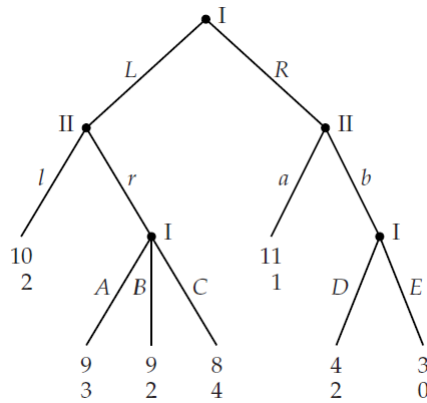


Algorithmic Game Theory

Problems 3

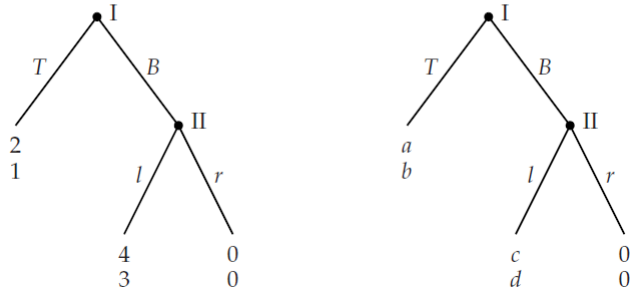
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Problem 1. Consider the following game tree. Note that the top payoffs at a leaf are for player I and bottom payoffs are for player II.



- (a) What is the number of strategies of player I and of player II?
- (b) How many reduced strategies do they have?
- (c) Give the reduced strategic form of the game.
- (d) What are the equilibria of the game in reduced strategies?
- (e) What are the subgame-perfect equilibria of the game?

Problem 2. Consider the following game trees.



- (a) Find all equilibria for the game tree on the left. Which of these are subgame-perfect?
- (b) In the game tree on the right, the payoffs a, b, c, d are positive real numbers. For each of the following statements (i), (ii), (iii), decide if it is true or false, justifying your answer with an argument or counterexample.
- (i) the game always has a subgame-perfect equilibrium (SPE);
 - (ii) the payoff to player II in any SPE is always at least as high as her payoff in any equilibrium;
 - (iii) the payoff to player I in any SPE is always at least as high as his payoff in any equilibrium.

Problem 3. In 1981, Robert Rosenthal introduced the so-called **centipede game**. The game in extensive form can be described as follows

- At stage 1, player I chooses between move **R** and **D**.
 - If she chooses **D**, player 1 gets 1 and player 2 gets 0;
 - If she chooses **R**, the game goes to round 2.
- At stage 2, player 2 chooses between **r** and **d**.
 - If he chooses **d**, player 1 gets 0 and player 2 gets 2;
 - If he chooses **r**, the game moves to round 3.
- At stage 3, player 1 chooses between **R** and **D**.
 - If she chooses **D**, player 1 gets 3 and player 2 gets 1;
 - If she chooses **R**, the game moves to round 4.
- At stage 4, player 2 chooses between **r** and **d**.
 - If he chooses **d**, player 1 gets 2 and player 2 gets 4.
 - If he chooses **r**, both players get 3.

Do the following

- Draw the tree representation of the game.
- Apply backward induction and find its outcome.
- Give the pure strategies of both players and the payoff matrix of the normal form of the game.
- Find all (sub-game perfect) Nash equilibria.

After its introduction, the **centipede game** has been extensively studied in experiments. What do you think: Were the people taking part in the experiments more inclined to play the Nash equilibrium or to cooperate?

Problem 4. Argue that in any sequential game, the backward induction strategy profile is in fact a Nash equilibrium.