TU Dresden, Fakultät Informatik
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Complexity Theory

Exercise 11: Randomised Computation and Quantum Computing

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Exercise 11.1. Let $0 < p < 1$ and let $(X_i \mid i \in \mathbb{N})$ be a sequence of independent random variables $X_i \colon \Omega_i \to \{0, 1\}$ such that $P(X_i = 1) = p$ for all $i \in \mathbb{N}$. Describe a way how to transform the sequence $(X_i \mid i \in \mathbb{N})$ into a sequence $(Y_i \mid i \in \mathbb{N})$ such that $P(Y_i = 1) = P(Y_i = 0) = 1/2$. The construction may have a zero probability to fail.

Exercise 11.2. Consider the following alternative definition of ZPP:
A language **L** is in ZPP if and only if there exists some polynomial time PTM $\mathcal{M}$ that answers Accept (A), Reject (R), or Inconclusive (I), and all of the following hold.

- For all $w \in \mathbf{L}$, $\mathcal{M}$ always returns A or I.
- For all $w \notin \mathbf{L}$, $\mathcal{M}$ always returns R or I.
- For all $w \in \Sigma^*$, $\Pr[\mathcal{M}(w) = \mathsf{I}] < \frac{1}{2}$.

Prove that the alternative definition is equivalent to the one given in the lecture.

Exercise 11.3. Prove Theorem 23.7 (see slide 18 of lecture 23).

Exercise 11.4. Let **UPath** be the set of all tuples $\langle G, s, t \rangle$ with G an undirected graph, and s and t are two connected vertices in G. Show that **UPath** $\in \mathrm{RL}$ using the following result.

Theorem. Let G be some undirected graph and let s and t be some vertices in G. If s is connected to t, then the expected number of steps it takes for a random walk from s to hit t is at most $10n^4$.

Exercise 11.5. Review the slides from Lecture 24, and especially slides 13–18 on calculating the odds in a quantum-based game strategy.

1. Can Bob use the experiment to learn about Alice's choice, possibly by repeating the experiment with many pairs of entangled particles? Compute the chances of Bob emasuring 0 in each case considered on the slides (no rotation, only Alice rotates, both rotate).
2. Suppose Alice measures her bit without any rotation happening before. Specify the possible states of Bob's remaining 1-qubit system after this.
3. Consider the case $x = 0$ and $y - 1$, and the following order of events: Alice measures, Bob rotates, Bob measures. What is the probability of the game being won in this case.