Exercise 10.1. Show that $\text{MajSat}$ is in PP.

$\text{MajSat} = \{ \varphi \mid \varphi \text{ is some propositional logic formula that is satisfied by more than half of its assignments} \}$

Exercise 10.2. Show $\text{BPP} = \text{coBPP}$.

* Exercise 10.3. Show $\text{BPP}^{\text{BPP}} = \text{BPP}$.

Exercise 10.4. Find the error in the following proof that shows $\text{PP} = \text{BPP}$: Let $L \in \text{PP}$. Then there exists a poly-time bounded PTM accepting $L$ with error probability smaller than $\frac{1}{2}$. Using error amplification, we can make this error arbitrarily small, and in particular smaller than $\frac{1}{3}$. Hence, $L \in \text{BPP}$.

Exercise 10.5. Let $M$ be a polynomial-time probabilistic Turing machine. We say that $M$ has error probability smaller than $\frac{1}{3}$ if and only if

$$\Pr[M \text{ accepts } w] < \frac{1}{3} \quad \text{or} \quad \Pr[M \text{ accepts } w] \geq \frac{2}{3}$$

for all inputs $w$. Show that deciding whether a polynomial-time probabilistic TM has error probability smaller than $\frac{1}{3}$ is undecidable.