P \,|\, Q \xrightarrow{\mu} P' \,|\, Q} \qquad \textsc{Par}_\mathrm{R}\ \frac{Q \xrightarrow{\mu} Q'}{P \,|\, Q \xrightarrow{\mu} P \,|\, Q'} \qquad \textsc{Comm}\ \frac{P \xrightarrow{\mu} P' \quad Q \xrightarrow{\overline{\mu}} Q'}{P \,|\, Q \xrightarrow{\tau} P' \,|\, Q'}\ \tau \notin \{\mu, \overline{\mu}\}$$

Task 1. — Some Algebraic Properties of Bisimilarity

The language $\mathsf{CCS}(\mathsf{Act}, \emptyset, \emptyset)$ is often referred to as finCCS.

a) Show that $|$ is not idempotent.

b) Given an example process P with $P \not\simeq \mathbf{0}$ for which $P \,|\, P \simeq P$ holds. Can P be a *finCCS* process?

c) Does it hold that $\mu.(P + Q) \simeq \mu.P + \mu.Q$? What about $(P + Q) \,|\, R \simeq (P \,|\, R) + (Q \,|\, R)$?

Task 2. — The Relabeling Operator

Milner introduced the additional process operator of relabeling *to CCS. A function $f : \mathsf{Act} \to \mathsf{Act}$ is called a* relabeling *if $f(\overline{\alpha}) = \overline{f(\alpha)}$ and $f(\tau) = \tau$. Process $P \in \mathsf{CCS}$ under relabeling f is the process $P[f]$ and its semantics is given be the following rule.*

$$\textsc{Rel}\ \frac{P \xrightarrow{\mu} P'}{P[f] \xrightarrow{f(\mu)} P'[f]}$$

In contrast, a name substitution *$\sigma : Names \to Names$ applied to process P replaces every occurrence of name $a \in Names$ by $\sigma(a)$. Consequently, $\sigma(\overline{a}) = \overline{\sigma(a)}$.*

a) Show that $\simeq$ is preserved by relabeling.

b) Show that $(P + Q)[f] = P[f] + Q[f]$.

c) Is it true that $(P \,|\, Q)[f] \simeq P[f] \,|\, Q[f]$? If not, impose conditions on f so that the property holds.

d) Same question for $(\boldsymbol{\nu} a\, P)[f] \simeq \boldsymbol{\nu} a\, P[f]$.

e) Consider relabeling function $f = \{b \mapsto a\}$ (identity on all other actions; note, $f\left(\overline{b}\right) = \overline{a}$) and name substitution $\sigma = \{b \mapsto a\}$. Show that for process $P \overset{\text{def}}{=} a \mid \overline{b}$, $P[f]$ is not bisimilar to $P\sigma = a \mid \overline{a}$.

f) Show that bisimilarity is not preserved by substitution. Therefore, find two *finCCS* processes being with no restriction, becoming differnt under name substitution.

Task 3. — CSP-like Parallel Composition

Let $A \subseteq Names$. Process $P \parallel_A Q$ is defined as

$$P \parallel_A Q \overset{\text{def}}{=} \boldsymbol{\nu a}\,(P \mid Q)$$

where $\boldsymbol{a}$ is the sequence of all names in A.

a) Show that $\parallel_A$ is commutative but not associative.

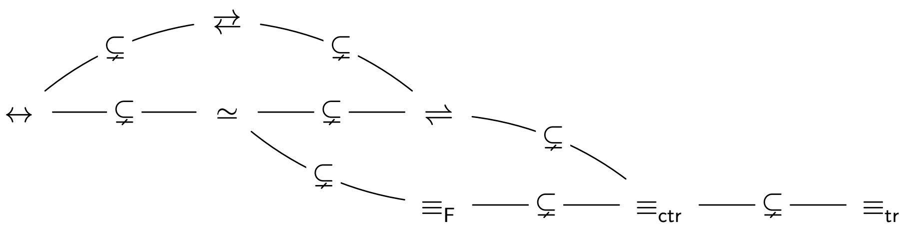

Figure 1: Our Linear-Time Branching-Time Spectrum

Task 4. — Congruence Properties in the Spectrum

Reconsider our spectrum in Figure 1 for compositionality and check, which of the equivalences is a congruence for CCS.