TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic ∴ Group

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Knowledge Representation and Reasoning

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Exercises 4

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Problem 1.
Write down the following:

1. An unsatisfiable $\mathcal{ALC}$-knowledge base $\mathcal{K}$ whose TBox is satisfiable and whose ABox contains only role assertions.
2. An unsatisfiable $\mathcal{ALC}$-knowledge base whose TBox is empty.
3. An unsatisfiable $\mathcal{ALC}$-TBox.
4. A satisfiable $\mathcal{ALC}$-TBox $\mathcal{T}$ such that all the atomic concepts occurring in $\mathcal{T}$ are unsatisfiable w.r.t. $\mathcal{T}$. Write down a model of $\mathcal{T}$.
5. A satisfiable $\mathcal{ALC}$-knowledge base such that all its models contain at least two domain individuals.

Hint: For this exercise, the top concept ($\top$) and the bottom concept ($\bot$) are of great help. In the first problem, for instance, you could choose as satisfiable TBox:

$$\mathcal{T} \quad = \quad \{\exists R.\top \sqsubseteq \bot\}$$

Problem 2.
Which of the following statements is true?

1. $A \sqcap \neg A$ is satisfiable.
2. $A \sqcup \neg A$ is satisfiable.
3. $A \sqcap \exists r.\mathsf{B} \sqcap \exists r.\neg \mathsf{B}$ is satisfiable.
4. $A \sqcap \exists r.\mathsf{B} \sqcap \forall s.\neg \mathsf{B}$ is satisfiable.
5. $A \sqcap \exists r.\mathsf{B} \sqcap \forall r.\neg \mathsf{B}$ is satisfiable.
6. $A \sqcap \neg A$ is subsumed by B.
7. B is subsumed by $A \sqcup \neg A$.
8. $A \sqcap \exists r.\mathsf{B}$ is subsumed by $\mathsf{A} \sqcap \exists r.\top$.
9. $A \sqcap \forall r.\mathsf{B}$ is subsumed by $\mathsf{A} \sqcap \exists r.\mathsf{B}$.

10. $A \sqcap \exists r.\mathrm{B}$ is subsumed by $\mathrm{A} \sqcap \forall r.\mathrm{B}$.

Problem 3.
Extend the knowledge base you built in Problem 2 from the previous Exercise Sheet to capture the following statements. You may use additional features such as number restrictions if needed; you may also need more than one axiom for some of the statements.

1. Cars have between three and four wheels.
2. Bicycles have exactly two wheels.
3. A human who legally controls a car holds a driving license and is an adult (this is a difficult one!).
4. A vehicle is controlled by exactly one human.
5. A thing's parts' parts are that thing's parts.
6. A car with a broken part is broken.
7. Bob controls a car with a wheel that has a broken axle.